Dear Editor:

Please consider for publication in *Nonlinear Processes in Geophysics* (*NPG)* the following revised manuscript entitled:

Revisiting Lorenz's and Lilly's Empirical Formulas for Predictability Estimates

by Bo-Wen Shen, Roger Pielke Sr., and Xubin Zeng.

We appreciate the valuable comments by the Editor and reviewers. The complexities of the scientific problems originate from studies conducted a long time ago (in the 1960s and 1970s) and related findings were inaccurately cited in the subsequent years. Consequently, to facilitate prompt discussions during the open discussion period, we have posted six short responses with a specific focus in each response. In the final version of the responses, we will summarize what has been discussed in the revised manuscript and the six responses.   Given the tremendous efforts that were made over the past years, we believe our revised manuscript will be of significant interest to NPG readers focusing on predictability.

The main changes in the revised manuscript and key points in the responses are as follows:

- Revised manuscript: The original Section 3.6 regarding the Lilly's formula for two discretization methods was moved into Appendix B to avoid repeated discussions of the scale factor Jacobian, as suggested by Reviewer 1. A few paragraphs were added and editorial changes were made to improve readability (see the manuscript with tracked changes).
- Responses Part 1A (R1A or AC1): "A reevaluation of Figure 3 in Zhang et al. (2019)"
- Responses to Editor (AC5): Additional discussions of Zhang et al. and the validity of the revised Logistic equation
- Responses Part 1B (R1B or AC2): "A Brief Note on Turbulence-based Turnover Time" (focusing on the relationship between kinetic energy, velocity, and turnover time.)
- Responses Part 1C (R1C or AC3): "Qualitative Predictability Estimates Using Lilly's Formula and Comparative Insights"
- Responses Part 2A (R2A or AC4): "A Brief Note on Turbulence-based Turnover Time" (focusing on the properties of scale invariance and self-similarity)
- Responses Part 2B (R2B or AC6): "A Brief Note on Bistability, Duality, and Dimensional Transitions in Recent Turbulence Studies"

We believe that the insights of our study, along with our recent chaos and predictability studies, suggest opportunities to explore larger predictability and extend weather forecasts using various approaches,

including sophisticated theoretical, real-world, and artificial intelligence-powered models. We hope our revisions and responses are acceptable. Thank you for your consideration of our work.

Best Regards,

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URL Links for the Posted Responses:

- Shen, Pielke Sr., and Zeng, 2024: Responses Part 1A (R1A): "A reevaluation of Figure 3 in Zhang et al. (2019)". https://doi.org/10.5194/egusphere-2024-2228-AC1

- Shen, Pielke Sr., and Zeng, 2024: Responses to Editor: Additional discussions of Zhang et al. and the validity of the revised Logistic equation. https://doi.org/10.5194/egusphere-2024-2228-AC5

- Shen, Pielke Sr., and Zeng, 2024: Responses Part 1B (R1B): "A Brief Note on Turbulence-based Turnover Time." https://doi.org/10.5194/egusphere-2024-2228-AC2

- Shen, Pielke Sr., and Zeng, 2024: Responses Part 1C (R1C): "Qualitative Predictability Estimates Using Lilly's Formula and Comparative Insights" https://doi.org/10.5194/egusphere-2024-2228-AC3

- Shen, Pielke Sr., and Zeng, 2024: Responses Part 2A (R2A): "A Brief Note on Turbulence-based Turnover Time" (this is different from R1B). https://doi.org/10.5194/egusphere-2024-2228-AC4

- Shen, Pielke Sr., and Zeng, 2024: Responses Part 2B (R2B): "A Brief Note on Bistability, Duality, and Dimensional Transitions in Recent Turbulence Studies" https://doi.org/10.5194/egusphere-2024-2228-AC6

Responses to Editor's on-line comments

Note to the Editor and all reviewers: We already posted six responses online and these responses will be summarized here. Following the comments and suggestions from the Editor and reviewers, we have revised the manuscript by

- moving the original Section 3.6 regarding the Lilly's formula for two discretization methods into Appendix B to avoid repeated discussions of the scale factor Jacobian,
- adding a few paragraphs, and
- making editorial changes to improve readability (see the manuscript with tracked changes).

> To answer first a request by the authors, I do not think any of the mathematical developments in their paper is flawed. I only think that many of those developments are too lengthy and in addition useless to ordinary readers of *NPG*, who can be expected to be familiar with the mathematical notions that are used there.
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> I had mentioned the article by Zhang *et al.* (2019), which also leads to the conclusion of an ultimate predictability limit of two or three weeks. I had also mentioned (without explicit reference to Zhang *et al.*) that the coincidence of the predictability limits of Lorenz and Lilly should be mentioned, and possibly discussed. The authors present a discussion of the validity of the approach taken by Zhang *et al.*, as well as of their conclusion. The authors may of course wish to include that discussion in a revised version of their paper, but that is by no means what I considered necessary, or even desirable.
>
> The authors finally present a discussion of the predictability of the modified Logistic Equation. That discussion seems to me to be totally irrelevant for their paper, and corresponds to nothing I had asked or suggested.

First, we appreciate the Editor's acknowledgment of the absence of discernible flaws in the mathematical analysis. To address the Editor's concerns, we have previously provided responses, which can be accessed from the following link: https://doi.org/10.5194/egusphere-2024-2228-AC5. For convenience, a concise summary of our responses is also provided here.

(1) To prevent repetitive mathematical discussions, we have moved the original Section 3.6 to Appendix B. Here's a summary of Section 3.6: we demonstrate that Lilly's series over a non-uniform grid ($k = 2^n k_L$) and Lilly's integral with respect to ln(k) (i.e., a varying scale factor of 1/k) are consistent.

(2) As discussed in the AC5 response file, Zhang et al.'s findings cannot be directly applied to validate Lorenz's and Lilly's formulas because they use different evaluation criteria.

(3) Zhang et al. failed to provide compelling justifications for choosing a new tunable parameter in the modified Logistic equation.

(4) While the findings of Zhang et al. have been cited to support the predictability limit in the middle latitude, Zhang et al. (2019) study also suggests greater predictability for certain variables and weather systems, such as MJOs. Therefore, the two-week predictability limit cannot be a robust theoretical limit for the intrinsic predictability of weather. In fact, a recent study employing an AI-driven model has unveiled remarkable 30-day ensemble simulations of the MJO (for instance, as depicted in Figure 11 of Lang et al. 2024).

Link for the Posted Responses:

- Shen, Pielke Sr., and Zeng, 2024: Responses to Editor: Additional discussions of Zhang et al. and the validity of the revised Logistic equation. https://doi.org/10.5194/egusphere-2024-2228-AC5