Response to Referee 1:

- Line 73, about the limitation of the technique to measure AoA above 2 degrees: Is this limitation due to exceeding 1 cycle (1 wavelength) of delay between the top and the bottom elements of the interferometer? Or/and due to the way it is produced (angle of conjugate product of both signals)? If it is due to ambiguity due to a number of integer wavelengths in the relative delay, this could be overcame by resolving this number of integer cycles, which maybe could be achieved with code delays (if the bandwidth of the ABS-D is broad enough – I do not know), and/or by cross-correlating both signals searching for the maximum, instead of measuring only the phase of the complex conjugate. Furthermore, if the ABS-D system uses more than one frequency, combining information from both could also help resolving this ambiguity.

The limitation to the measured AoA being below an elevation of 2 degrees is due to significant bending of the signal occurring only at grazing incidence. Signals can be received at higher elevations, although the influence of refraction is significantly smaller and less variable. We use the expected AoA of the broadcasts (using a radiosonde refractivity profile nearby in time) to solve the AoA ambiguity arising from wrapped phases (i.e. to figure out the correct multiple of 2pi to add to the measured phase). Since the measured AoA typically only deviates from the predicted AoA by a few/several hundredths of a degree, this is a very reliable method. A disagreement greater than 0.57 deg. would be required to cause problems. Even in this case, the continuity of the flightpaths could be used to infer the correct solution.

- Did the authors analyse areas in Europe where the 2 deg topographic constraint would apply? Are there many zones where the interferometers could be located? Or very few?

No specific regions in Europe were analysed except for the Clee Hill site in Shropshire, UK. Ideally, the interferometer would be located with a clear view of the horizon with airports visible (to allow ADS-B transmissions to be received throughout the extent of the lower atmosphere). A single receiver on a hill-top site can receive ADS-B signals from over 500 km away, so the requirement for a dense network is reduced.

- More about measurement of AoA: are the Dopplers induced by the aircraft motion affecting the AoA? Are these Doppler shifts accounted for somewhere in the processing chain?

The doppler shift expected for aeroplane speeds is too small to have an effect of any importance on the received frequency (and therefore on the measured AoA).

- Line 79, '… beta is the incident AoA …' : as mentioned for epsilon later on, this angle tends to be called elevation angle rather than incidence (incidence would be the complementary to beta).

The AoA has been defined as the elevation angle of the signal for consistency.

- Line 100: the techniques implemented here (simulation and retrieval) are all based on geometric optics. Have the authors checked potential alternatives using wave optics?

No specific analysis using wave optics have been performed in this study, although work has been started to model the impact of atmospheric multipath and ducting and what signatures this might have in the observational data (the principal impact being the violation of the proportional relationship between the phase difference and the AoA, as multiple rays arrive at the receiver from the same source).

- Line 119, equation in-line $\epsilon_i = \sin^{-1} (u_i)$: it would help having this equation in a separate line and with a number. This would help understanding equation 10b and text in line 134.

This has been corrected.

- Figure 3: it would help a lot to have indication in the graphic about h_i, \epsilon_i and s_i (as used in equation mentioned above). This is specially true for the variable 's', as the explanation in lines 116-117 is not clear.

Figure 3 has been updated for clarity.

- Line 135: 'The initial position and direction… by h_0 and beta respectively': the equations above do not use beta but u_0, perhaps add in brackets the link between beta and u_0.

An added statement that u_0 = sin(beta) has been added. When integrating the ray path, beta is the initial (observed) AoA of the signal, and throughout the ray path the elevation angle above the local horizon is epsilon.

- Figure 5: the plots on the right-column of this figure are not discussed. The sensitivity to bottom layers is deduced from the left-column. What is the right-column informing about? About the along-ray resolution needed to be applied? (it seems to require better than 100 m, but later it is said that the actual processing is done at 100 m)

The right-hand side demonstrates the convergence of the value of the gradient computed using finite differences and the adjoint state method. A trade-off between the accuracy of the gradient calculation and the computational cost of integrating thousands of rays was needed. The following has been added to the discussion:

"The right panels of Fig. 5 show the magnitude of the relative difference between the gradients calculated using finite differences and the adjoint state method for 1 m and 100 m ray integration step sizes respectively. The value of the gradients computed using finite differences and the adjoint state method converge as the ray step size decreases, however, the computational cost of integrating rays with a very small step size increases significantly. For this study the 100 m integration step size was used as the gradient calculation was sufficiently accurate without being prohibitively expensive to compute for thousands of rays."

Figure 7a: this shows only the +-5 deg azimuth around the antenna boresight. The configuration of the antenna arrays shown in the picture does not seem to limit much on azimuth (but elevation). Therefore, it is likely that the interferometer was able to collect data from a broader range of directions. I think it would be interesting to show them all, too, as a way to illustrate the potential of the technique from a single interferometer. For example, add a panel to the left of panel (a), showing all the measurements, then the central would be the current (a) with only those selected for this study.

A panel has been added to the left of the original Figure 7(a) showing the entire observed region.

Caption of Table I: please indicate that these are results from synthetic observations (it is explained in the body of the manuscript, but not in the caption). Also, the three angular values are not explained in the caption (nor the table itself), so perhaps the caption could add a sentence to explain that 0.00 deg, 0.01 deg and 0.05 deg refer to the standard deviation of the added noise (added to the AoA or to the relative phases? Please clarify, too).

The caption of Table I has been updated to clarify that these are synthetic retrievals and that the angular values refer to the standard deviation added to the observed AoA: "The RMS values for the initial refractivity profile and the synthetic retrieved refractivity profile (relative to the reference radiosonde profile). The three angular values refer to the standard deviation of the added noise to the observed AoA"

Last paragraph of section 3.1, starting at line 258: are these results shown in some of the figures or tables? I could not find them. If they are not shown but only explained in the text, perhaps add at the end 'not shown in the figures' or similar indication.

The following has been added: "The figures for the synthetic refractivity retrievals using the 15 December and 18 July 2022 refractivity profiles are not shown."

Line 276-277 and black points in Figure 10: it is not clear what the black points are. One would think they are the measured AoA (using the interferometric technique), but then they would be noisy, too (while they are absolutely smooth). So I assume they are the line-of-sight AoA, that is, the angle produced assuming the straight line between the interferometer location and the aircraft location. Could this be clarified? Perhaps a set of bullets could summarize all the AoA involved in the analysis, and then relate to these bullets when showing or discussing about one or the other. For example AoA_interf: (measured by interferometer); AoA_LOS (for line-of-sight); AoA_mod0 (result of the synthetic observations modelled based on the initial refractivity profile); AoA_modR (result of the synthetic observations modelled based on the final retrieved refractivity profile).

A new set of bullet points has been added to clarify the variables analysed:

- AoA_LoS: The line-of-sight (LoS) AoA determined from the reported position of the aircraft;

- AoA_init_model: The synthetic LoS AoA determined from the end position of the ray traced through the initial refractivity profile;
- AoA_ret_model: The synthetic LoS AoA determined from the end position of the ray traced through the retrieved refractivity profile;

Line 296: ‘and and’ (twice, typo)

This has been corrected.

Caption Figure 10 (and/or text around Line 274): please add the time window of this 60 minutes batch. Is it centred at 12:00? or from 11:00 to 12:00? Perhaps part of the disagreement in Figure 11, for time 0-15 min and 15-30 min is due to time variations (Fig 11 shows good agreement for the last two quarter hours, so I wonder if these were closer to the radiosonde).

A statement has been added that the retrieval windows were from 08:36 to 09:36 UTC (the first hour of observations).

Discussion starting at Line 322: how easy it would be to deduce a ‘bending’ and ‘impact’ observables as in radio occultation? If this were easy from these measurements, then I believe the 2D forward operators developed for radio occultations and used for their assimilation into NWP could be adapted to this technique. On the one hand, the formulation for bending-impact of reflected signals is already in place, while the geometry in the interferometer is half of the reflected signal in radio occultation. I believe the forward operator for this technique is half (0.5*) Eq.3 in Aparicio et al., 2018 (doi: 10.5194/amt-11-1883-2018), that is, without the 2 factor in both terms. It should not be difficult to adapt the 2D operators to such equation.

This was a very interesting analogy to explore. An adapted geometry of the radio occultation technique was explored in Lewis et al 2023(b). The challenge behind this approach is the unknown emission angle of the radio transmission from the aircraft. This prevents the bending angle being measured directly between the receiver and the aircraft. It was assumed that an estimate could be determined from the model state (since the emission angle can be computed as arcsin(a / nr) where a is the impact parameter, n and r are the refractive index and radial distance of the aircraft respectively), where n is determined from the model. This has not been tested yet so it is not known what impact this would have.

Section 4.3 Multipath: the multipath that comes from nearby constant features (topography, buildings, etc), keeps a constant pattern. Therefore, collecting data for several days, from as many aircraft as possible in as many AoA and Azimuth angles as possible would results in a constant pattern for the oscillations (e.g., after some de-trending to keep only the multipath effect). Once this is well characterized, it can be corrected. See a similar example done in a cliff-based campaign with GPS radio occultations, very much affected by multipath (Fig 3 in doi: 10.5194/acp-16-635-2016).

This was a very useful insight and an additional statement has been added to the manuscript:

“The multipath due to foreground reflections could also be mitigated using a long time series of observations. The impact of reflections from a static foreground would be a systematic perturbation to the observed phase difference. With enough observations from aircraft traversing at a range of elevations and distances the constant multipath interference could be well characterised after accounting for atmospheric variability. The non-constant variability, such as atmospheric refractivity changes, could be de-trended out with sufficient observations. A similar technique is used in ground-based GNSS radio occultation, where Padulles et al (2016) describes how time series of the observations can be used to remove a significant contribution of the multipath effect on the uncertainty in the measurements of the atmospheric polarimetric effects on observables.”

More recent experiments have implemented a calibration table of the multipath interference as a function of AoA and azimuth that uses the same idea as above.

Instrumental uncertainties: have you considered the effect of different subsystems that link to only one of the two ends (top/bottom) of the interferometer, such as length and properties of the cables, different RF elements, antenna phase patterns? All these aspects might introduce uncertainties and should be carefully calibrated.

Each array was constructed from identical off-the-shelf components (antennas, attenuators, cables). Some aspects of the design: sub-array cable lengths, antenna placement were dependent on human interaction. Ideally, the individual gains and beam patterns of the antennas would be precisely calibrated but this has not yet been implemented.

Additional changes

While making changes to some figures using the suggested comments, the corresponding author discovered a couple of mistakes in the original preprint we would like to correct before publication:

- Figure 5: In the caption, the description for the ray was an AoA of 0.11 deg., an altitude of 11.2 km and a distance of 409.5 km. This should read instead to be an observed AoA of 0.2 deg., an altitude of 8.8 km and distance of 350 km to correctly correspond to the shown figure.

- Figure 8 and Table 1: The synthetic retrievals were generated using an earlier (incorrect) estimate receiver height. After running the synthetic retrievals with the correct receiver height, the plot looks almost identical, although the values in Table 1 are changed slightly as follows (new values and original values in parentheses):
    - 22 Sep 2023: 8.21 (8.64) 0.76 (0.96) 1.42 (1.51) 3.11 (3.59)
    - 18 Jul 2022: 9.63 (9.77) 1.89 (1.81) 3.42 (3.44) 7.09 (8.39)
    - 15 Dec 2022 4.84 (5.10) 0.70 (0.54) 0.88 (1.03) 1.18 (1.73)

Figure 12: The plots shown were computed with the previously mentioned incorrect receiver height. Using the correct receiver height changes the plots slightly, although the conclusions are unchanged.

On line 317, it was stated that no significant frontal activity was present that could have resulted in the observation noise. This was incorrect, there was in fact some frontal activity that could have contributed to the observed noise in the retrievals.

The corresponding author would like to apologise for not catching these errors in the earlier preprint.

Response to Referee 2:

1 (a) In line 213 of the manuscript a sentence starts saying that "Approximately 263,500 refracted ADS-B transmissions were received between an AoA of 0.0° to 2.0° over an ~ 80-minute period between 08:36 and 09:58 UTC on the 22 September 2023. The synthetic and real retrievals were obtained using the central 10° azimuthal sector of received transmissions". A little later, starting in line 219, it is said that "The adjoint inversion algorithm was tested on synthetic data generated with ADS-B data recorded using the prototype interferometer at Clee Hill, Shropshire. A subset of 5000 ADS-B transmissions were chosen randomly from within the central 10° azimuthal sector". But this subset comes from the whole set of observations (if not, this must be clarified), so I don't understand what the authors mean by "synthetic data".

Thank you for pointing this out, we apologise for the confusion. The observations throughout the paper were from the central 10 deg. azimuthal sector. For the synthetic retrievals, 5000 observed AoAs and reported ADS-B positions were used from this central sector. The following has been added to the manuscript to clarify the process of generating synthetic observations:

"The observations used in the analysis were restricted to the central 10 deg. azimuthal sector of the entire observation dataset. A subset of 5000 ADS-B transmissions were chosen randomly from within the central azimuthal sector to use in the synthetic refractivity retrieval experiments. Radiosonde data from the nearby Watnall station in Nottinghamshire (position shown in Fig. 7a) was used to generate a refractivity profile to simulate the refraction of the subset of ADS-B transmissions. Three radiosonde soundings were used to generate the synthetic observations - 12:00 UTC 22 September 2023 (see Fig. 2) 12:00 UTC 18 July 2022 (during an intense heatwave) and 12:00 UTC 15 December 2022 (during a cold spell) (see Fig. 9). The synthetic observations were generated by tracing the 5000 rays out from the receiver to the reported distances of each of the ADS-B transmissions using the refractivity profiles determined using the radiosonde data. The interferometrically-measured observed AoAs were used as the initial directions of the rays and the height of the ray endpoint was used as the "true" height of the synthetic ray origin (i.e. equivalent to the height of a synthetic aircraft). The method is equivalent to that shown in Fig. 3, except the heights of the synthetic aircraft are given by h_f The heights of the synthetic aircraft are treated as the target heights h_t in the synthetic retrievals."

1 (b) Starting in line 227, it is said "Three radiosonde soundings were used to generate the synthetic observations - 12:00 UTC 22 September 2023 (see Fig. 2), 12:00 UTC 18 July 2022 (during an intense heatwave) and 12:00 UTC 15 December 2022 (during a cold spell) (see Fig 8). The simulated samples of refracted ADS-B

transmissions were injected with varying degrees of noise to model the impact of experimental uncertainties on the retrieved refractivity profiles". How are these samples simulated? Are they the same as the 5000 ADS-B transmissions mentioned before? But these were, as far as I understand, real observations, not simulations. This question also applies to table 1: it looks like the authors assume that the simulated AoAs do not have noise, but then, again, I don't see how the simulations are performed. I think this deserves further explanation. A similar comment applies to the conclusions, where it is said that "Three AoA measurement noise cases were simulated" (lines 373-374), but in line 375 it is said that the noise was "injected into the observed AoA measurements".

We apologise for the confusion. The real ADS-B transmissions and observed AoAs are used to generate synthetic observations using the process described in the response to 1 (a). The impact of noise in the observed AoA measurement was simulated by adding a random perturbation to each of the 5000 AoAs from a normal distribution with varying standard deviations.

1 (c) It's a little distracting that the refractivity profiles derived from the radiosoundings on 18 July 2002 and on 15 December 2022, shown in figure 8, are discussed after the results in fig. 9. In addition, unless I'm missing something, I don't see a quantitative support to the assertion in lines 261-262, referred to the profile of fig 8a, that "However, in the presence of measurement noise exceeding a standard deviation of 0.01° the complex structure cannot be resolved". What's the contribution of the refractivity profiles in fig. 8 to the method assessment?

Thank you for this helpful suggestion. Figure 8 has been moved to the end of the section discussing the synthetic refractivity retrievals. The wording has been made more specific to refer to the inability to capture positive refractivity gradients in the presence of significant observational noise: "However, in the presence of measurement noise exceeding a standard deviation of 0.01 deg. the positive refractivity gradient cannot be resolved."

1 (d) How are the RMS errors in table 1 computed? I miss an equation for that.

We apologise for missing this. An equation has been added showing the computation of the RMS.

1 (e) In figure 9a, the retrievals of the synthetic refractivity profile (whatever it means) for the two cases of noise in the AoA (note by the way that the units (°) are missing in the standard deviation of the angles in the figure legend), do not seem to be affected by that noise. Shouldn't there be error bars or shaded areas around those retrievals? Likewise, what do the shaded areas in fig. 9a and b represent? Do they mark the limits of the refractivity profiles retrieved with ±5 m and with ±10 m error in the receiver position?

We apologise for this error. The units have been added to the plot. Error bars have been added as the range in retrieved profiles for retrievals using twenty different random normally-distributed observed AoA noise cases. The shaded areas represent the limits of the refractivity profiles retrieved using ±5 m and with ±10 m error in the receiver position (this has been added to the figure caption).

2 (a) In line 275 it is said “Figure 10 shows the reported AoA as a function of horizontal angle”. I suppose that the horizontal angle is the elevation angle an aircraft should be seen from the receiver were not for refracting effects; in any case, the meaning of “horizontal angle” should be explained. Perhaps that could be illustrated in fig. 3, which could also be used to illustrate the “local elevation angle”, in a similar way as do several figures in reference Zeng et al. 2014, in Lewis et al. 2023a od in Lewis et al. 2023b (sorry for jumping from a section to another).

The corresponding author apologises for the confusing terminology. In Figure 10 the “horizontal angle” actually refers to the “azimuthal angle” (although 0 deg. is not due north, but in the direction of the central beam of the receiver). This has been clarified in the caption of Fig. 10.

2 (b) Without any explanation, the statement in lines 276-277, “The black points (obs) indicate the reported AoA determined from the ADS-B positional data (i.e. the "true" reported flight paths)” seems strange at first thought, since the AoA is not reported (at least directly) by the ADS-B system, but given by interferometer. Perhaps the authors mean that the phase ambiguity resulting from a large baseline/wavelength ratio is resolved using the height reported by the ADS-B, as explained in Lewis et al. 2023a. If this is the case, I think that a brief explanation would be helpful.

We apologise for the confusing terminology. A new set of bullet points has been added to clarify the variables analysed:

- AoA_LoS: The line-of-sight (LoS) AoA determined from the reported position of the aircraft;
- AoA_init_model: The synthetic LoS AoA determined from the end position of the ray traced through the initial refractivity profile;
- AoA_ret_model: The synthetic LoS AoA determined from the end position of the ray traced through the retrieved refractivity profile;

The LoS AoA is now referred throughout, being the angle above the horizontal of a straight line drawn between the receiver and the aircraft (for the reported ADS-B position) or the ray end position (for the simulated cases).

2 (c) Line 277: How are the “retrieved flight paths using the initial (prior) exponential profile” obtained? I understand, from the explanations in section 2.3, that the retrieved ray is essentially obtained by propagating from the receiver to the aircraft horizontal position and adjusting iteratively the refractive indices of the layers the atmosphere is divided into until the quadratic difference between the ray endpoint height and the reported aircraft height is minimized, but how is the AoA determined from just the initial refractivity profile? Is it done as suggested in section 5.1 of Lewis et al. 2023a? If this is the case, I think a brief explanation should be included in the present paper or, at least, an indication to the reader to consult that reference close to the mention of the “retrieved flight paths using the initial (prior) exponential profile”.

We apologise for the confusing terminology. The AoA referred to here is the line-of-sight (LoS) AoA. This is determined by tracing a ray out from the receiver to the aircraft horizontal position and computing the angle above the horizontal of the straight line joining the receiver and end point of the ray (this is equivalent to the observed AoA if no refraction occurred). An additional statement has been added referring to Lewis et al 2023a.

2 (d) Line 285: Is the “observed AoA measurement” the same as the “true reported AoA”? Is the use of “reported” in “retrieved reported AoA” necessary? Why? Related to this, is the “retrieved modelled reported AoA” mentioned in the caption of fig. 12 the same as the “retrieved reported AoA” mentioned in line 285? Maybe a set of definitions of the different AoAs considered would be helpful, even if these definitions are found in Lewis et al 2023a, in which case this should emphasized.

We apologise for the confusing terminology. The “observed AoA” refers to the AoA determined using the interferometer. The “true reported AoA” referred to the LoS AoA determined using the ADS-B positional information. These have been updated in the bullet point list described in response to 2(b), with the true reported AoA refereed to as AoA_LoS.

2 (e) Is it possible to assign uncertainties (error bars) to the retrieved refractivity profiles shown in fig. 11?

At this stage it is very difficult to assign uncertainties to the retrieved refractivity profile. The retrieved profile is generated using thousands of individual ADS-B signals, where the uncertainty in the observed AoA for each is unknown. We attempt to quantify the uncertainty using the LoS AoA of the end point of the ray through the retrieved refractivity profile and the LoS AoA determined using the ADS-B reported position, where the large standard deviation in Figure 13 demonstrates the uncertainty in the retrieval in terms of measurable quantities. A better estimation of the actual uncertainty in the refractivity will likely require sensitivity/assimilation experiments using a numerical weather prediction model, with initial experiments being started using the Data Assimilation Research Testbed (DART).

A final, not fundamental, remark on something that can mislead a reader: were not for the paper title, reading only the abstract and especially the introduction, one could think that the paper is going to deal with humidity profiles. I understand that the refractivity profile provides humidity information, but somehow the introduction should make clear that the paper deals with refractivity, not humidity, profiles

We apologise for the lack of clarityThe following has been added on line 47 of the original manuscript:

"High resolution direct/indirect measurements of humidity are challenging to obtain. However, the high spatial and temporal variability of water vapour in the lower atmosphere results in significant changes in the optical properties of the atmosphere. A valuable source of indirect humidity information for use in NWP is provided through observations of refractivity…”

Additional changes

While making changes to some figures using the suggested comments, the corresponding author discovered a couple of mistakes in the original preprint we would like to correct before publication:

- Figure 5: In the caption, the description for the ray was an AoA of 0.11 deg., an altitude of 11.2 km and a distance of 409.5 km. This should read instead to be an observed AoA of 0.2 deg., an altitude of 8.8 km and distance of 350 km to correctly correspond to the shown figure.

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