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Statistical estimation of probable maximum precipitation

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Abstract. Civil engineers design infrastructures exposed to hydrometeorological hazards, such as hydroelectric dams, using the estimation of probable maximum precipitation (PMP). The World Meteorological Organization (WMO) defines PMP as *the maximum amount of water that can physically accumulate over a given time period and region, depending on the season and without considering long-term climate trends*. Current methods for calculating PMP have many flaws: some variables used are not directly observable and require a series of approximations to be used; uncertainty is not always taken into account and can sometimes be complex to determine; climate change, which exacerbates extreme precipitation events, is difficult to incorporate into the calculations and subjective choices increases estimation variability. The goal of this work is to propose a statistical and objective method for estimating PMP that meets the WMO definition and allows for uncertainty estimation and climate change incorporation. This novel approach leverages the Pearson Type I distribution, a generalization of the Beta distribution over an arbitrary interval. The proposed method is applied to estimate the PMP at two meteorological stations in Québec, Canada.

1 Introduction

1.1 Context

Dams play a crucial role in regulating streamflow and generating hydroelectricity, providing essential water management and renewable energy resources. Over-sizing these infrastructures during construction or renovation can lead to unnecessary costs. Conversely, under-sizing them can pose risks to dam safety, the environment, and surrounding populations, and may result in excessive costs. In the province of Québec (Canada) alone, there are over 6,000 dams over one meter in height spread across the region (CEHQ, 2023). According to regulations derived from the Dam Safety Act, civil engineers must design these structures using an estimation of the local extreme flood conditions. Depending on the risk in case of breakage, various flood estimations are used, such as the millennial, decamillennial, or probable maximum flood (PMF). The latter is the greatest theoretically possible flood on a specific watershed and is computed based, among other factors, on the probable maximum precipitation (PMP). According to the World Meteorological Organization (WMO, 2009), the PMP corresponds to the maximum precipitation accumulation over a fixed duration in a given region.

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1.2 moisture maximization approach for PMP estimation

In its 2009 manual, the WMO details several PMP estimation approaches, with hydrometeorological methods combining algorithms of storm selection, transposition, and maximization being the most popular in Canada. Regarding storm selection, some authors use all rain events where the precipitation height exceeds a given threshold (Beauchamp et al., 2013), while others utilize all observed precipitation data over a given period (Ben Alaya et al., 2018). This selection process is usually carried out by meteorologists and depends on physical factors (CEHQ and SNC-Lavalin, 2004; DTN and MGS Engineering, 2020; Environmental Water Resources Group Ltd., 2020). Since the number of selected storms is small and varies from one calculation to another and among different meteorologists, this selection process introduces significant variability in the estimation of the PMP.

To increase the number of storms used in PMP estimation, a common practice is to include storms from neighboring areas that are likely to also affect the region of interest. Over the past decades, meteorologists have developed various techniques considering the orography and other features of the areas to realistically transpose storms (WMO, 2009). This storm transposition can be incorporated into the storm selection process of PMP estimation methods. While it increases the sample size for PMP estimation, it also introduces additional sources of variability.

The moisture maximization approach estimates the PMP using the relationship between the amount of precipitation and the humidity of the air. Let Y_i be the precipitation of storm $1 \leq i \leq n$ among the n selected storms. The PMP estimation is based on moisture maximization (WMO, 2009) as follows:

$$PMP = \max_{i \in \{1,\ldots,n\}} \left\{ Y_i \times \frac{PW_{\max}}{PW_i} \right\}; \quad (1)$$

where PW_i corresponds to the precipitable water of storm i, and $PW_{\max}$ to the maximum precipitable water of any storm in the considered region. The quantity $Y_i \times \frac{PW_{\max}}{PW_i}$ is often referred to as the maximized precipitation of event i if the maximal precipitable water were available at the moment of the storm. The PMP then corresponds to the maximum of the maximized precipitation. The ratio $\frac{PW_{\max}}{PW_i}$ is referred to as the maximization ratio and is sometimes arbitrarily set to a numerical value between 1.5 and 2.5 to avoid the overestimation of the PMP (Schreiner and Riedel, 1978; Hansen, 1988; WMO, 2009; Beauchamp et al., 2013). The use of this threshold is subjective and lacks physical or mathematical justification (Rouhani and Leconte, 2016).

The moisture maximization expressed in Eq. (1) can also be rewritten as follows, given a slightly different interpretation:

$$PMP = \max_{i \in \{1,\ldots,n\}} \left\{ \frac{P_i}{PW_i} \times PW_{\max} \right\}. \quad (2)$$

In this last expression, the ratio $\frac{P_i}{PW_i}$ corresponds to the ratio of precipitation to precipitable water and is referred to as the precipitation efficiency of storm i. The PMP occurs when the maximum precipitation efficiency coincides with the maximum precipitable water. Ben Alaya et al. (2018) utilize this definition to model the dependence between extreme values of precipitation efficiency and precipitable water. They demonstrate that the comonotonicity imposed by Eq. 2 leads to overestimation of the PMP in North America.

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In practice, the precipitable water PW_i at the moment of storm i and the maximum amount of precipitation for the considered region, $PW_{\max}$, are unknown and must be estimated for using the moisture maximization approach. The amount of precipitable water can be estimated using the specific humidity of the air column above the area (WMO, 2009). However, for the majority of meteorological stations, specific humidity is neither observed nor recorded. The recommended estimation of precipitable water by the WMO (2009) uses the dew point, which is usually recorded, and requires pseudo-adiabatic conditions (United States Weather Bureau, 1960; Miller, 1963). Viswanadham (1981) observed that the relation between surface dew point and precipitable water is greater when the latitude is over 25° than in lower latitude zones. A study conducted by Chen and Bradley (2006) in the Chicago region indicates that the pseudo-adiabatic conditions hypothesis could lead to overestimation of precipitable water, and Rouhani and Leconte (2020) noted that PMP estimates vary greatly depending on how the precipitable water was approximated. The uncertainty of these estimations is often neglected in PMP estimation. It is not uncommon for the uncertainty of PW_i to lead to a precipitation efficiency larger than 1, which is physically impossible.

1.3 Empirical approach for PMP estimation

The Hershfield method (Hershfield, 1961a, b, 1965) is an alternative to moisture maximization for PMP estimation. The method relies on a series of m precipitation annual maxima. The PMP estimates is as follows:

$$PMP = \bar{x} + Ks \tag{3}$$

where $\bar{x}$ and s correspond respectively to the mean and the standard deviation of the series of annual maxima and K corresponds to the frequency factor for estimating PMP at that location. Hershfield (1961a) proposed a method to estimate this factor. This approach is widely employed for its simplicity and ease of use but can only estimate PMP over smaller watersheds (WMO, 2009). It also has the advantage of not requiring additional hydrometeorological data such as specific humidity or dew point. Only precipitation series from which annual maxima are extracted are required.

This method is often classified as a statistical technique, but in this paper, it is considered empirical, as the link between the PMP and the empirical moments is empirical. It should be noted that the PMP durations considered by Hershfield (1965) and available in WMO (2009) are all less than or equal to 24 hours, which is inadequate for calculating longer-duration PMP. The relevance of the procedure can also be questioned : Eq. 3 simply defines PMP as an extreme quantile of the distribution of annual maximum precipitation. Moreover, Koutsoyiannis (1999) demonstrates that the values estimated by the Hershfield method correspond on average to return periods of 60,000 years from a fitted Generalized Extreme Value (GEV) distribution of annual maximum rainfall series.

1.4 PMP estimation using simulated data

The methods of estimation of the PMP presented by the WMO (2009) only consider precipitation data observed at hydrometeorological stations. Methodologies adapted for regional simulations have been developed for regions in Canada (Beauchamp et al., 2013; Rousseau et al., 2014; Clavet-Gaumont et al., 2017; Rouhani and Leconte, 2018, 2020), North America (Kunkel et al., 2013; Ben Alaya et al., 2018, 2020a, b) and other parts of the world (Sarkar and Maity, 2020; Visser et al., 2022).

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The use of these climate simulations not only allows for the consideration of a greater number of extreme rainfall events but also enables the estimation of futures PMP. Indeed, the WMO (2009) defines the PMP as stationary values and CC isn't taken into account in the calculations. However, it is widely acknowledged that CC has a direct impact on extreme precipitation events and should therefore be considered in their estimation. Using projected climate simulations, Kunkel et al. (2013) demonstrate a global increase of water vapor concentration in the atmosphere, without a sufficient evolution in values of upward vertical motion or horizontal wind speed, factors that could counterbalance the rise in air humidity. This increase implies larger future values of PW_{max}, and consequently, an increase in PMP. Several papers conclude that PMP will generally increase in future climate (Beauchamp et al., 2013; Rousseau et al., 2014; Clavet-Gaumont et al., 2017; Rouhani and Leconte, 2018, 2020; Ben Alaya et al., 2018, 2020a, b; Sarkar and Maity, 2020; Visser et al., 2022).

1.5 Objectives of the paper

The PMP estimations using the approaches described in the previous paragraphs are very sensitive to arbitrary choices such as the storm selection procedure, fixed maximization ratio, and frequency factor specification, and are provided without uncertainty. The objective of the present paper is to develop a formal statistical model for PMP estimation based on actual recorded precipitation from which (1) uncertainty is readily quantifiable and (2) subjective choices are removed to increase the reproducibility of the estimation. The remainder of the paper is organized as follows: Section 2 describes the data to which the proposed method is applied and the current PMP estimations at these locations. Section 3 presents the proposed statistical model for estimating the PMP, and Section 4 provides a simulation study assessing the model's performance in PMP estimation. The PMP estimations on the real dataset are presented and discussed in Section 5. Finally, the proposed methodology is discussed and compared with other approaches in Section 6, and the conclusion is provided in Section 7. The data and code for reproducing all figures and results are available at the following public repository: https://github.com/JuliaExtremes/PMP.jl.

2 Data

2.1 Observations

The proposed method for estimating the PMP is demonstrated using data from two meteorological stations in Québec (Canada) located 26 km apart: the Montreal Pierre-Elliott-Trudeau International Airport and St-Hubert Airport stations. The data are available from the Environment and Climate Change Canada (ECCC) website. Daily precipitation in mm and dew point in Celsius were extracted between May 1 and October 31 for each year to focus on liquid rainfall. Figure 1 shows the histogram of non-zero daily rainfall for each station, and Table 1 compiles several precipitation statistics for both stations.

2.2 PMP estimates

As a point of comparison, the PMP estimates for both stations are calculated using the usual approaches: moisture maximization and the Hershfield method. For the moisture maximization approach, daily precipitation amounts that exceeded the 90th

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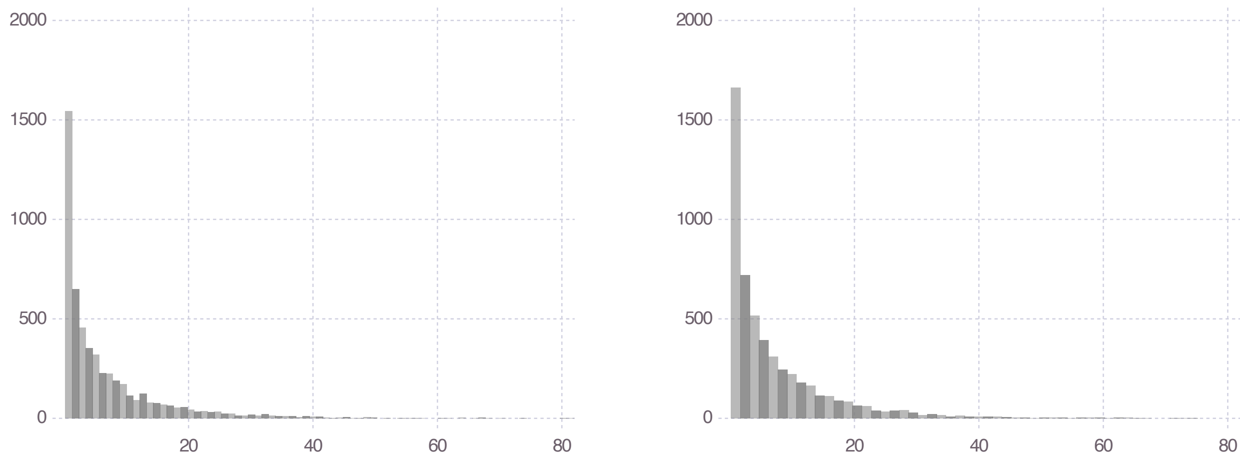

(a) Montréal

(b) St-Hubert

Figure 1. Histogram of the non-zero summer precipitation in mm for (a) Montréal (QC) and (b) St-Hubert (QC).

	Montréal	St-Hubert
Period	1953–2024	1949–2024
Number of days with precipitation	5321	5303
Mean of non-zero precipitation	6.9 mm	7.4 mm
Maximum precipitation	81.9 mm	106.5 mm
Mean of precipitation annual maxima	44.9 mm	49.9 mm
Standard deviation of precipitation annual maxima	14.3 mm	18.0 mm

Table 1. Summer (May to October) daily precipitation statistics for the Montréal and St-Hubert stations.

percentile for each year are selected. The precipitable water is estimated using the maximum persistent dew point over twelve hours, as described by WMO (2009). Corresponding PMP estimates for both stations are provided in Table 2. The PMP estimates are 282 mm and 436 mm for Montréal and St-Hubert respectively. Note that it is also possible to estimate the PMP using instead the 100-year return level of the precipitable water instead of the sample maxima (e.g. Ben Alaya et al., 2018), but with our data, the estimated PMP were similar: 284 mm and 427 mm for Montréal and St-Hubert respectively.

For the Hershfield approach, the frequency factor of $K = 15$ is employed as proposed by Hershfield (1961a). The adjustment based on the number of data points, as suggested by WMO (2009), is not considered. Hershfield's PMP estimates are 261 mm for Montréal and 322 mm for St-Hubert, and are also provided in Table 2.

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Approach	Montréal	St-Hubert
Moisture maximization	282 mm	436 mm
Hershfield methods using $K = 15$	261 mm	322 mm

Table 2. Estimated PMP at Montréal and St-Hubert using the standard approaches.

125 ## 3 Methodology

In this section, a statistical model is developed for PMP estimation based on the definition expressed in Eq. (1). Statistical inference methods for this proposed model are also described.

3.1 Statistical model

From Eq. (1), let $\tilde{Y}_i$ denotes the maximized daily precipitation of day i:

130 $$\tilde{Y}_i = \frac{Y_i}{EP_i} \times EP_{\max}. \tag{4}$$

Factoring for the actual precipitation of day i gives the following expression:

$$Y_i = \frac{EP_i}{EP_{\max}} \times \tilde{Y}_i. \tag{5}$$

Since every maximized precipitation is less than or equal to the PMP, as the PMP corresponds to the maximum of these maximized precipitations, it is possible to formulate the actual daily precipitation as follows:

135 $$Y_i = \frac{EP_i}{EP_{\max}} \times r_i \, PMP, \tag{6}$$

with $0 < r_i \leq 1$ corresponding to the fraction of the maximized precipitation of day i to the PMP.

The ratio $0 < \frac{EP_i}{EP_{\max}} \times r_i \leq 1$ can be modeled with the Beta distribution. Actual precipitation Y_i, which corresponds to this last ratio to the PMP, can be modeled with the Beta distribution rescaled on the interval $(0, PMP)$. The Beta distribution on the interval (a, b) for $a < b$ is referred to as the Pearson Type I distribution (Johnson et al., 1995, Chap. 24).

140 Therefore, we propose to model the actual precipitation of day i with the Pearson Type I distribution as follows:

$$Y_i \sim PearsonType1(0, \psi, \alpha, \beta); \tag{7}$$

where the Beta parameters $\alpha > 0$ and $\beta > 0$ govern the ratio $\frac{EP_i}{EP_{\max}} \times r_i$ and where $\psi > 0$ corresponds to the PMP. The lower bound is set at 0 because only non-zero precipitation events are considered.

With this modelling, the PMP constitutes a distribution parameter to be estimated with the data. Uncertainty can then be provided using the usual statistical methods, as described in the next section.

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3.2 Parameter estimation

3.2.1 Method of moments

The first four central moments, namely the mean m, the variance v, the skewness s, and the kurtosis k, of the Pearson Type I distribution $PearsonTypeI(0,\psi,\alpha,\beta)$ are given by the following expressions (adapted from Johnson et al., 1995, Chap. 24):

$$m = \frac{\alpha\psi}{\alpha+\beta}; \tag{8}$$

$$v = \frac{\alpha\beta\psi^2}{(\alpha+\beta)^2(\alpha+\beta+1)}; \tag{9}$$

$$s = \frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}; \tag{10}$$

$$k = \frac{6(\alpha^3-\alpha^2(2\beta-1)+\beta^2(\beta+1)-2\alpha\beta(\beta+2)}{\alpha\beta(\alpha+\beta+2)(\alpha+\beta+3)}+3. \tag{11}$$

The skewness and kurtosis depend only on the shape parameters α and β, and not on the upper bound ψ. It is possible to invert these equations to factorize for the distribution parameters (adapted from Johnson et al., 1995, Chap. 24):

$$\alpha = -\frac{s(k+3)+\left(\frac{-s(k+3)-\sqrt{s^2(k+3)^2-4(2k-3s^2-6)(4k-3s^2)}}{2(2k-3s^2-6)}\right)(10k-12s^2-18)}{\sqrt{(s(k+3))^2-4(2k-3s^2-6)(4k-3s^2)}}+1; \tag{12}$$

$$\beta = -\frac{s(k+3)-\left(\frac{-s(k+3)+\sqrt{s^2(k+3)^2-4(2k-3s^2-6)(4k-3s^2)}}{2(2k-3s^2-6)}\right)(10k-12s^2-18)}{\sqrt{(s(k+3))^2-4(2k-3s^2-6)(4k-3s^2)}}+1; \tag{13}$$

$$\psi = a+\sqrt{v}\left(\frac{\sqrt{s^2(k+3)^2-4(2k-3s^2-6)(4k-3s^2)}}{(2k-3s^2-6)}\right). \tag{14}$$

To estimate the parameters of the Pearson Type I distribution using the method of moments from a random sample, the empirical moments of the sample–sample mean, sample variance, sample skewness, and sample kurtosis– are plugged into Eqs. (12)–(14) to obtain the parameter estimates. The uncertainty of the parameter estimates can be assessed through non-parametric bootstrap (Efron, 1979).

3.2.2 Maximum likelihood

The density of precipitation Y_i distributed as the $PearsonType1(0,\psi,\alpha,\beta)$ is given as follows (Johnson et al., 1995):

$$f_{(Y_i|\psi,\alpha,\beta)}(y_i) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\,\frac{(y_i)^{\alpha-1}(\psi-y_i)^{\beta-1}}{\psi^{\alpha+\beta-1}}, \quad 0<y_i<\psi. \tag{15}$$

Assuming that the n non-zero daily summer precipitation are independent, the likelihood can be written as follows:

$$f_{(\boldsymbol{Y}|\psi,\alpha,\beta)}(\boldsymbol{y}) = \prod_{i=1}^{n} f_{(Y_i|\psi,\alpha,\beta)}(y_i), \tag{16}$$

where $\boldsymbol{Y}=(Y_i,\ldots,Y_n)$ denotes the vector of the n non-zero precipitations.

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Maximizing the likelihood expressed in Eq. (16) is a non-regular problem (Wang, 2005). When $\beta > 1$, a local maximum exists, and parameter estimates can be obtained. Additionally, parameter uncertainty can be estimated using the Fisher information matrix. However, when $\beta \leq 1$, the local maximum does not exist, and the estimation procedure fails. In this case, several solutions have been proposed, but they are not relevant to the present paper since, for precipitation, the parameter β is strictly greater than 1.

The Pearson Type I distribution is continuous, as expressed in Eq. (15). However, precipitation measurements are discrete. For our data, the precipitation measurement resolution is 0.1 mm, and no precipitation less than 0.2 mm can be measured. This discretization of precipitation measurements has a larger impact on small amounts. Discrepancies appear between the continuous distribution and the discrete measurements, which places mass on points of measurement. One approach to tackle this problem is to censor the likelihood function for small precipitation amounts below a given threshold $u > 0$ (e.g., Naveau et al., 2016) as follows:

$$f^c_{(\boldsymbol{Y}|\psi,\alpha,\beta)}(\boldsymbol{y}) = \prod_{\{i:y_i \leq u\}} I_{\frac{u}{\psi}}(\alpha,\beta) \prod_{\{i:y_i > u\}} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(y_i)^{\alpha-1}(\psi - y_i)^{\beta-1}}{\psi^{\alpha+\beta-1}}, \tag{17}$$

where $I_y(\alpha,\beta)$ denotes the regularized incomplete beta function of parameter (α,β) evaluated at y. Precipitation smaller than u still counts in the likelihood, but their actual values are not considered. Parameter estimates can be obtained by using this censored likelihood.

Another approach would be to set the lower bound of the Pearson Type I distribution to $(0.2 - \epsilon)$ where $\epsilon > 0$. This would maintain some mass at the measurement points but could sufficiently de-emphasize the issue, allowing the continuous likelihood to serve as a good approximation of the discrete measurements. One of these methods could be used if parameter estimation by maximum likelihood is affected by the discretization of precipitation measurements.

3.2.3 Bayesian method

Estimation of the Pearson Type I distribution can also be performed under the Bayesian paradigm. The benefit of using the Bayesian method lies in its ability to describe uncertainty. Unlike the non-parametric bootstrap and the asymptotic Gaussian convergence of maximum likelihood estimates, Bayesian inference directly provides parameter uncertainty based on the data at hand, without relying on asymptotic arguments.

Bayesian methods require a prior distribution for the model parameters. For the Pearson Type I distribution expressed in Eqs. (15) and (16), an improper non-informative prior distribution for the upper bound $\psi > 0$ and the shape parameters $\alpha > 0$ and $\beta > 0$ can be defined as follows:

$$f_{(\psi,\alpha,\beta)}(\psi,\alpha,\beta) \propto \frac{1}{\psi}\frac{1}{\alpha}\frac{1}{\beta} \quad \text{for } \psi > 0,\ \alpha > 0 \text{ and } \beta > 0. \tag{18}$$

The same prior distribution can be used with the censored likelihood expressed in Eq. (17) or with a positive lower bound.

If prior information on the upper bound (the PMP) is available, an improper semi-informative prior can be used as follows:

$$f_{(\psi,\alpha,\beta)}(\psi,\alpha,\beta) \propto f_\psi(\psi)\frac{1}{\alpha}\frac{1}{\beta} \quad \text{for } \alpha > 0 \text{ and } \beta > 0, \tag{19}$$

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where the prior information on ψ is modelled with the proper density f_ψ. If $\beta \leq 1$, the problem is non-regular and the informative prior proposed by Hall and Wang (2005) can be used to solve this issue.

The posterior distribution of the parameters is not available in analytical form for either of the proposed prior distributions. A sample from the posterior distribution can be obtained, for example, using a Gibbs sampling scheme, and inference can be performed using the generated sample.

3.3 Identifiability issues

When $0 < \alpha < 1$ and $\beta \geq 1$, i.e., when the density is convex, non-identifiability issue occurs between β and ψ. Indeed, these parameters can compensate for each other. For example, let $Z_1 \sim \text{PearsonType1}\left(10, \frac{1}{10}, \frac{99}{10}\right)$ and $Z_2 \sim \text{PearsonType1}\left(100, \frac{1}{10}, \frac{999}{10}\right)$ be two random variables with very different upper bounds–10 for Z_1 and 100 for Z_2–but whose moments are similar, as shown in Table 3. Both variables have the same mean and approximately the same variance. Although there are slight differences in skewness and kurtosis, these differences are not large enough to overcome the sampling uncertainty of these higher-order moments estimates.

Variable	mean	variance	skewness	kurtosis
Z_1	0.1	0.09	5.44	40.58
Z_2	0.1	0.10	6.22	57.45

Table 3. Moments for the variables Z_1 and Z_2.

This non-identifiability issue is even more critical for parameter estimation using the model likelihood (both maximum likelihood and Bayesian methods). For example, consider the variable Z_1 again and generate a large random sample of size 5000. The log-likelihood of the model evaluated at the true parameter vector $\left(10, \frac{1}{10}, \frac{99}{10}\right)$ is 35678.3. The log-likelihood evaluated at another parameter vector $\left(100, \frac{1}{10}, \frac{999}{10}\right)$ is 35678.0, which is practically the same, even though the parameters are quite different. The impact of this non-identifiability issue is assessed for parameter estimation with the method of moments, maximum likelihood, and Bayesian method with a simulation study provided in the following section.

4 Simulation study

In this section, a simulation study is conducted to assess the performance of parameter estimation methods for two different distribution behaviors: concave and convex density. The Pearson Type I distribution with parameters $(0, 50, 2, 2)$ is used for the concave distribution, while the Pearson Type I distribution with parameters $(0, 50, \frac{1}{10}, 6)$ is used for the convex distribution. For each of these distributions, 100 random samples of various sizes were generated, and parameter estimation was performed on each sample.

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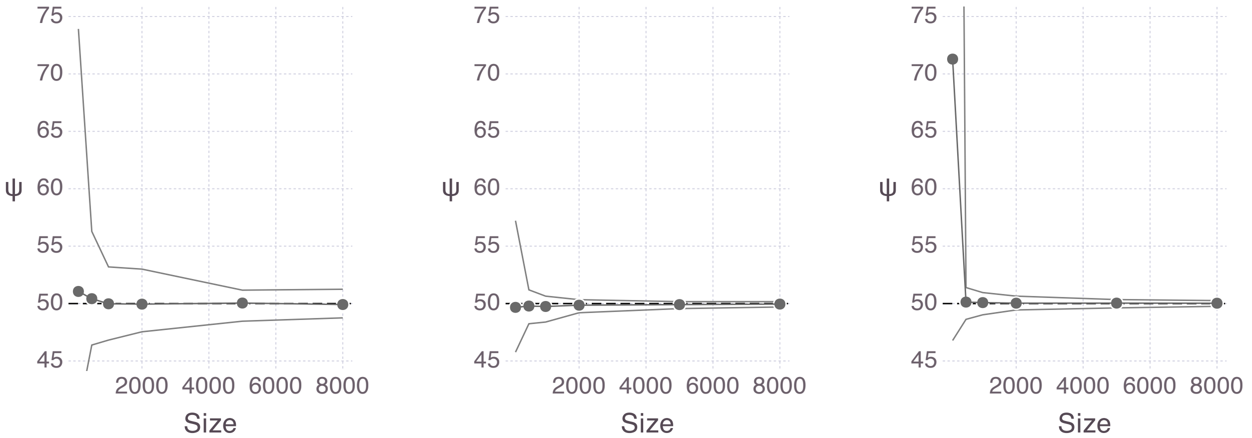

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(a) Method of moments (b) MLE (c) Gibbs sampling

Figure 2. Mean and 95% empirical confidence interval for the ψ estimates of the 100 samples of the $PearsonType1(0,50,2,2)$ distribution obtained with (a) the method of moments, (b) the maximum likelihood and (c) the Bayesian methods using Gibbs sampling.

4.1 Pearson Type I with concave density

For each of the 100 random samples, with sizes ranging from 100 to 8000, parameters were estimated using the method of moments, maximum likelihood, and Bayesian methods. Figure 2 displays the mean of the 100 parameter estimates for the upper bound ψ as a function of the sample size, as well as the 95% empirical confidence interval. For the Bayesian method, both a Gibbs sampling scheme and the No-U-Turn Sampler (NUTS) algorithm were implemented, yielding similar results.

The three estimation procedures perform very well in estimating the upper bound, which is the parameter of interest in this paper. The mean estimate hovers around the true value of 50, and the confidence intervals include the true value. Estimation remains accurate even for relatively small sample sizes of 2000, which corresponds to approximately 20 years of precipitation data. However, the methods based on likelihood yield more precise results than the method of moments.

4.2 Pearson Type I with convex density

Figure 1 shows the mean and the 95% empirical confidence intervals for the samples generated from the Pearson Type I distribution with a convex density. For the method of moments, the estimation of the upper bound is close to the true value of 50. The confidence intervals, wider compared to those associated with the concave density, include the true value. However, estimation variability is very large. It is very sensitive to the sample. For example, for moderate sample sizes around 4000, the upper bound estimate average is around 50, but for some samples, the estimate exceeds 100, which is two times larger than the true value. For other samples, the estimate is smaller than 25, which is half the true value.

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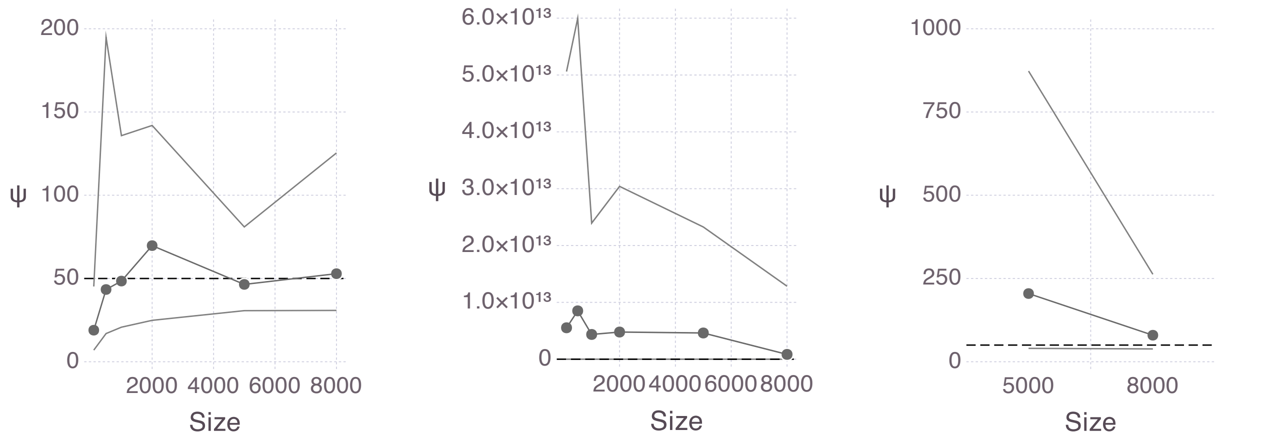

(a) Method of moments (b) Maximum likelihood (c) Bayesian

Figure 3. Mean and 95% empirical confidence interval for the ψ estimates of the 100 samples of the $PearsonType1(0,50,1/10,6)$ distribution obtained with (a) the method of moments, (b) the maximum likelihood and (c) the Bayesian methods using Gibbs sampling.

Upper bound estimates using the maximum likelihood and Bayesian methods are not useful, as shown in Figure 1. The non-identifiability issue arises because the shape parameter β compensates for the larger upper bound. While an informative prior for the upper bound could be introduced to control this issue, it would need to be highly informative. However, this approach was not pursued because using such a restrictive prior defeats the purpose of removing subjectivity in PMP estimation.

The sensitivity to the sample and the non-identifiability issue are resolved with very large sample sizes as shown in Figure 4. The estimates are well stabilize around a sample size of 40,000. For precipitation in Canada, it corresponds to approximately 400 years of data.

4.3 Key findings from the simulation study

For the Pearson Type I distribution with a concave density, parameter estimates are precise with all three estimation methods considered. For a convex density, the non-identifiability issue in the likelihood is too severe to obtain usable upper bound estimates for common sample sizes using the maximum likelihood and Bayesian methods. However, these two methods could be used with very large sample sizes.

Although the method of moments is sensitive to the data, it provides realistic estimates of the upper bound, even for common sample sizes. This method should be favored for parameter estimation of non-concave Pearson Type I distributions.

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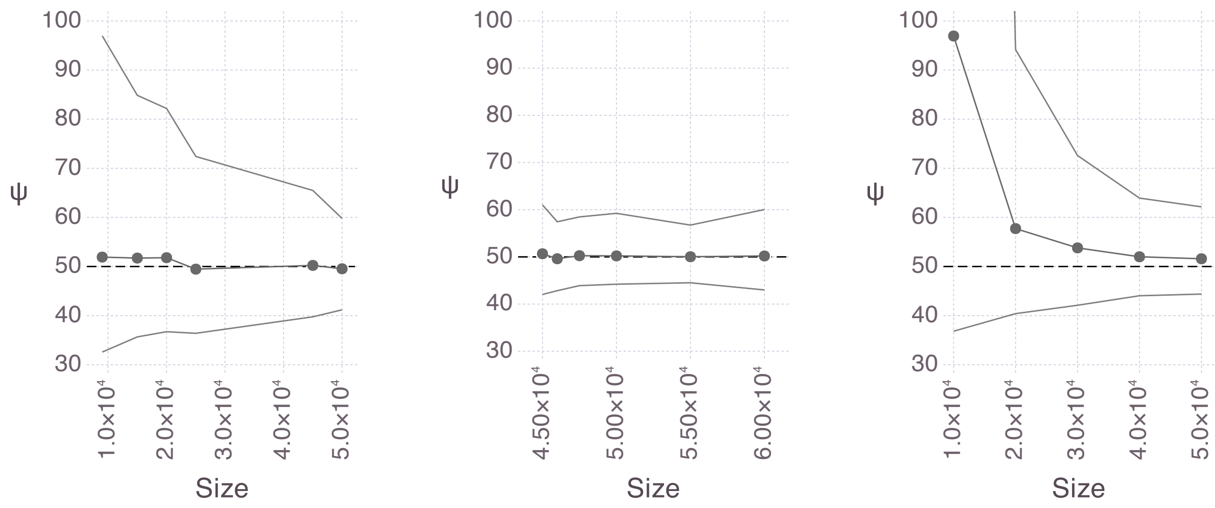

(a) Method of moments (b) Maximum likelihood (c) Bayesian

Figure 4. Mean and 95% empirical confidence interval for the ψ estimates of the 100 very large samples of the $PearsonType1(0,50,1/10,6)$ distribution obtained with (a) the method of moments, (b) the maximum likelihood and (c) the Bayesian methods using Gibbs sampling.

5 Probable maximum precipitation estimation

The Pearson Type I distribution is fitted to the non-zero precipitation data recorded at Montréal and St-Hubert, with the upper bound estimate assumed to represent the PMP. As shown in Figure 1, the non-zero precipitation density appears to be convex, so the model is fitted using the method of moments.

5.1 PMP estimation for Montréal

The Pearson Type I distribution has been fitted to the 5321 non-zero daily summer precipitation with the method of moments. The parameter estimates are as follows:

$$\hat{\psi} = 270.0 \quad (141.6\,, 938.9)$$

$$\hat{\alpha} = 0.4577 \quad (0.3881\,, 0.5349)$$

$$\hat{\beta} = 18.81 \quad (9.014\,, 71.99)$$

The values in parentheses correspond to the 95% confidence intervals estimated by non-parametric bootstrap using 10,000 samples. Figure 5 shows the upper bound estimate for each bootstrap sample. The uncertainties of the second shape parameter β and the upper bound ψ are very large, which is expected given the simulation study results for a convex density. Note that using the maximum likelihood and Bayesian methods does not yield valid estimates due to identifiability issues.

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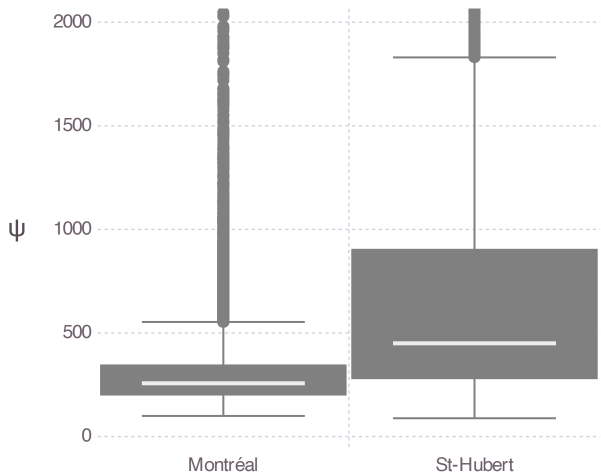

Figure 5. PMP estimates (in mm) obtained by non-parametric bootstrap at Montréal (QC) and St-Hubert (QC).

The PMP estimate given by the fitted Pearson Type I distribution is consistent with the estimate obtained using the moisture maximization method based on Eq. (1). The former estimates the PMP at 270 mm, while the latter estimates it at 284 mm. Unlike the proposed method, the moisture maximization method does not provide uncertainty estimation.

5.2 PMP estimation for St-Hubert

For St-Hubert, the Pearson Type I distribution has been fitted to the 5303 non-zero daily summer precipitation events, and the parameter estimates obtained with the method of moments are as follows:

$$\hat{\psi} = 416.5 \quad (165.0\,,9006)$$

$$\hat{\alpha} = 1.463 \quad (0.4381\,,1.566)$$

$$\hat{\beta} = 34.75 \quad (15.98\,,645.9)$$

where the values in parentheses correspond to the 95% confidence intervals estimated by non-parametric bootstrap using 10,000 samples. Uncertainties in the upper bound and the second shape parameter are exceedingly high, indicating that the non-identifiability issue is more pronounced for these data. Figure 5 shows the upper bound estimates for each bootstrap sample. The PMP estimate given by the fitted Pearson Type I distribution, 417 mm, is consistent with the estimates obtained using the moisture maximization method, 436 mm. As with the Montréal data, maximum likelihood and Bayesian methods do not yield valid parameter estimates.

It should be noted that the estimate for the first shape parameter α with the method of moments is larger than 1, which is inconsistent with the convex form of the distribution. However, the confidence interval includes values smaller than 1.

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6 Discussion

6.1 Pros and cons of the proposed approach

The proposed statistical approach to estimate the PMP translates the usual definition of the PMP into a statistical distribution for the recorded precipitation. The PMP constitutes one of the three parameters, and the remaining two concern the shape of the distribution. By estimating the parameters using standard statistical approaches, such as the moment, maximum likelihood, and Bayesian methods, it is possible to adequately describe the uncertainty, particularly for the PMP parameter. Additionally, the proposed approach uses all the precipitation recorded at the station rather than only a subset from the stations and neighbouring stations. This reduces the subjectivity present in standard approaches.

Another benefit that we did not exploit in this paper concerns non-stationarity. With the Pearson Type I distribution, it is relatively straightforward to model non-stationary parameters to incorporate climate changes and seasonality. For example, precipitation Y_{tsi} of year t, season s, and event i can be modelled as a function of the year and the event i as follows:

$$Y_{tsi} \sim PearsonType1(0, \psi_{ts}, \alpha_{ts}, \beta_{ts});$$

where the year t and the season s could constitute a covariate.

The major drawback of the proposed approach lies in the non-identifiability issue when the data distribution is convex, as is the case for precipitation. Maximum likelihood and Bayesian methods become very unstable, and this issue also affects the method of moments, although to a lesser extent. Regularized maximum likelihood or informative priors could be used to address the non-identifiability, but the constraints that have to be added to control it are quite narrow. We felt that this added too much subjectivity to the proposed approach and that it would lose its benefits compared to the standard PMP estimation approaches.

In the simulation study, it is shown that the non-identifiability issue vanished with very large sample sizes. Figure **??** shows that the maximum likelihood estimation is stable from a sample size of 45,000. If we consider that 100 storms occurs during a year, such sample size would correspond to 450 years of observation. Of course, no meteorological record is that long, but it could be possible to have such a sample size of synthetic storms generated with a storm generator.

Another alternative to increase the sample size would be to include information from nearby stations. This could be achieved within the Bayesian framework described in Section 3.2.3 by replacing the parameter prior distribution with a spatial prior. However, the dependence between stations would need to be modeled in the likelihood, as a single storm can generate precipitation across multiple stations. Accounting for this dependence would decrease the effective sample size of the pooled stations, and we believe that this effective sample size might not reach the level where the estimates are stable. However, we could be wrong.

PMP estimation, whether with the proposed method in this paper or with more standard approaches, is very sensitive to the data due to non-identifiability. For the two nearby meteorological stations considered, i.e., the Montréal Pierre-Elliott-Trudeau International Airport and the St-Hubert Airport stations located 26 km apart, the PMP estimates are very different. However, these two stations do not experience significantly different climates and storms. Furthermore, in several estimates

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provided by engineering consulting firms, storms from even more distant stations are combined to estimate the PMP, a practice known as storm transposition. Moreover, the extreme-value analysis of the precipitation at these two stations, presented later in Section 6.3, yields consistent return level estimates. Therefore, the difference in the PMP for these two stations is more a numerical problem related to the PMP definition than a genuine difference in the PMP.

6.2 Reparametrisation

To reduce the impact of model non-identifiability, we also developed a new parametrization for the Pearson Type I distribution, replacing the shape parameters with a location parameter $\mu > 0$ and a concentration parameter $\nu > 0$:

$$\mu = \frac{\alpha\psi}{\alpha+\beta} \quad \text{and} \quad \nu = \alpha+\beta.$$

We therefore have $\alpha = \frac{\mu\nu}{\psi}$ and $\beta = \frac{\nu}{\psi}(\psi - \mu)$. However, even with this parametrization, non-identifiability remained an issue for maximum likelihood and Bayesian inference.

6.3 Comparison with extreme value analysis

For the purpose of comparison, an extreme value analysis have been performed on the precipitation data of Montréal and St-Hubert. The Peaks-Over-Threshold (POT, Davison and Smith, 1990) extreme-value model has been fitted by maximum likelihood to the Montréal data, with the threshold of 30 mm chosen using the mean residual life plot method as described by Coles (2001, Chap. 4). The estimated parameters of the generalized Pareto distribution modelling the excesses above the threshold are as follows:

$$\hat{\sigma} = 9.95 \quad (7.96\,, 12.45)$$
$$\hat{\xi} = 0.0421 \quad (-0.1288\,, 0.2131)$$

where the values in parentheses correspond to the 95% confidence intervals estimated using the Fisher information matrix. The model fits the data very well, as shown by the return level plot in Figure 6a. Note that the shape parameter estimate is positive, indicating an unbounded heavy-tailed distribution, which is typical for precipitation but inconsistent with the PMP existence assumption. Nevertheless, a short-tailed distribution cannot be excluded, as the shape parameter confidence interval includes negative values. As an indication, the 10,000-year return level estimated with the POT model is 156 mm ($58\ mm\,, 255\ mm$), and the PMP value of 270 mm corresponds to a return period longer than 10 million years.

The POT model has also been fitted to the St-Hubert data. Parameter estimates are as follows:

$$\hat{\sigma} = 13.1594 \quad (10.5917\,, 16.3494)$$
$$\hat{\xi} = 0.0259 \quad (-0.1335\,, 0.1854);$$

and Figure 6b shows the model fit to the data. Again, the model fits the data very well, and the shape estimate is positive, indicating a heavy-tailed distribution. Using the fitted POT model, the 10,000-year return level estimate is 170 mm

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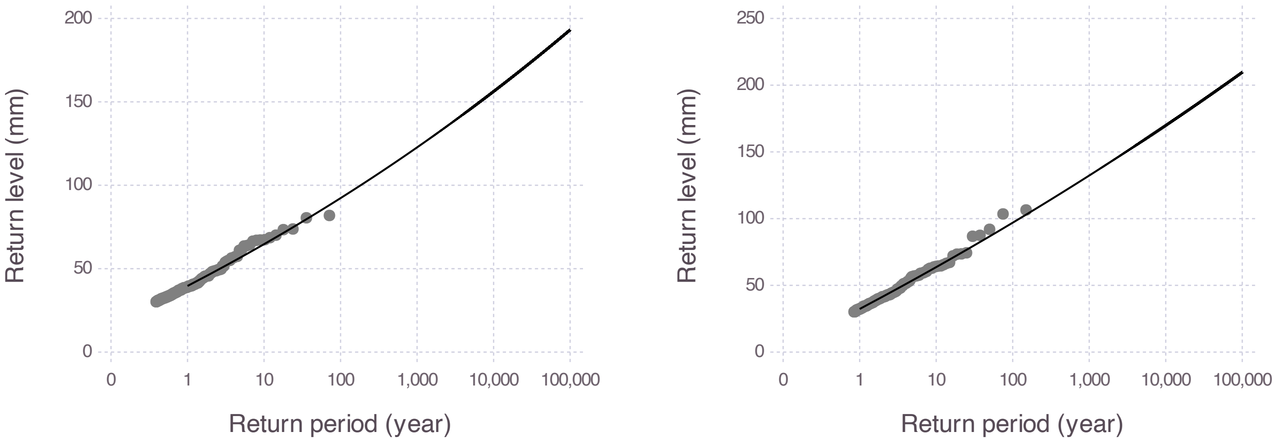

(a) Montréal data

(b) St-Hubert data

Figure 6. Return level plot of the fitted Peaks-Over-Threshold model for (a) the Montréal data and (b) the St-Hubert data.

$(80\ mm\,,\,259\ mm)$, which is consistent with the corresponding estimate of $156\ mm$ for Montréal, located $26\ km$ apart. The PMP estimate of $416\ mm$ corresponds to a return period longer than 1 billion years.

6.4 Recommendations

Although translating the definition of the PMP into a statistical model is interesting, and despite the possibility of including non-stationarity and estimating uncertainty, we do not recommend using the Pearson Type I distribution to estimate the PMP. The non-identifiability makes the model too sensitive to the data, and the PMP estimate becomes too volatile. This problem is also present in the standard moisture maximization method. Therefore, we align with the conclusions of the National Academies of Sciences, Engineering, and Medicine (2024), which recommends an extreme value analysis instead. Additionally, the extreme-value theory allows for a genuine estimation of the uncertainty of extreme values, even when extrapolating to return periods that exceed the range of the data. Furthermore, it is easily generalizable to non-stationary cases, allowing the integration of the effects of climate change.

More generally, the fact that PMP estimates using either moisture maximization, Hershfield's method, or the Pearson Type I method are so sensitive to the data is a critical concern from an engineering standpoint. Specifically, for the Pearson Type I method with a convex density, depending on the data, the PMP estimate can range from half to more than twice the true value. In the former case, using the estimate would result in under-dimensioning the infrastructure, putting the public at risk. In the latter case, using it would result in over-dimensioning the infrastructure, thereby increasing costs and environmental impacts. Although uncertainty estimates are not available with the moisture maximization and Hershfield's methods, the fact that the corresponding PMP estimates for Montréal and St-Hubert were so different is an important indication of the methods'

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Approach	Montréal	St-Hubert
Moisture maximization	282 mm	436 mm
Moisture maximization using PW_{100}	284 mm	427 mm
Hershfield method using $K = 15$	261 mm	322 mm
Pearson Type I	270 mm	417 mm
10,000-year return level (POT)	156 mm	170 mm

Table 4. Estimated PMP at Montréal and St-Hubert using the standard approaches and the Pearson Type I distribution. The second line corresponds the PMP estimation using the 100-year return level of the precipitable water instead of the empirical maximum as proposed by Ben Alaya et al. (2018).

sensitivity. Table 4 compiles all the PMP estimates for Montréal and St-Hubert, along with the 10,000-year return levels estimated with the POT model.

In the case of this article, we have seen that the POT model fits the data from both stations very well and that the estimates of the 10,000-year precipitation were consistent. Moreover, the extreme value analysis indicates an unbounded and heavy-tailed distribution of precipitation, which is consistent with numerous results in the literature (e.g. Papalexiou and Koutsoyiannis, 2013). Therefore, it is better to design infrastructure by setting an appropriate level of risk and evaluating the uncertainty of the estimate.

7 Conclusions

In this study, we developed a new statistical model for estimating the PMP based on its definition. The model involves modelling daily precipitation with the Pearson Type I distribution, where the upper bound corresponds to the PMP. As a proper statistical model, parameter and uncertainty estimations can be derived using well-known statistical methods.

Our analysis demonstrates that while the proposed statistical approach offers potential benefits—such as translating the PMP definition into a statistical model, incorporating non-stationarity, and providing uncertainty estimates—significant drawbacks limit its practical application. The major challenge lies in the non-identifiability issue, which renders the model highly sensitive to data and leads to volatile PMP estimates. This issue persists despite attempts at reparametrization and the use of regularized maximum likelihood or informative priors, which introduce subjectivity that undermines the model's advantages.

Given the inherent challenges and limitations of the Pearson Type I distribution for precipitation modelling, we recommend using extreme value analysis for PMP estimation. This approach aligns with the findings of the National Academies of Sciences, Engineering, and Medicine (2024), which advocate for extreme value analysis due to its robustness and applicability, even in the context of non-stationary conditions brought about by climate change. With our data, the 10,000-year return level estimates of daily precipitation at the two considered locations were consistent, in contrast to the PMP estimates for those two locations. Moreover, the extreme value analysis indicated a heavy-tailed distribution, consistent with existing literature, which invalidates the concept of PMP.

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Future work may involve estimating the PMP of storms instead of daily precipitation. In this paper, we estimated the daily PMP, but precipitation accumulation over several days could also be of interest. However, accumulation over several days would decrease the sample size and exacerbate the non-identifiability issue. Future work may also focus on PMP estimation based on a large sample of synthetic storms provided by a storm generator.

In conclusion, while innovative statistical methods offer promising avenues for PMP estimation, traditional extreme value analysis remains, in our opinion, the most practical and reliable approach for assessing precipitation extremes and guiding infrastructure design.

Code and data availability. The data and code for reproducing the results are provided in the public repository: https://github.com/JuliaExtremes/PMP.jl.

Author contributions. Authors' Contribution statement using CRediT with degree of contribution:

Anne Martin: Formal Analysis (lead), Investigation (lead), Methodology (lead), Software (equal), Validation (equal), Visualization (equal), Writing – Original Draft Preparation (equal).

Élyse Fournier: Conceptualization (equal), Funding Acquisition (equal), Investigation (supporting), Methodology (supporting), Supervision (supporting), Writing – Original Draft Preparation (supporting).

Jonathan Jalbert: Conceptualization (equal), Funding Acquisition (equal), Investigation (supporting), Methodology (supporting), Software (equal), Validation (equal), Visualization (equal), Supervision (lead), Writing – Original Draft Preparation (equal).

For more information, please see the taxonomy website.

Competing interests. The is no competing interest to declare.

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