



## Inverse Differential Equation Modeling of ENSO Prediction Based on Memory Kernel Functions

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10 **Abstract.** The ENSO system is a complex climate pattern that is crucial in global climate systems and plays a key role in climate prediction. Both statistical methods and numerical models have been dedicated to achieving accurate predictions of ENSO variations; however, there is still a considerable gap in practical applications. Therefore, we proposed a memory kernel function-based approach to solve the inverse problem of ENSO time-varying systems.

15 We attempted to establish differential equations by constructing memory vectors composed of multiple initial values to describe the evolutionary characteristics of this complex system. Unlike traditional inverse problem-solving methods, our research scheme delved into the inherent properties exhibited by ENSO, such as memory and periodicity, and embedded these properties as specific targets in differential equations. By leveraging the flexibility of

20 evolutionary algorithms to solve mathematical problems, we achieved targeted modeling of the ENSO system. The results demonstrate that this scheme overcomes the limitations of traditional differential equations with a single initial value and extends these equations to memory vector equations based on multiple initial values. This not only enhances our ability to describe the evolutionary laws of complex systems but also improves the timeliness and reliability of ENSO

25 predictions, achieving encouraging results.

### 1 Introduction

The urgency to predict short-term climate change with precision is critical given the increasing frequency and intensity of extreme weather events. Accurate climate predictions play a vital role in formulating disaster-prevention and mitigation policies (Hwang et al., 2021;

30 Usman, 2017). As one of the countries most severely affected by meteorological disasters, China experiences approximately 55% of the deaths and 87% of the direct economic losses caused by natural hazards worldwide. Furthermore, if weather-related secondary geological



disasters occur, these figures will increase to 81% and 89%, respectively (Wu et al., 2014).  
Therefore, improving the accuracy of short-term climate predictions has considerable  
35 socioeconomic and scientific importance (Guan et al., 2015).

Climate models have become sophisticated and prevalent tools for simulating the  
interactions between the atmosphere, oceans, land surface, and ice (Eyring et al., 2019; Masson  
and Knutti, 2011). Based on physical principles, dynamic models use equations that describe  
atmospheric and oceanic processes (Palmer, 2001). These models require substantial  
40 computational resources and detailed initial conditions, making them powerful for long-term  
projections but often less effective for short-term predictions involving rapid changes and  
smaller scales (Alizadeh, 2022). Additionally, statistical models rely on historical data, are  
generally less computationally intensive, and can be useful for short-term forecasts (Sunyer et  
al., 2012). However, they are typically restricted to historical data and have a limited ability to  
45 predict unprecedented or highly variable events (Kjellström et al., 2010). Therefore, combining  
the strengths of dynamic and statistical approaches has emerged as a promising strategy for  
enhancing short-term climate prediction (Mittal et al., 2015; Feng et al., 2020). These  
advancements have substantially improved short-term rainfall and temperature predictions in  
eastern China (Wei, 2011; Tian et al., 2015; Zhang et al., 2018).

When utilizing current modeling capabilities and statistical theories, making substantial  
improvements in prediction accuracy through external optimization alone and addressing the  
biases in climate predictions at small spatial scales remain challenging in the short term.  
Numerical climate predictions inevitably have considerable biases that are challenging to  
address (Eyring et al., 2019). Thus, relying solely on numerical model outputs to address local  
55 climate prediction issues in different regions remains unsatisfactory. Statistical methods have  
been used for more than half a century (Sedki et al., 2009; Feng et al., 2019). These methods  
rely on the assumption that time series are stationary, making them less suitable for dealing  
with nonstationary nonlinear problems (Mirikitani and Nikolaev, 2010; He et al., 2012).  
However, climatic evolution processes often exhibit nonlinear and nonstationary characteristics.  
60 Ma et al. (2023) emphasized that the differential prediction method based on time series is  
nonlinear, combining the advantages of numerical models and the convenience of traditional  
statistical methods. This method has the functionality of continuous multi-step iterative  
prediction, like numerical models, and it can handle complex relationships between variables.

Therefore, our research was based on the memory characteristics of the climate system  
65 and addressed the limitations of a single initial value in numerical model methods. Specifically,



differential equation modeling of multi-initial memory kernel functions (MIMKFs) has emerged as a novel approach with considerable potential for improving the accuracy of short-term climate forecasts. The MIMKFs are a sophisticated class of differential equations that incorporate multiple initial conditions and memory effects into their framework (Ma et al., 70 2023). These equations were designed to capture the dynamic behavior of climate systems, where the future state depends not only on the present conditions but also on a series of past states. Furthermore, MIMKFs are particularly suitable for modeling complex time-dependent systems (Wan et al., 2012). This multi-initial approach mitigates the limitations of traditional single-initial-condition models, which may overlook large variations, leading to less reliable 75 predictions (Feng et al., 2001). Moreover, the self-memory aspect of these differential equations allows for the incorporation of past climatic events and trends, enhancing the ability of the models to predict future states. Targeted modeling can utilize limited resources and data more effectively to improve modeling and prediction (Olivetti and Messori, 2023). This is particularly crucial for short-term climate predictions, where recent weather patterns and 80 anomalies greatly influence future conditions (Dong et al., 2004; Chen et al., 2009). Thus, by leveraging the strengths of MIMKFs and targeted modeling, predictive tools can provide timely and accurate forecasts, which are essential for disaster preparedness and mitigation efforts.

For complex climate systems, the main challenge in prediction is the complexity of the constituent components of the system. The system evolution process typically includes not only 85 major cycles but also various high-frequency disturbance components (Feng et al., 2020; Wang et al., 2021). For example, sea surface temperature (SST) is a complex composite signal formed by the superposition of multiple cycles. Chen et al. (2004) used a hybrid statistical–dynamic modeling system and SST data to discuss the relative role of atmospheric noise and the initial ocean state in determining the predictability of the El Niño–Southern Oscillation (ENSO) 90 system. Lima et al. (2009) developed a two-stage statistical ENSO forecasting model to understand the possible sources of longer-range ENSO predictability. Statistical models, such as principal component analysis, have been widely used to capture the main modes of ENSO and to reduce the dimensionality of the predictors in forecasting (Drosdowsky, 2006; Lu et al., 2017). However, most statistical models assume that the embedding space is a linear subspace 95 of the original data.

Therefore, in this study, we aimed to produce improved statistical forecasts of ENSO as represented by the Niño3.4 index via the nonlinear MIMKFs method. To this end, we first studied the periodicity of the SST time series, performed wavelet transforms to obtain two-



dimensional time-frequency variation plots, extracted specific periods, and laid the groundwork  
100 for constructing periodic memory kernel functions (MKFs). Second, we conducted an inertial  
memory analysis of non-periodic components to seek theoretical bases for constructing inertial  
MKFs. Finally, based on the two memory principles, the basic differential forms of the MKFs  
were predefined in the modeling process, dynamic system target modeling of SST was  
conducted, and the actual performance of the MIMKFs' predictability of ENSO was examined.

## 105 2 Material and Methods

### 2.1 Data

We used the indices for ENSO, particularly the Niño3.4 index (Reynolds and Smith, 1994)  
provided by the National Oceanic and Atmospheric Administration. The data are available at:  
[https://www.esrl.noaa.gov/psd/gcos\\_wgsp/Timeseries/Nino34/](https://www.esrl.noaa.gov/psd/gcos_wgsp/Timeseries/Nino34/).

### 110 2.2 Inverse differential equation modeling of MKFs

The classical approach to solving inverse problems, which relies on simplified differential  
equations with a single initial value, is inadequate for characterizing complex climate systems.  
Inverting complex climate systems requires the construction of sophisticated differential  
equations, thereby demanding advanced parameter estimation techniques. To capture the  
115 complex characteristics of ENSO, we proposed using MKFs based on multiple initial values  
and employed flexible evolutionary algorithms for parameter estimation.

We assumed that the dynamic differential equation of the ENSO system was:

$$\frac{dx}{dt} = F(x, p, q, t) \quad (1)$$

Where  $x \in R^n$  represents the state vector of the system;  $p, q \in R^n$  denotes unknown  
120 parameter vectors;  $t$  is time; and  $F$  represents an unknown vector function. Using inverse  
differential equation modeling, the characteristics of ENSO and the related parameter  $p$  can be  
expressed using observational data  $y(t_i)$  (where  $t_i$  denotes the observation time points,  $i =$   
 $1, 2, \dots, N$ ). On assuming that  $F$  possesses a specific structure or form that includes the  
undetermined parameter  $p$ , the function  $F_a$  can be defined as follows:

$$125 \quad F_a(x, p, t) = m(x, p, t) + f(q, t) \quad (2)$$



Where  $p$  and  $q$  are unknown parameters and their values are estimated using evolutionary algorithms. Notably, functions  $m$  and  $f$  contain self-memory information (MKFs or basis functions), and their specific forms are dynamically determined using evolutionary algorithms.

To estimate the undetermined parameters  $p$  and  $q$  and the functions  $m$  and  $f$ , an error  
130 function  $E(m, f, p, q)$  that measures the discrepancy between the assumed model and  
observation must be constructed. Then, the error function can be defined using the least-squares  
method as follows:

$$E(m, f, p, q) = \sum \| x(t_i) - y(t_i) \|^2 \quad (3)$$

Where  $x(t_i)$  is the simulated state vector obtained from the assumed model  $F_a$  through  
135 numerical integration using the Runge–Kutta method. Subsequently, we employed a genetic  
algorithm to minimize the error function, thereby estimating the parameters  $p$  and  $q$  and the  
functions  $m$  and  $f$ . We generated an initial population containing multiple parameter vectors  
for  $p$  and  $q$  and the gene expressions of functions  $m$  and  $f$ . For each parameter vector  
( $p_j, q_j, m_j, f_j$ ) in the population, we computed the corresponding simulated state vector  $x(t_i)$   
140 and calculated the error function. The fitness of each parameter vector was evaluated based on  
the values of  $E(p_j, q_j, m_j, f_j)$ . Furthermore, a subset of the parameter vectors was selected based  
on their fitness for crossover and mutation. Parts of the two parameter vectors were combined  
through crossover operations, and genes were randomly altered through mutation operations to  
generate new parameter vectors. This process was repeated until the parameter vector with the  
145 highest fitness was produced.

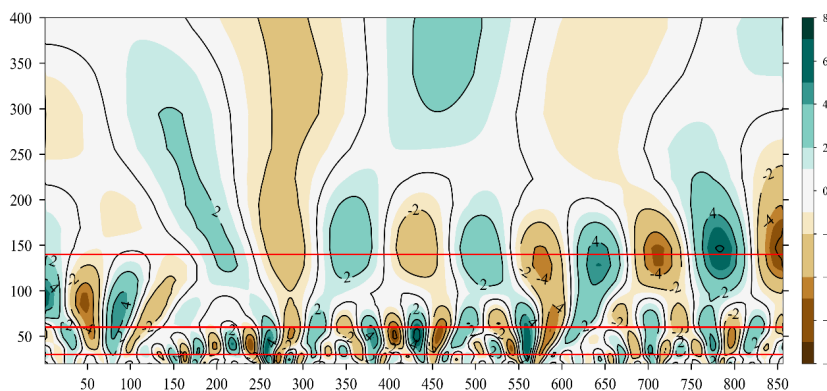
### 3 Results

#### 3.1 Analysis of SST memory patterns

Analysis of SST memory patterns involve more than simply randomly distributing the  
initial value vector for evolutionary modeling. It is also necessary to study the memory patterns  
150 of the target system prior to modeling. Herein, the monthly-scale SST corresponding to  
Niño3.4 was chosen as the research subject, with SST anomaly data available from January 1951  
to July 2022, spanning approximately 860 months. Typically, complex systems exhibit memory  
characteristics through two main approaches: periodic oscillations, referred to as "periodic  
memory" and inertial motion, termed "inertial memory". Initially, a periodic analysis was  
155 conducted on SST data to determine the characteristic parameters for constructing specific



MKFs. We employed wavelet analysis to obtain a time-frequency plot of monthly SST anomalies (Fig. 1).

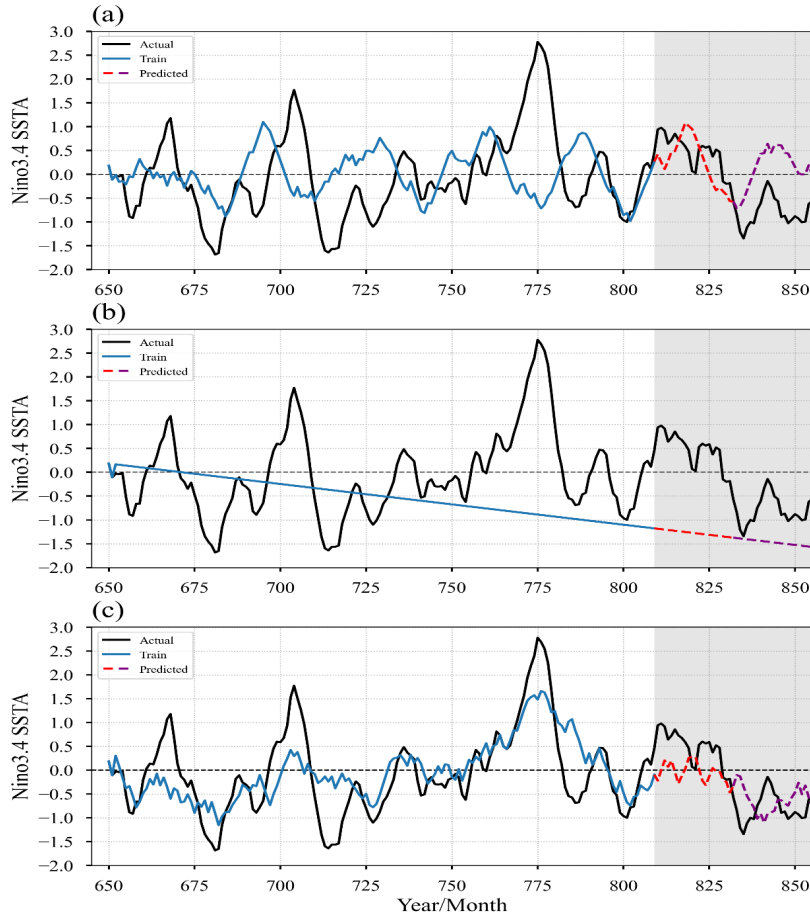


**Figure 1: Time-frequency plot of wavelet transform for SST anomaly sequence.**

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As shown in Fig. 1, the SST exhibits predominant cycles ranging from large to small scales, primarily at 140, 60, and 30 months. When converted to annual scales, these correspond to distinct cycles of approximately 11 years, noticeable cycles of approximately 5 years, and less-apparent cycles of approximately 2.5 years. These cycles appear to be relatively stable after the 210th month, corresponding to the period from 1969 to 2022. The remaining components represent less-apparent cycles characterized by high-frequency oscillations, which are considered to be inertial motions.

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170 **Figure 2: (a) Integral curve of the memory kernel function (MKF) ( $t$ ,  $sst$ ), (b) high-frequency component, and (c) integral curve of the SST modeling based on MKFs. The black line represents observed values, the blue line indicates fitting, and other colors represent the testing interval.**

175 Based on the time-frequency plot of the wavelet analysis, the earlier periods of SST exhibit relatively complex periodic patterns. Therefore, it was chosen to model the later observational SST data. The training dataset consisted of 180 months (sample range: 650–838, from April 2005 to December 2018), while the testing dataset comprised 26 months (830–856, from January 2019 to July 2022). The specific expression for the  $MKF(\sin t, X)$  obtained from evolutionary modeling is as follows:

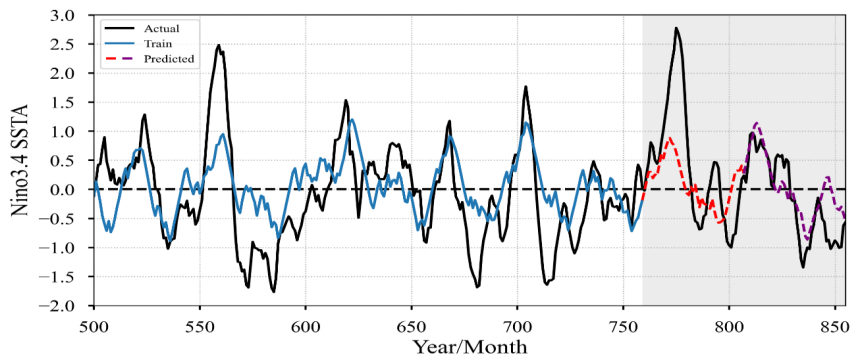
180 
$$MKF(t, sst) = 24.118 \sin\left(\frac{2\pi t}{60} + \frac{2945.3}{sst[-3]}\right) \quad (4)$$



where  $t$  represents time,  $X$  vector corresponds to the retrospective initial points of the SST, and  $sst[-3]$  denotes the observed value at the third previous time step of the current initial point  $sst[0]$ . It can be observed that the kernel function describes a memory signal with a period of 60 months and includes memory information from the SST of the third previous month (Fig. 185 2a). The high-frequency component  $f(t,X)$  is expressed as follows:

$$f(sst) = \frac{40.2}{sst[-5] + sst[-6] - sst[0] + 135.45} - 1.156 \quad (5)$$

Figure 2b shows that the higher-frequency component is described using information from SST observations of the fifth and sixth previous months. According to Fig. 2a and 2b, the kernel function and high-frequency component illustrate the characteristics and evolving trends of the SST periodic memory, respectively. Combining the periodicity of the former and the trend characteristics of the latter, the organically integrated training fitting and testing results are shown in Fig. 2c. Memory changes in SST have been described systemically. The ODE-MKF model achieved a correlation coefficient of 0.71 and a mean squared error of 0.63 for the training fitting, while for the testing part, the correlation was 0.68 with a mean squared error of 190 0.59. From the mathematical expression of the model, the ODE-MKF is relatively simple to calculate by embedding targeted kernel functions, yet it achieves a high accuracy in fitting SST data. Experiments conducted outside the sample have also indicated that the ODE-MKF can describe periodic changes in SST through periodic kernel functions, while the non-periodic component was described using a high-frequency component. 195



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**Figure 3: Integral curve of ODE-MKF for the long sequence.**

Furthermore, by increasing the size of the training and testing samples, the difficulty of fitting can be enhanced. The inversion results for the setting [Model Parameter 2]: 260 months

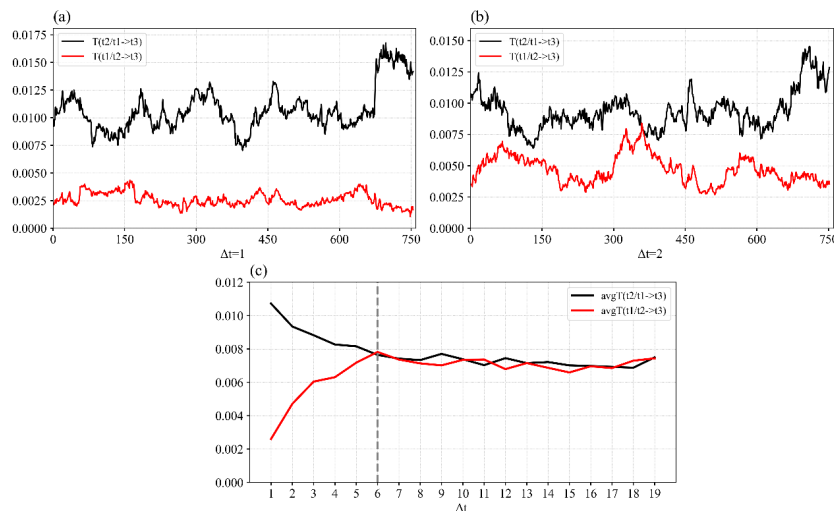




205 of training samples (sample range: 500–760, from October 1992 to June 2014) and 95 months  
for testing the model (761–856, from July 2014 to July 2022) are shown in Fig. 3. Extending  
the model training interval increases the difficulty of fitting and describing the memory with  
kernel functions. As shown in Fig. 3, the blue line indicates a fitting correlation coefficient of  
0.611 and mean squared error of 0.686. For the validation interval represented by the red part,  
210 the corresponding parameters were 0.63 and 0.693. The ODE-MKF performed well during the  
validation period and accurately predicted the main SST fluctuations. The reconstructed MKF  
involved two cycle ranges (140 and 30 months), indicating longer memory descriptions. The  
high-frequency component primarily comprised retrospective initial values that included  
certain nonlinear terms.

### 215 3.2 Inertial MKF

Random disturbances or processes in complex systems are not only difficult to predict but  
also tend to obscure the main periodic variations, thereby interfering with predictions. The  
transfer entropy method was employed as a diagnostic tool for assessing the inertial memory  
scales. In the following analysis, fluctuations in SST at the seasonal scale were examined to  
220 establish a theoretical basis for constructing the MKF. Using SST as an example, the inertial  
memory was analyzed using two methods.



**Figure 4: (a) Lagged transfer entropy of SST, (b) ( $\Delta t = 1$ , sample length=100), and (c) lagged transfer entropy of SST with sampling interval:  $\Delta t = 1-20$ .**

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The SST sequence was partitioned into lagged subsequences (with a sample size of 100), each consisting of three sequences (X, Y, and Z) corresponding to sampling time points  $t_1$ ,  $t_2$ , and  $t_3$ . For the third sequence group, the sampling interval was  $\Delta t = t_3 - t_2 = t_2 - t_1 = 1 - 20$ . According to definition of transfer entropy, the transfer entropy  $T(t_1/t_2 \rightarrow t_3)$  and  $T(t_2/t_1 \rightarrow t_3)$  values can be obtained. Based on Fig. S1, the two transfer entropy values for each sample group were obtained (see Fig. 4a and 4b), and their areas were represented as two numerical values, yielding a transfer entropy area sequence for  $\Delta t = 1 - 20$  as shown in Fig. 4c.

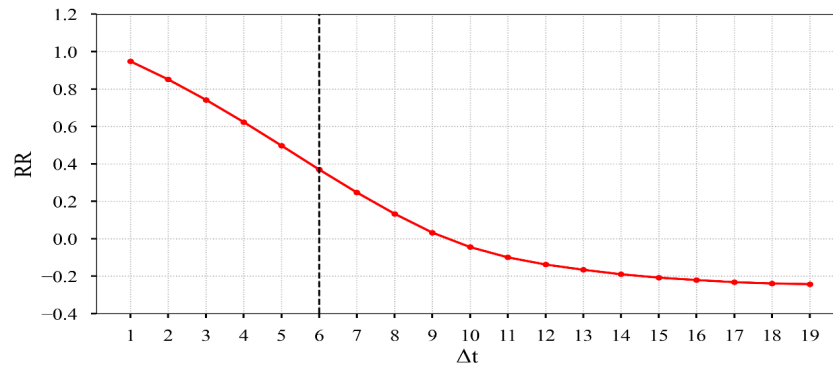
When ignoring the absolute values of the transfer entropy, an interesting phenomenon was observed between the two transfer entropy values. The time interval between  $t_2$  and  $t_3$  was always smaller than that between  $t_1$  and  $t_3$ , i.e.,  $t_3 - t_2 = t_2 - t_1 = 1 - 20$ , and  $t_3 - t_1$  was twice that of the former. Before  $\Delta t < 7$ , the contribution of sampling points  $t_2$  to  $t_3$  gradually decreased under condition  $t_1$ , whereas the contribution of sampling points  $t_1$  to  $t_3$  gradually increased under condition  $t_2$ . Both converged to a constant value at  $\Delta t = 6$ . It is clear that the blue line initially starts above the red line, as the time distance between  $t_2$  and  $t_3$  is always less than that between the latter, indicating that the contribution of  $t_2$  to  $t_3$  (or the percentage of  $t_3$ ) is higher than that of the former. After  $\Delta t > 6$ , the contribution of  $t_2$  to  $t_3$  decreases to a basic constant value. Corresponding to  $t_3 - t_1 = \Delta t = 2, 4, 6, 8, 10, 12$ , the contribution of  $t_1$  to  $t_3$  gradually increases and maintains the same state as the blue line. As the time span from  $t_1$  to  $t_3$  increases, the contribution of  $t_1$ , which is further away from  $t_3$ , increases. According to the definition of the transfer entropy calculation formula, this phenomenon occurs because the influence of  $t_2$  as a precondition decreases as the time span increases, resulting in a reduced impact on  $t_3$ . However, although the influence of  $t_1$  as the driving component also decreases, its rate of decrease is slower than that of  $t_2$ . This relationship resembles the combination of numerator and denominator relationships in conditional probability. Therefore, the contribution of  $t_1$  to  $t_3$ , as indicated by the value of the transfer entropy based on  $t_2$ , inevitably increases.

From the perspective of constructing the MKF, the two transfer entropies as mutual conditions indicate that the scale of memory changes occurs within 6–12 months. After 12 months, the memory remains almost unchanged. The changing state also indicates the existence of memory relationships between the sampling points. Based on this, the 6-month retrospective time scale can be used as the memory scale to construct the MKF, indicating that SST has relatively reliable inertial memory within 6 months.

### 3.3 Correlation coefficient

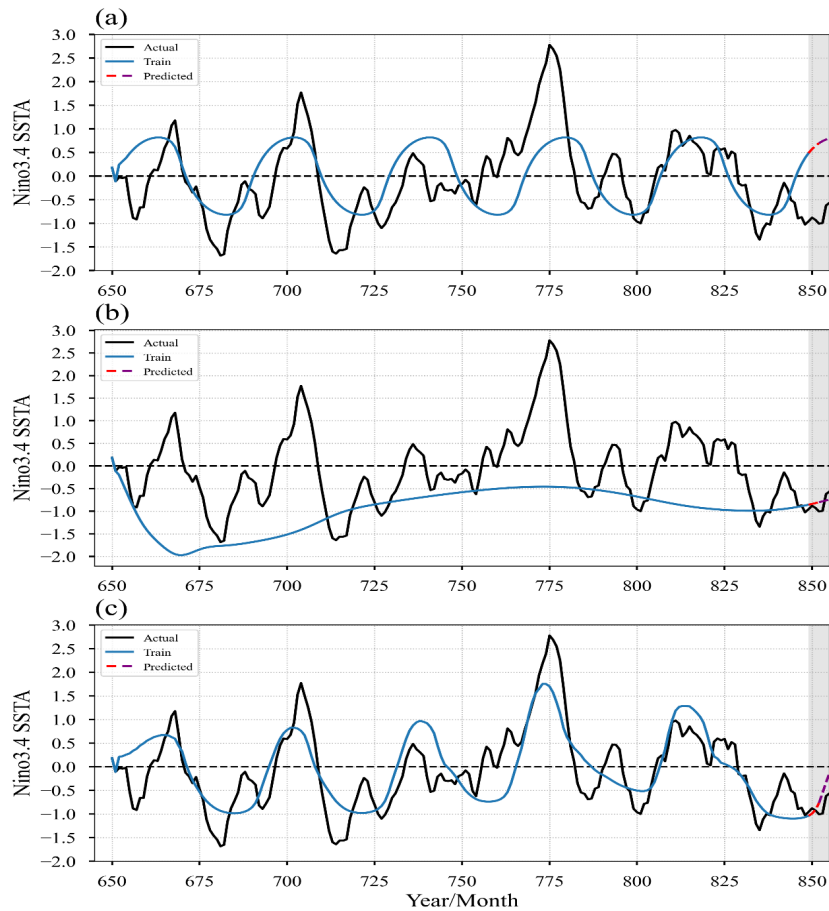


The correlation coefficient is a descriptive statistic measuring the degree to which changes in one variable correspond to those in another. According to its definition, the correlation coefficient only reflects synchronous changes between two variables and does not indicate causality between them. This does not affect its use in analyzing the problem of relationship memory. Following the lagged calculation scheme described above, the sliding correlation coefficient between the time series of the two sampling points was calculated with a subsequence length of 100. Figure 5 illustrates the variation in the average value of the lagged total sliding correlation coefficient.



**Figure 5: Lagged correlation coefficients.**

As shown in Fig. 5, as the time between sampling points increases, the correlation coefficient gradually decreases and becomes negative after a gap of approximately 10 months. If the confidence level for the correlation coefficient is set at 0.01, the corresponding correlation coefficient is approximately 0.39, with a lagged time span of approximately 6 months. With a further increase in the time span, the correlation is lost near 9.5 months, and thereafter, the correlation fluctuates randomly. It is evident that this inertial memory length aligns closely with the transfer entropy results. Therefore, when constructing the MKF, the maximum number of initial values considered was set to seven (including the current initial value). Consequently, compared to the previous method of randomly selecting the past 24 initial values, this initial value scale can substantially narrow the range of numerical solutions searched in evolutionary modeling, thereby potentially improving the modeling efficiency.



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**Figure 6: (a) Integral curve of inertial MKF, (b) integral curve of high-frequency disturbance component, and (c) integral curve of ODE-MKF based on inertial memory kernel function.**

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Through the characterization of memory using the tanh function of the inertial switch, the individual integral curve of the kernel function, as shown in Fig. 6a, describes relatively regular cyclical characteristics. Meanwhile, the high-frequency component  $ff$ , composed of retrospective value vectors and trigonometric functions, also exhibits certain nonlinear characteristics, as reflected in its individual integral curve in Fig. 6b, indicating a certain trend change.

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The ODE-MKF combines the advantages of both, further enhancing the understanding of oscillations in SST evolution details, with specific fitting parameters, including correlation coefficients and mean square errors. The test interval parameters are specified. Compared to the aforementioned cyclical memory, the ODE-MKF still exhibits considerable differences,



with the former showing more frequent small random fluctuations and the latter reflecting  
295 smoother cyclical patterns, as shown in Fig. 6c. Both demonstrated good predictive capabilities  
for trends.

#### 4 Discussion and Conclusion

We used wavelet analysis to detect both periodic and non-periodic features in SST data  
and constructed periodic MKFs and inertial MKFs to record the two types of motion in the  
300 system. The results provide important novel insights into the temporal dynamics of complex  
systems. Researchers have estimated and evaluated the link between observable variables and  
model parameters using evolutionary algorithms, transfer entropy and correlation coefficients,  
which aids in the selection and improvement of models. Moreover, focused modeling with the  
backtracking of multiple initial values provides a strong foundation for improving the accuracy  
305 and dependability of differential equation models and overcomes the inverse problem of  
complex and nonlinear systems more efficiently. Using these methods to construct and solve  
the periodic and inertial MKFs for ENSO, we obtained better ENSO predictions.

Using multiple initial values to extends the single initial value inverse problem to  
backtracking multiple initial values and utilizing more historical information from the time  
310 series. By incorporating evolutionary algorithms into the model with improved equations, the  
model parameters and initial conditions were estimated. Improving the operation of equations  
and using evolutionary algorithms, researchers can quickly explore the solution space and find  
the configurations that best fit the observed data. The model parameters can be improved  
through this iterative optimization process such that they match the underlying dynamics of the  
315 system more closely, additionally, improves the precision and dependability of differential  
equation models (Chen et al., 2022).

From these findings, the temporal dynamics of complex systems can be better understood  
by examining the short space–time memory patterns. Two separate techniques for modeling  
temporal dependencies: inertial memory, which is characterized by the persistence of prior  
320 states for a specific length. This tool can use to quantification the ENSO interannual Extended  
cyclo-stationarity quasi-cycles and consider the possible ENSO memory (Gauchere, 2010).  
Periodic memory, which is reflected by repeated cycles in the states of the system (Lustig et al.,  
2017). The variability of the Niño 1-2 and Niño 3 has large component of long-memory  
behavior associated with the quasi-biennial and the semi-annual frequency. This is particularly  
325 true when analyzing targeted modeling, which involves the retracing of several initial values.



Understanding the characteristics of periodic and inertial memory is essential for understanding the behavior of a system in the long run. The selection of suitable MKFs for modeling can be greatly aided by examining SST memory patterns. Prevalent cycles in SST data were identified using wavelet analysis, which directs the creation of MKFs specifically designed to capture the memory properties of a system (Brown and Garcia, 2021). Characteristic factors, such as inertial motion and periodic cycles, were identified to guide the creation of MKFs that efficiently capture temporal dependencies in the observed data.

To represent the memory dynamics of the system over time, inertial MKFs are essential for capturing the system's inertial motion (Wang and Zhang, 2021). To develop the inertial MKFs, the magnitude of memory changes in the system was ascertained by examining the lagged transfer entropy. Researchers can increase the fidelity of differential equation models to capture the inertial memory properties of a system by including inertial MKFs in the modeling framework. As a gauge of the linear link between variables, the correlation coefficient sheds light on the synchronous changes that occur between two variables (Johnson and Smith, 2019). The correlation coefficient makes it easier to evaluate the link between the observed data and model parameters in the context of focused modeling. The choice of suitable model structures, inertial memory ranges and parameterizations is guided by the observed correlations between variables, which improves the precision of the differential equation models.

This study highlights the advantage of using focused modeling with backtracked multiple initial values to understand the temporal dynamics of complex systems. Researchers can advance state-of-the-art modeling approaches and obtain deeper insights into the dynamics of complex systems by examining SST memory patterns, utilizing correlation coefficients, combining evolutionary algorithms, and applying inertial MKFs.

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### **Code availability**

The code supporting the results of this study can be obtained from the corresponding authors according to reasonable requirements.



### Data Availability

The original contributions presented in the study are included in the article. Further enquiries can be directed to the corresponding authors.

### Author contributions

360 SW: Conceptualization; Methodology; Writing–original draft; HL: Formal analysis;  
Writing–original draft; LZ: Investigation; Supervision; XZ: Investigation; Supervision; SX:  
Investigation; Supervision; MQ: Investigation; Supervision; YC: Investigation; Supervision.  
QM: Investigation; Supervision; Writing–original draft.

### Competing interests

365 The contact author has declared that none of the authors has any competing interests.

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