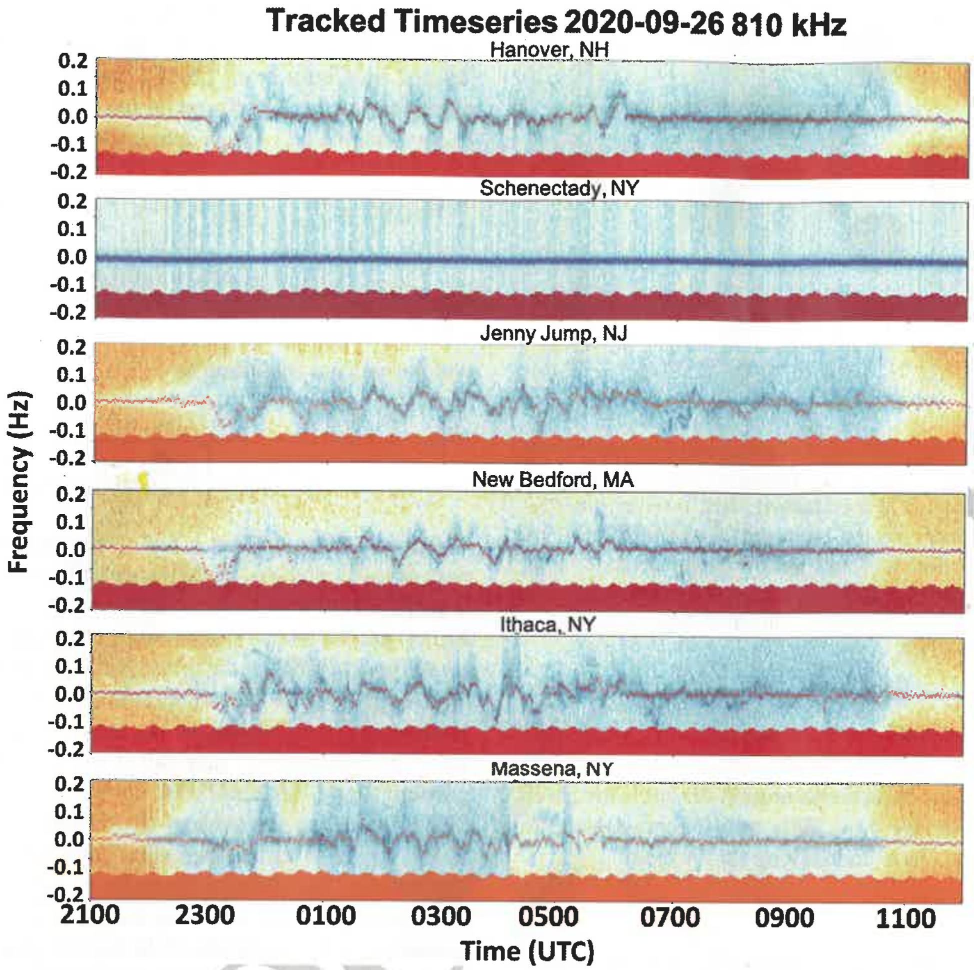

Figure 3. Spectrograms of the 810 kHz signal received at each location in the network covering 0.9 Hz bandwidth from 22:00 UTC on 25 September to 12:00 UTC on 26 September 2020. The solid, colored blocks at the base of each spectrogram illustrate the corrections made to account for variations in the frequency of the transmitted signal. The overlaid (red) trace is the extracted waveform generated by the semi-automated tracking algorithm with manual correction, as described in the text.

bance causing the Doppler shifts is consistent in shape and velocity across the entire array. Figure 4 shows the stationary, high-correlation time delays calculated with respect to the Hanover (light green) receiver for the event on the night of 26 September 2020 between 01:30 and 02:00 UTC. From the given time delays, it is immediately evident that the observed TID is propagating in the S/SE direction across the array as indicated by the red arrow.

We use three methods for determining the phase velocity from the time delays, denoted as triad, slowness, and sine fit. The triad method is a vector geometry approach to phase velocity which can be statistically analyzed because our experiment includes many baselines, i.e., many combinations of receiver triplets. A phase velocity can be calculated for each triplet as follows: for two pairs of receivers composing the triplet, the distance between reflection points is divided by the time delay between the received signals and inverted to obtain vector components, and adding these vectors provides an estimate of inverse phase velocity pertinent to the triplet. This calculation is repeated for all possible combinations of three receivers and averaged to obtain a final estimate for the phase velocity:TS2

1/3

$$V_{\text{phase}} = \frac{N}{\sum_{n=1}^{N!/3!(N-3)!} \frac{2}{3} \cdot \left[\sum_{i=1}^{3}\sum_{j=i+1}^{3} \frac{lag_{ij}}{d_{ij}}\right]}. \qquad (1)$$

TS3The “sine method” takes advantage of the arrangement of the reflection points, which lie approximately on a circle of radius $R = 100$ km centered on the Schenectady transmitter, indicted by a dashed line in Fig. 1. Using a Cartesian coordinate system with origin at Schenectady and with x directed eastward and y directed northward, this circle is described by $x^2 + y^2 = R^2$. Suppose a plane wave approaches this circle of receivers from the north, propagating in the $-y$ direction with velocity V. If the plane wave reaches the top of the cir-

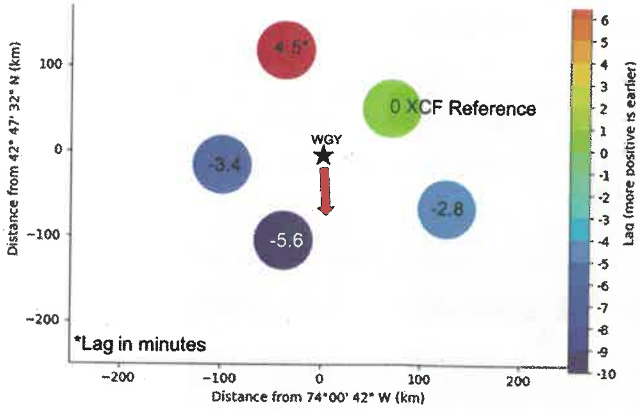

Figure 4. Map showing the location of each reflection point along with the time lag between frequency variations detected at each receiver and those detected at the Hanover, NH, receiver (arbitrarily chosen as reference) for the event detected at 02:00 UTC on 26 September 2020. Red indicates positive lags (signals arriving earlier), and blue indicates negative lags (signals arriving later). The observed lag times indicate a southern direction of propagation (arrow) for the observed TID.

cle ($x = 0$, $y = R$) at time $t = 0$, the equation for a wave front is $y = R - Vt$. Defining θ as the around-the-circle angle off of the y axis, x values of receivers around the circle are given by $x = R\sin\theta$. Inserting these TS4 x and y expressions into the equation for the circle and solving for t yields

$$t = (R/V)(1 - \cos\theta). \tag{2}$$

The time delay as a function of receiver position θ is therefore a sine wave; in this case, the receiver having minimum delay is that at $\theta = 0$ (at the northern edge of the circular array of receivers), and the receiver having maximum delay is that at $\theta = \pi$ (at the southern edge of the array). To treat plane waves coming in at other angles requires merely rotating the entire problem by the appropriate angle; for a plane wave directed at an angle φ off of the y axis, the time delay as a function of angular receiver position around the array would be

$$t = (R/V)\,(1 - \cos(\theta - \varphi)). \tag{3}$$

This function is fit to the measured delays of the five receivers arrayed at known angles θ around the origin (Schenectady) to determine the angle φ and the magnitude R/V, from which the direction of propagation and the horizontal phase velocity of the plane wave can be determined, since R is known.

The slowness method for tracking activity captured by a receiver array was originally developed in the seismology community (Lacross et al., 1969) and also applied to the location of thunder sources in storms (Johnson et al., 2011). Chum et al. (2014) and Chum and Podolska (2021) adapted this method for TIDs with a Doppler sounding array. As implemented with our array, the slowness parameter is defined as the inverted phase velocity. Sampling a range of x and y components of the slowness generates an energy map, as defined by Chum et al. (2014), which is a measure of the alignment of the waveforms at the center of the array. The energy map, $W(s_x, s_y)$, is defined by

$$W\left(s_x, s_y\right) \propto \sum_{t_i=-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \left[\frac{\sum_{n=1}^{N} f_{\mathrm{Dn}}(t_i) + s_x\Delta x_n + s_y\Delta y_n + s_z\Delta z_n}{N}\right]^2 \tag{4}$$

and normalized according to

$-\frac{\Delta t}{2}$

$$C\left(s_x, s_y\right) \propto \frac{W(s_x, s_y)}{\frac{1}{N} \times \sum_{n=1}^{N}\left[\sum_{t_i=\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \text{TS5}\ f_{\mathrm{Dn}}(t_i)^2\right]}, \tag{5}$$

where s_x and s_y are the x and y components of the slowness velocity and f_{Dn} is the Doppler shift measured at time t_i at the location of each reflection point. The Δx_i values in Eq. (4) are the distance from each reflection point to the center of the array. Since the array uses only the 810 kHz WGY signal for this study's analysis, the altitudes of reflection are assumed to be equal and the Δz term can be neglected. Figure 5 shows the energy map calculated using the slowness algorithm for 01:30–02:00 UTC on the night of 26 September 2020. We define the peak of the energy map (black circle) as the top 97 % of values, called peak slowness correlation coefficients. The distance from the center of the map to the centroid of the peak slowness correlation coefficients is the estimate of the magnitude of the inverse phase velocity, and the azimuthal direction of that centroid relative to the center of the map is the estimate of the direction of propagation. The inverse phase velocities associated with the extreme points of the region of peak slowness correlation coefficients can be taken as estimates of the relative uncertainty in magnitude (based on the inner and outer extrema relative to the origin) and direction (based on the azimuthal extrema). These relative uncertainty values are useful in comparing one phase velocity to another calculated with the slowness method, but they bear no obvious relation to the absolute uncertainty of the measurement.

Table 2 lists the events detected between 4 February 2020 and 31 March 2021, using the Schenectady station (WGY, 810 kHz) as the transmitter. These events were initially identified by manual inspection of reflected skywave plots similar to those shown in Fig. 3. The semi-automated tracking described above was applied to them. Phase velocities for each event, calculated using each of the three methods, are shown in the table.

The triad and sine fit methods both depend on the time delays determined from the normalized cross-correlation, making them especially sensitive to tails/outliers in the distribution of those delay times. Furthermore, both of these methods, but especially the sine method, become significantly less