

1 Introduction

 The advent of Geographic Information Systems (GIS) marks a milestone in the evolution of geography. As a core function of GIS software, spatial statistics provide powerfulmethods and tools that enable researchers and decision-makers to analyze spatial patterns and associations on the Earth's surface comprehensively and accurately. Spatial heterogeneity and positional dependence are two fundamental characteristics to beconsidered in spatial data processing (Goodchild and Haining, 2004). Correspondingly, Zonal Statistics and Focal Statistics are two essential methods of spatial statistical analysis. The former can be achieved through a model that involves partitioning raster data into several zones based on predefined rules or attributes, performing statistical analyses on the rastercells within each zone, and then outputting the results as a mosaic rasterlayer (Singla and Eldawy, 2018; Haag et al., 2020; Winsemius and Braaten, 2024). The latter, also known as neighborhood or local window statistics, takes each raster cell as the center and extends a specified range surrounding the center to form a local window according to the designated window size; it performs statistical analyses on the rastercells within this window and then outputs the results as a mosaic rasterlayer (Mathews and Jensen, 2012; Kassawmar et al., 2019; Zhang et al., 2021). The calculated statistics for both zonal and focal methods are similar, including the mean, maximum, minimum, sum, and so on.

 Currently, the mainstream GIS software platforms including ArcGIS and QGIS provide tool modules such as Focal Statistics and Zonal Statistics, which have promoted the usage of these two methods. From an application perspective, Zonal Statistics primarily address spatial stratified heterogeneity (SSH), which can be detected by dividing the target variable though environmental characteristic classified variables (Wang et al., 2016; Wang and Xu, 2017; Gao et al., 2022). For instance, the actual or potential growth of vegetation may vary significantly due to different environmental conditions such as slope and aspect (Zhang et al., 2018, 2019;

53 Xu et al., 2020). With respect to Focal Statistics, it focuses on spatial position dependence (SPD), which can be addressed or at least weaken by introducing the local windows or geographic weights (Tobler, 1970; Wolter et al., 2009; Wagner et al., 2018). For example, even soils or rocks with the same texture generally exhibit variations in geochemicalelement content due to their different spatial locations; however, these differences diminish with decreasing distance, indicating that these attributes are dependent on spatial position (Krige and Magri, 1982; Trangmar et al., 1986; Zuo, 2014).

 In our real world, SSH and SPD may coexist, with the former exhibiting abrupt changes and the latter exhibiting gradual changes. For example, due to variations in land-sea distribution, solar radiation, and altitude, terrestrial vegetation exhibits strong meridional, latitudinal, and vertical zonal distribution patterns respectively (Qiu et al., 2013; Dong et al., 2019; Eddin and Gall, 2024), which explains the significant SPD in vegetation coverage. Meanwhile, due to the influence of local topography, microclimate, and human activities, the vegetation coverage differences caused by these factors do not entirely manifest as gradual changes. Typical evidence includes phenomena such as vegetation on shady and sunny slopes generally shows SSH (Álvarez-Martínez et al., 2014; Zhang and Zhang, 2022;) and significant differences between urban and rural landscapes (Zhang et al., 2023b). Furthermore, due to differences in formation age, there are significant variations in material across strata, which is a majorreason for the SSH of mineral resources distribution (Zhao Pengda, 2006; Zuo, 2020). Subsequently, under the influence of internal and external geological processes, the distribution of mineralization elements often exhibits SPD characteristics (Cheng, 2006, 2012), and Geostatistics and Kriging methods were developed to explain this phenomenon (Krige, 1951; Goovaerts, 1997; Müller et al., 2022). Therefore, when dealing with problems involving spatial statistics, it is necessary to consider both SSH and SPD simultaneously.

Some scholars have noted this issue and developed certain improved models in their

 respective fields to overcome the challenges posed by solely considering SSH or SPD. Professor Zhu and his group expanded upon traditional spatial interpolation methods, which typically focus solely on spatial dependence, by introducing constraints derived from environmental similarity (Zhu et al., 2019). They further proposed the "Third Law of Geography", which states that the more similar the geographic configurations of locations, the more similar the values (processes) of the target variable at these locations (Zhu et al., 2018; Zhu et al., 2020). Meanwhile, Professor Zhang and his group enhanced traditional vegetation potential assessment models, which typically only consider similar habitat conditions, by incorporating spatial sliding window techniques (Zhang et al., 2019). This development led to a model for assessing vegetation restoration potential based on local windows, simultaneously 88 considering spatial proximity and environmental similarity (Xu et al., 2020; Zhang, 2023a). The most recent attempt at spatial statistical modeling that considers both SSH and SPD is by Lessani and Li (2024), who developed similarity and geographically weighted regression model. This new model integrates distance weights and similarity weights to address the limitations of traditional geographically weighted modeling that only considers spatial dependency.

 These studies focused on specific issues such as spatial interpolation, regression, and extreme values. Although these models effectively address the combination of both SSH and 96 SPD, there is currently a lack of a universal spatial statistics tool similar to Focal Statistics and Zonal Statistics. This study aims to develop a spatial statistical model, termed the Focal-Zonal Mixed Statistics, within the framework of GIS spatial statistics.The newly developed toolbox, FZStats v1.0, integrates traditional Focal Statistics and Zonal Statistics, as well as Focal-Zonal Mixed Statistics. In terms of algorithm design, we employ multiprocessing and batch processing techniques, which promise to enhance operational efficiency and user experience. We believe that the FZStats v1.0 toolbox, especially the newly

- proposed Focal-Zonal Mixed Statistics, has the potential to offer methods and tools to better
- understand and address SSH and SPD issues.

2 Models

2.1 Focal Statistics model

- The modeling of Focal Statistics involves three functional methods: (1) defining the
- neighborhood window, (2) identifying the cells located within the neighborhood, and (3)
- calculating the neighborhood statistics.
- **2.1.1 Defining the neighborhood window**
- Defining the neighborhood window is a crucial prerequisite for Focal Statistics. There are two parameters to define the neighborhood window: its shape and size. These can be adjusted based on the spatial characteristics of the data and the objectives of the research. Commonly used shapes include circular, square, and rectangular, while the size is typically specified in terms of the number of cells.

116 Formally, let NW denote the neighborhood window, the following expression can be obtained.

$$
118 \quad NW = f(Shape, Size) \tag{1}
$$

119 where $f(.)$ represents the function used to characterize the neighborhood window, Shape 120 refers to the geometric configuration of the window, while *Size* specifies its extent.

2.1.2 Identifying cells within the neighborhood

 Once the neighborhood window is determined, the spatial sliding window technique can be used to identify the cells located within the neighborhoods defined by the neighborhood window centered around given cells (Hyndman and Fan, 1996). For each current location $Cell(i, j)$, the neighborhood can be expressed as:

$$
126 \quad Nbh(i, j) = nbh(Cell(i, j), NW) \tag{2}
$$

127 where i and j denote the row and column number of current cell at location (i, j) , respectively;

- 128 *nbh*(.) is the function for determining the neighborhood of $Cell(i, j)$, and NW represents the
- 129 neighborhood window.
- 130 Then cells located within $Nbh(i, j)$ form a cell set, which can be described as follows:

131
$$
CS_F(i,j) = \{Cell(i',j') \in \mathbf{R}_v \mid is_in_nbh(Cell(i',j'), Nbh(i,j)) == TRUE\}
$$
 (3)

132 where $is_in_nbh(.)$ is the indicator function used to identify whether $Cell(i, j')$ is located 133 within the neighborhood $Nbh(i, j)$; i and j are for the row and column number of the 134 input value raster \mathbf{R}_v , respectively.

135 In Eq. (3), the detailed form of $is_in_nnbh(.)$ depends on the shape of the neighborhood 136 window. For example, when the window is circular, $is_in_nbh(.)$ can be expressed as:

137
$$
\sqrt{(i'-i)^2 + (j'-j)^2} \le d
$$
 (4)

138 where d is the radius of the circular window, i.e. window size, and *i* and *j*, and *i* and *j* are 139 as explained above.

140 **2.1.3 Calculating the focal statistics**

141 Suppose that $ST_F(Type, Set)$ denotes the statistical function of Focal Statistics, and Type 142 and *Set* are for the statistical parameter and the cell set to be processed. At the location of 143 Cell(i, j) and under the Focal Statistics model, Set can be specified as $CS_F(i, j)$. Then the 144 output of the Focal Statistics for $Cell(i, j)$ can be expressed as:

$$
145 \t OF(i, j) = STF(Type, CSF(i, j))
$$
\n(5)

146 Expressed in terms of raster layer operations, Eq. (5) can be further formulated as:

$$
147 \t R_{F,out} = Focal_Statistics(R_v, NW, Type) \t (6)
$$

148 where R_v and $R_{F \text{ out}}$ represent the input value raster and the output raster for Focal 149 Statistics, respectively, while NW and $Type$ denote the functions for neighborhood window 150 and statistical type in that order.

151 **2.2 Zonal Statistics model**

152 Unlike Focal Statistics, which require only a value raster as input, Zonal Statistics require two

- 153 input raster layers: one as the value raster and the other as the zone raster. The zone raster
- 154 defines the shape and distribution of the zones, and each cell can only belong to a single zone.
- 155 Zonal Statistics calculates the statistics for each zone based on the corresponding cells from
- 156 the value raster, and the calculated statistic is assigned as the output value for all cells within
- 157 the zone. Finally, the output values of different zones are assembled into the output raster.
- 158 Zonal Statistics modeling involves two functional methods, which are for identifying the
- 159 cells in the value raster by zone and calculating zonal statistics respectively.

160 **2.2.1 Identifying cells in the value raster falling into each zone**

- 161 In Zonal Statistics, spatial overlay analysis can be used to find the zone code for each cell in
- 162 the value raster (Hyndman and Fan, 1996):

$$
163 \t Z_k(i,j') = \text{Zone}(\text{Cell}(i,j')) \tag{7}
$$

164 where $Z_k(i, j')$ represents the zone code at location (i, j') , and $Zone(.)$ is the function that 165 returns the zone code for the value raster cell at location (i, j) .

166 For a given zone Z_k , the corresponding cells in the value raster form a cell set $CS_Z(Z_k)$, 167 which can be expressed as:

168
$$
CS_Z(Z_k) = \{ \text{Cell}(i,j') \in \mathbf{R}_v \mid \text{Zone}\left(\text{Cell}(i,j')\right) == Z_k \}
$$
 (8)

169 **2.2.2 Calculating the zonal statistics**

170 The calculation of statistics for a given zone Z_k can be represented as:

$$
171 \tOZ(Zk) = STZ(Type, CSZ(Zk))
$$
\n(9)

- 172 It is important to note that the calculated statistics are assigned to all cells within each 173 zone, and the statistics for allzones are ultimately mosaicked into the output raster.
- 174 Using R_v and R_z to denote the input layers of value raster and zone raster,
- 175 respectively. Zonal Statistics can be expressed as:

$$
176 \t R_{Z,out} = Zonal_Statistics(R_v, R_z, Type) \t(10)
$$

177 where $\mathbf{R}_{Z,out}$ represents the output raster, and $Type$ is for the statistic type.

178 **2.3 Focal-Zonal Mixed Statistics**

- 179 Similar to Zonal Statistics, Focal-Zonal Mixed Statistics also require two input raster layers,
- 180 and the specific modeling process involves the following two functional methods.

181 **2.3.1 Identifying cells within the neighborhood that belong to the same zone**

182 Actually the determination of the target cell set in Focal-Zonal Mixed Statistics combines 183 both the spatial proximity condition from Focal Statistics, and the environmental 184 characteristic similarity condition from Zonal Statistics. For $Cell(i, j)$ at the current location, 185 if its neighborhood is $Nbh(i, j)$ and its zone code is $Z_k(i, j)$, then its cell set consists of all 186 cells within the neighborhood that belong to the same zone as the cell in Focal-Zonal Mixed 187 Statistics. Mathematically, this can be expressed as:

188
$$
CS_{F-Z}(i,j) = \{Cell(i',j') \in \mathbf{R}_v \mid \begin{cases} \text{is_in_nbh}(\text{Cell}(i',j'), \text{Nbh}(i,j)) == \text{TRUE} \\ \text{Zone}(\text{Cell}(i',j')) == Z_k(i,j) \end{cases} \}
$$
(11)

189 **2.3.2 Calculating the focal-zonal mixed statistics**

190 Still using *Type* to represent the statistical type, the output result of Focal-Zonal Mixed

191 Statistics for the current $Cell(i, j)$ can be expressed as:

192
$$
0_{F-Z}(i,j) = ST_{F-Z}(Type, CS_{F-Z}(i,j))
$$
\n(12)

193 In the form of raster layer operations, Eq. (12) can be further expressed as:

194
$$
R_{FZ_{out}} = Focal_Zonal_Statistics(R_v, R_z, NW, Type)
$$
 (13)

- 195 where R_v , R_z , and $R_{FZ,out}$ represent the value raster, zone raster, and output raster for
- 196 Focal-Zonal Mixed Statistics, respectively; NW is the neighborhood window, and $Type$ is
- 197 for statistical parameter.
- 198 **3 Module design**

199 **3.1 Modeling process for Focal-Zonal Mixed Statistics**

- 200 The flowchart for the newly proposed Focal-Zonal Mixed Statistics is presented in Fig. 1, and
- 201 the detailed modeling process is described as follows.

Figure 1. Flowchart for the modeling of Focal-Zonal Mixed Statistics

(1) Preparation of the value raster and the environmental factor rasters

205 This initial step involves collecting and preprocessing the spatial data required for the analysis. The value rastertypically represents the primary variable of interest, i.e., the target layer, such as temperature, pollution levels, or vegetation indices. Environmental factor rasters include various influencing factors, such as elevation, slope, land cover, and other relevant geographical features that may contribute to the heterogeneous distribution of the target layer. Preprocessing methods may include resampling, reprojecting, and normalizing the data to ensure consistency and compatibility among the rasterlayers, so that they share the same spatial extent, resolution, and reference system.

(2) Construction of unique-value environmental characteristic zonal raster (UV-ECZR)

 This process can be achieved using the "Reclassify" tool in ArcGIS totransform continuous or categorical environmental factor rasters into discrete classes based on predefined criteria. Subsequently, the UV-ECZR is generated through spatial overlay analysis and unique-value encoding. Cells in the UV-ECZR that share the same unique-value

- environmental characteristic code (UV-ECC) form a similar environmental unit (SEU). A
- detailed implementation of this process is described in the following Sect. 3.2.1.
- (3) Determination of neighborhood window and statistical parameters
- This process involves defining the neighborhood window and specifying the statistical
- parameters for Focal-Zonal Mixed Statistics.
- (4) Preparation of output raster
- This step involves creating an output raster with the same spatial extent, resolution, and reference system as the input rasters. This output raster will store the results of the
- Focal-Zonal Mixed Statistics calculations.
- 227 (5) Calculation of the statistics
- In this step, the moving window technique is applied to locate each current cell and its local window. For each current cell, identify the neighborhood cells based on the defined neighborhood window parameters (refer to Sect. 2.1.1). Within this neighborhood, isolate the cells within the same SEU as the current cell. Subsequently, calculate the specified statistic for these cells in the value raster that correspond to those isolated cells.
- 233 (6) Save of output raster
- Write the statistical result to each corresponding cell in the output raster one at a time,
- 235 and save the raster file after all cells have been processed.
- The core algorithm involved in the above steps is described in the following section.
- **3.2 Core algorithm design for Focal-Zonal Mixed Statistics**
- **3.2.1 Algorithm design for the UV-ECZR construction**
- 239 Assume that there are p continuous environmental variables, i.e., E_1, E_2, \ldots, E_p , with their
- 240 corresponding reclassified variables being CE_1, CE_2, \ldots, CE_p . The number of categories and
- 241 the digit lengths of these categories are denoted as S_1, S_2, \ldots, S_p and D_1, D_2, \ldots, D_p ,
- respectively. The method for calculating the digit lengths of the categories is as follows:

(1) Mask matrix for elliptical window

267 An elliptical window is defined by three key parameters: the length of majoraxis, the 268 ratio of the minor axis to the major axis, and the deflection angle of major axis. Let (x_0, y_0) 269 represent the center of the ellipse, i.e., the current location, α denotes the semi-major axis 270 length, r be the minor-to-major axis ratio, and θ be the deflection angle. Then the elliptical 271 window can be mathematically expressed as:

272 Ellipse
$$
((x_0, y_0), a, r, \theta) = \frac{[(x-x_0)\cos\theta + (y-y_0)\sin\theta]^2}{a^2} + \frac{[-(x-x_0)\sin\theta + (y-y_0)\cos\theta]^2}{(ra)^2}
$$
 (17)

273 Based on Eq. (15), the bounding box of the elliptical window can be represented as 274 BBo $x_{ellipse}(minX, maxX, minY, maxY)$, where $minX, maxX, minY, maxY$ are as follows:

$$
275 \quad \begin{cases} \min X, \max X = x_0 \pm \sqrt{\frac{4CF}{B^2 - 4AC}} \\ \min Y, \max Y = y_0 \pm \sqrt{\frac{4AF}{B^2 - 4AC}} \end{cases} \tag{18}
$$

276 here,

$$
\begin{cases}\nA = a^2 (\sin^2 \theta + r^2 \cos^2 \theta) \\
B = 2a^2 (r^2 - 1) \sin \theta \cos \theta \\
C = a^2 (\cos^2 \theta + r^2 \sin^2 \theta) \\
F = -\frac{1}{2} (Dx_0 + Ey_0) - r^2 a^4\n\end{cases}
$$
\n(19)

278 The bounding box $BBo x_{ellipse}$ provides a simplified and direct spatial reference for 279 constructing a Boolean mask matrix for the elliptical window, i.e., $Matrix_{Ellipse\ mask}$, where 280 cells inside and outside the $BBox_{ellipse}$ are assigned values of "True" and "False", 281 respectively. In Focal Statistics, this mask is used directly to define the area of interest for 282 statistics, see Fig. 2a.

283

Figure 2. Heatmaps for the Boolean mask matrix: (a) the elliptical window of Focal Statistics, (b) the similar environmental unit (SEU) of Zonal Statistics, and (c) the elliptical window similar environmental unit (EW-SEU) of Focal-Zonal Mixed Statistics

(2) Mask matrix for similar environment in the bounding box

289 SEU is the basic object of Zonal Statistics. In Focal-Zonal Mixed Statistics, for the current cell, the elliptical window similar environmental unit (EW-SEU) is established according to the environmental characteristic code within the initial neighborhood window 292 defined by the bounding box. Using $Matrix_{Similarity_mask}$ to represent this unit, cells with

- 293 the same environmental characteristic code as the current cell are assigned a value of "True", 294 while others are assigned a value of "False", as shown in Fig. 2b. 295 (3) Mask matrix for similar environment in the elliptical window 296 The matrices of steps (1) and (2) shares the same dimensions, and thus the similar 297 environment mask matrix for the current cell in the elliptical window can be constructed using 298 a logical "AND" operation between these two matrices, as expressed in the following 299 equation: 300 $Matrix_{E S \; mask} = Matrix_{Similarity \; mask} \wedge Matrix_{Ellipse \; mask}$ (20) 301 where Λ denotes the logical "AND" operator. $Matrix_{ES\;mask}$ serves as the basis for 302 determining the valid range forFocal-Zonal Mixed Statistics, as illustrated in Fig. 2c. 303 **3.2.3 Algorithm design for the statistics calculation** 304 The algorithm for the statistics calculation is designed as follows: 305 (1) Determination of valid statistical cells in the value raster 306 Using *Matrix_{Value}* to represent the cell array from the value raster within the bounding 308 *Matrix*_{Value}, the final valid statistical value matrix *Matrix*_{Valid} is obtained: 309 $Matrix_{Valid} = Matrix_{ES \; mask} \; \otimes Matrix_{Value}$ (21) 310 where⊗denotes bitwise multiplication. This operation collects cells from the value raster that 311 are located within the neighborhood and share the same UV-ECC as the current cell, while 312 masking out other cells that could interfere with the statistical results. In $Matrix_{Valid}$, the 313 masked cells can be represented with "NaN". 314 (2) Design of the calculation function for the statistics 315 Taking $Matrix_{Valid}$ as the final input, the calculation functions for Focal-Zonal Mixed 316 Statistics can bedesigned based on scientific computing tools such as NumPy. This library
	-
-

307 box defined above, then by performing a bitwise multiplication of $Matrix_{E S \; mask}$ with

317 provides a range of statistical methods, including minimum, maximum, mean, standard

50th, 75th, or 98th percentiles.

(4) Optimization settings

 In this section, users can fine-tune various parameters to optimize the calculation performance. Key settings include:

 Chunk processing: Users can divide the input raster into smaller chunks, which can enhance performance by reducing the memory load and making it easier to handle large datasets.

 Parallel processing: Users can configure the number of processors used for parallel processing to reduce computation time. On computers with higher configurations, increasing the number of processors allows for the utilization of more cores, enabling simultaneous task execution and significantly reducing processing times.

 Threshold setting: Users can specify a minimum sample threshold for statistical calculations, which defines the minimum number of cells required for performing the 354 statistical measure. This threshold ensures that the statistical computations are based on a sufficient sample size, thereby enhancing the reliability and robustness of the results.

 Additionally, to further improve multitasking efficiency and achieve a certain degree of automation, a batch processing feature is provided in the toolbox. Users can define parameters in an INI-format configuration file (config.ini), which simplifies the process by eliminating repetitive configurations. This feature allows users to set up and execute multiple tasks in a single operation, supports parameter reuse, and provides a means fortracking errors.

Figure 3. User interface design of FZStats v1.0

4 Experimental study

4.1 Background of thecase

 Geothermal, like coal, oil, and natural gas, is a valuable energy mineral resource, and its development and utilization play a significant role in alleviating energy supply pressure and improving the global environment (Huang and Liu, 2010; Goldstein et al., 2011). The most important indicator for geothermal resource exploration is thermal anomalies (Romaguera et al., 2018; Gemitzi et al., 2021). In recent years, with the rapid development of remote sensing, Land Surface Temperature (LST) derived from thermal infrared bands has become a key

 method for identifying geothermal anomalies. However, LST is influenced by various factors, including not only geothermal activity but also slope, aspect,and surface vegetation cover, among other environmental factors (Tran et al., 2017; Duveiller et al., 2018; Zhao and Duan, 2020).

 To effectively extract LST anomalies caused by geothermal activity, it is necessary to suppress the influence of surface environmental variables. Within the analytical framework of the Focal-Zonal Mixed Statistics developed in this study, terrain features are incorporated into environmental zoning, and the spatial sliding window technique is employed to mitigate environmental interference and enhance the abnormal information from geothermal activity.

4.2 Data preprocessing

4.2.1 Spatial distribution of LST

 Landsat 8 images (Orbit Number: 116031) observed on September 16, 2013 covering the study area, i.e., Changbai Mountain region, were used for LST mapping and geothermal exploration in this study. After preprocessing operations such as radiometric calibration and 385 atmospheric correction, the Universal Single-Channel Algorithm (Jiménez-Muñoz et al., 2009, 2014; Zhang et al., 2016b) was employed to retrieve the LST of the study area, as shown in Fig. 4. By comparing Figs. 4 and 5, it can beseen that there is a strong spatial correlation between LST and terrain factors, especially the slope aspect. Since the local time of the satellite passing over the study area was 10:43 AM, and the solar azimuth angle was 153°, the LST exhibited significantly higher values on the southeast-facing slopes than on the northwest-facing slopes.

Figure 4. Spatial distribution of land surface temperature (LST)in the study area

4.2.2 Mapping of unique-value environmental characteristic zones

 The slope and aspect were used as environmental factors to construct the UV-ECZR (Fig. 5a 396 and b). As previously mentioned, these two factors have a strong spatial coupling relationship with LST. Although elevation and vegetation coverage were not directly applied in environmental zoning, they can be considered similar within the neighborhood window (Zhang et al., 2019). Therefore, their effects are indirectly suppressed. In other words, in Focal-Zonal Mixed Statistics modeling, sample heterogeneity caused by long-range spatial variables can be controlled through spatial proximity, while that brought by short-range spatial variables can be suppressed through environmental similarity.

 Figure 5. Maps ofenvironmental factors: (a) slope aspect, (b) Slope degree, and (c) the composite unique-value environmental characteristic zonal raster (UV-ECZR).

4.3 Enhancement of geothermal anomalies based on Focal-Zonal Mixed Statistics

 In mineral prospectivity mapping, standard deviation standardization is often employed to assist in constructing indicator variables for prospecting. This process involves subtracting the mean from the original value and then dividing the result by the standard deviation. This indicator reveals how many standard deviations the original value deviates from the mean. The essence of this method lies in determining the appropriate range for calculating the mean and standard deviation, enabling a comparison of the current value against the mean and using

 the standard deviation to quantify this difference. In this study, Focal-Zonal Mixed Statistics was used for this purpose, i.e., defining the comparable sample range based on both spatial proximity and environmental similarity. Specifically, in this case, the level of LST at each current location is assessed within the range determined by both the local window and the similar terrain features. This approach mitigates the influence of factors such as terrain and vegetation, thereby producing a distribution map of LST anomalies that predominantly reflects geothermal activity.When the current window is a circle with a radius of 7.2 km, the final enhanced geothermal anomaly is shown in Fig. 6.

Figure 6. Enhanced geothermal anomaly map based on Focal-Zonal Mixed Statistics with a local window

radius of 7.2 km.

- Comparing Figs. 5 and 6, it is evident that the LST anomalies enhanced using Focal-Zonal Mixed Statistics exhibit a betterspatial correlation with known geothermal wells (obtained from Yan et al., 2017), and their high values indicate known geothermal wells more effectively. Therefore, we have reason to believe that the high-value areas in Fig. 6 have a higher probability of revealing new geothermal resources.
- **5 Discussion**

5.1 Advantages of the new statistics

 Based on the standard deviation standardization approach described above, we also employed 432 Zonal Statistics and Focal Statistics to enhance geothermal anomalies for further model comparison. Specifically, the Receiver Operating Characteristic (ROC) curve was used to compare the performance of LST itself and its three enhancement indices in geothermal prospectivity mapping.

 The ROC curve is plotted with the False Positive Rate (FPR) and True Positive Rate (TPR) as the x-axis and y-axis, respectively (Fawcett, 2006; Hanczar et al., 2010), and the resulting Area Under Curve (AUC) is used for quantitative evaluation of certain indices or models. AUC values range from [0.5, 1], where higher values indicate better predictive performance and accuracy of the model, and vice versa.

 The ROCs of LST and its three enhancement indices obtained by Focal Statistics, Zonal 442 Statistics, and Focal-Zonal Mixed Statistics, respectively, are depicted in Fig. 7. It can be 443 observed that the enhancement effect based on Focal-Zonal Mixed Statistics is significantly better than that based on the other two models, as the AUC of Focal-Zonal Mixed Statistics is 0.731, which is much higher than that of Zonal Statistics (0.638) and Focal Statistics (0.657). 446 Moreover, the AUC values of the latter two are also higher than that of LST, although marginally.

 Figure 7. The ROCs of Land Surface Temperature (LST) and its three enhancement indicators obtained by Focal Statistics, Zonal Statistics, and Focal-ZonalMixed Statistics, respectively. Parameter Settings: the local window for Focal Statisticsand Focal-ZonalMixed Statistics is a circle with a radius of 7.2 km; the categories used for Zonal Statistics are the same as those for Focal-Zonal Mixed Statistics; and a geothermal well represents an area of 0.1 km surrounding it.

5.2 Robustness of the new method

 To ensure that the superior performance of the new model in Sect. 5.1 is not coincidental, it is necessary to adjust parameters such as the local window size and the geothermal well representative area and conduct multi-scenario comparison experiments. This will help analyze the robustness of the new model's advantages.

 When the representative area for a geothermal well is determined by 0.1km, 0.2km, and 0.3km buffers, respectively, the AUC values for LST and its enhancement indices are calculated. These values, obtained through different models under various local window radii, are plotted on a Cartesian coordinate system, as shown in Fig. 8.

 Figure 8. The changes in AUC values with the window size of Land Surface Temperature (LST) and its three enhancement indicators obtained by Focal Statistics, Zonal Statistics, and Focal-Zonal Mixed Statistics, when a geothermal well represents circles with a radius of (a) 0.1km, (b) 0.2km, and (c) 0.3km, respectively.

 Overall, the two enhancement models incorporating neighborhood windows, i.e.,Focal Statistics and Focal-Zonal Mixed Statistics, perform better than the Zonal Statistics model and the LST without enhancement. The poor performance of Zonal Statistics is due to the strong spatial variability of LST and the simplicity of the classification scheme used. Additionally, since local window methods are sensitive to spatialscale, the performance of Focal Statistics and Focal-Zonal Mixed Statistics varies with the window size. However, regardless of whether the geothermal well representative area is 0.1km, 0.2km, or 0.3km, the performance of Focal-Zonal Mixed Statistics consistently surpasses that of Focal Statistics.

5.3 Advancements of the Toolbox

 The FZStats v1.0 developed in this study not only integrates traditional Focal Statistics and 478 Zonal Statistics, which deal with SPD and SSH respectively, but also innovatively implements Focal-Zonal Mixed Statistics based on spatial proximity and environmental similarity, addressing both SPD and SSH. Therefore, this toolbox is expected to provide a novel solution to spatial statistics.

 A variety of parameter setting interfaces are provided to enhance the statistical applicability of the developed toolbox, ensuring it meets the requirements of different application scenarios and computing conditions. In terms of neighborhood window settings, in addition to rectangular and circular windows, an elliptical window is also available, allowing users to express spatial anisotropy in the neighborhood through elliptical parameters. Regarding statistical parameters, the new toolbox supports traditional metrics like mean, standard deviation, minimum, and maximum values, as well as calculations for arbitrary percentiles. To make the best use of memory and CPU capabilities, the toolbox supports raster data chunk processing and multi-process modes, accommodating different computer memory capacities and enabling parallel processing on multi-core CPUs. Additionally, users can set a minimum cell number of samples for statistics through the "Threshold" parameter to avoid low statistical precision and unreliable results due to insufficient sample size.

 Lastly, to enhance automation and efficiency in multitasking, the toolbox provides a batch processing solution. Users can write parameters into an INI-format multi-section configuration file, which avoids repetitive and tedious manual operations. This can not only enable one-time setup and automatic execution of multiple tasks, but support parameter reuse and error tracing.

6 Conclusions

 This study developed the FZStats v1.0 toolbox using Python3 and QT5. The new toolbox 501 integrates Focal Statistics, Zonal Statistics, and the newly developed Focal-Zonal Mixed Statistics. We provided detailed algorithm implementations and modeling processes for these methods and evaluated their performance in geothermal anomaly identification. The main conclusions are asfollows: First, the development of the Focal-Zonal Mixed Statistics is essential, as it addresses gaps that traditional Focal Statistics and Zonal Statistics cannot fill. 506 Second, FZStats v1.0 offers extensive parameter setting options, supporting different window

- 507 shapes and types of statistics; simultaneously, by adjusting processing parameters, it can ensure efficient performance on computers with varying configurations. Third, case analyses show that Focal-Zonal Mixed Statistics significantly enhance geothermal anomalies compared to Zonal Statistics and Focal Statistics methods, with this advantage being robust.
- In summary, FZStats v1.0 not only innovates spatial statistical methods theoretically but also demonstrates powerful functionality and flexibility in practical applications, making it a promising tool in the field of geothermal anomaly identification and other areas requiring spatial statistical solutions.

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- *Code availability.* The source code for FZStats v1.0 is available on GitHub at https://github.com/Renna11/FocalZonalStatistics. The latest version of the software can be obtained at https://zenodo.org/records/13208114.
- *Data availability.* The sources of the original data supporting the case study in this paper are as follows: (1) Landsat 8 images used in this research were downloaded from https://earthexplorer.usgs.gov (last accessed: 3 August 2024); (2) Land Surface Temperature 525 data can be obtained from http://databank.casearth.cn (last accessed: 3 August 2024); (3) original elevation data used for calculating slope and aspect is from the Shuttle Radar Topography Mission (SRTM) Global 1 Arc-Second Product, provided by NASA, available at https://earthexplorer.usgs.gov (last accessed: 3 August 2024); and (4) geothermal well data is sourced from Yan et al. (2017). Sample data can be found at https://zenodo.org/records/13766015. Readers can refer to the instructions provided in the "README.md" file on the code repository (https://github.com/Renna11/FocalZonalStatistics)

- for guidance on software use, which allows for the reproduction of the case analysis using the
- aforementioned original data.
- *Author contributions.* DZ and QC conceived the original idea. NR and DZ developed the
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- *Competing interests.* The authors declare no competing interests.
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