Response to Reviewers' Comments

We thank the Editors very much for handling of our manuscript entitled "**Inverse Differential Equation Modeling of ENSO Prediction Based on Memory Kernel Functions**" (egusphere-2024-2181) by Wan et al. First and foremost, I would like to express my sincere gratitude for the thorough review and insightful comments provided on our manuscript. Those comments are all valuable and very helpful for revising and improving our paper, as well as the important guiding significance to our research. We have carefully considered each point raised and have made the following revisions to our manuscript. Please see our point-by-point response (in blue normal font) below.

Reply to Reviewer#1:

We thank the reviewer for recognizing the value of our work and providing helpful comments. In the following, the text with normal font are the reviewer's original comments, and those with blue normal font are the authors' response.

Major comments:

1. The introduction lacks sufficient background on ENSO. As a primary subject of the study, ENSO's specific characteristics and importance should be more thoroughly introduced.

Response: Thanks for the kind and insightful comments and suggestions! We have added the ENSO's specific characteristics and importance. Please refer in the revised version. As follows:

Lines 90-100: For instance, the El Niño-Southern Oscillation (ENSO) is the most influential driver of global climate variabilities (Philander, 1983) and exhibits complex composite signals. El Niño events lead the central and eastern equatorial Pacific warming, which reduces the west-to-east zonal sea surface temperature (SST) gradient (Trenberth, 1997). The decreased SST gradient causes the western Pacific, the Inter-Tropical Convergence Zone, and the South Pacific Convergence Zone migrate toward the eastern equatorial Pacific (Cai et al., 2014). As a result, atmospheric convection becomes established in patterns that signal the likelihood of extreme precipitation or droughts globally (Stephens et al., 2015). What's worse, under transient greenhouse warming, ENSO variability is projected to increase pre-2100 (Geng et al., 2024). However, given the nonlinear relationship between ENSO and the magnitude and frequency of extreme events, enhancing ENSO prediction is critical.

2. The paper would benefit from a more comprehensive literature review and additional citations. In particular, recent advancements in ENSO prediction methods should be cited to provide a stronger context for this work. This will help readers understand how the proposed method compares to and builds upon existing approaches.

Response: We appreciate the reviewer's insightful comments. We have carefully incorporated recent advancements in ENSO forecasting methods to provide context for the main objectives of this paper and to highlight the advantages of our research. We have revised the text as suggested and the updated version is as follows:

Lines 106-119: Ham et al. (2019) utilized global SST anomalies and upper 300 m ocean heat content anomalies as predictor variables in a convolutional neural network (CNN) model, achieving an effective 18-month advance prediction of the Niño 3.4 SST index with an accuracy of 80%. Building on this foundation, Hu et al. (2021) enhanced the model performance and stability by incorporating dropout technology and residual connection modules, leading to the development of the Res-CNN model, which further refines the prediction capability up to 20 months in advance. The extended nonlinear recharge oscillator model demonstrates proficient ENSO forecasts with lead times of up to 16–18 months (Zhao et al., 2024). In addition, a neural network model that employs self-attention mechanisms, achieving multivariate predictions that extend from two-dimensional SST grid points to three-dimensional upper ocean fields related to ENSO (Zhou and Zhang, 2023). However, most statistical models assume that the embedding space is a linear subspace of the original data and machine learning techniques mainly achieving high prediction accuracy, which leads to ENSO prediction ignoring its own physical variabilities.

3. The theoretical foundation for using MKFs needs more explanation. How do these functions specifically relate to ENSO dynamics? A clearer link between the mathematical formulation and physical processes would enhance the paper's impact.

Response: Thanks for the reviewer's insightful comments. We have explained the theoretical foundation for using MKFs and stressed clearer link between the mathematical formulation and physical processes. Please refer to the revised version as follows:

Lines 138-144： A climate sequence test represents a set of state information about the climate system, wherein observed values at adjacent time points exhibit a degree of correlation. This correlation diminishes over time, reflecting a weakening of information memory. To capture the complex features of ENSO, we propose a multi-initial value-based MKFs approach that incorporates multiple backtracking observations to gather more original information about the system (Mazzola et al., 2010). Additionally, this approach employs a flexible evolutionary algorithm for parameter estimation.

4. The comparison with existing ENSO prediction methods is lacking. How does this approach compare to current state-of-the-art models in terms of accuracy and computational efficiency?

Response: Thank you for your comprehensive review and helpful comments. We added the comparison with existing state-of-the-art ENSO prediction methods. Please refer to the revised version as follows:

Lines 406-419: ENSO forecasts are critical, yet most operational forecasts typically do not extend beyond 6 months. Our results capture the system's nonlinear characteristics and reflect its trend changes. From January 1951 to July 2022, we successfully identified the SST change trend and forecasted ENSO trend changes up to 26 months in advance. For comparison, Petrova et al. (2024) employed a statistical unobserved dynamic components model to predict ENSO events 11 to 17 months in advance. Additionally, the extended nonlinear recharge oscillator model demonstrates proficient ENSO forecasts with lead times of up to 16–18 months (Zhao et al., 2024).

Ham et al. (2019) used a CNN model to achieve an effective 18-month advance prediction. Hu et al. (2021) developed the Res-CNN model, incorporating dropout technology and residual connection modules to enhance performance, stability, and prediction capability up to 20 months in advance. Overall, our results demonstrate a superior capability to capture ENSO trend changes, allowing us to predict ENSO trends up to 26 months in advance. However, the prediction accuracy for some anomalously high and low SST values needs improvement, which we aim to address in future work.

5. The discussion of results (Section 4) is quite brief. A more in-depth analysis of the implications of these findings for ENSO prediction and climate modeling, in general, would be valuable.

Response: Thank you for the detailed and thoughtful feedback. As suggested, we have added in-depth analysis of the implications of ENSO prediction. Please refer to:

Lines 384-397: ENSO has significantly influenced various geophysical processes and climate variability, often triggering extreme events and natural disasters. The ENSO events have notably impacted the extreme events such as heatwaves (Oueslati et al., 2017; Wei et al., 2020) and precipitation (Liu et al., 2024). Much of the skill in predicting temperature and precipitation extremes arises from the consistency of large-scale drivers. In some regions, most of this predictive skill is derived from ENSO, while changes in forcing can become sources of skill within a decade (Hanlon et al., 2013). Additionally, processes driving and influencing the frequency, duration, and intensity of long-duration extreme events include tropical SST forcing associated with ENSO (Sillmann et al., 2017). Our method can predict ENSO trends up to 26 months in advance, enhancing the understanding of decadal and long-term changes in ENSO variability and predictability, and aiding policymakers in responding more effectively to ENSO-related extreme events and disasters. Moreover, current linear inverse models cannot fully capture ENSO's nonlinear dynamics and seasonality (Alexander et al., 2022). Our method captures these nonlinear changes and trends, further enhancing and improving climate modeling.

6. The overall structure of the paper is disorganized, particularly in the Results section. The "Correlation coefficient" subsection should not be presented as a standalone section on par with other subsections. This structural issue makes it difficult to follow the logical flow of the analysis and results.

Response: We appreciate the reviewer's valuable insights Our research primarily focuses on analyzing the memory of SST fluctuations at the annual scale to establish a theoretical basis for constructing MKF. Using SST as an example, we analyze inertial memory through two methods: lagged transfer entropy and correlation coefficient. Generally, we aim to use the correlation coefficient results to validate the findings from lagged transfer entropy, thereby identifying the SST memory cycle. To simplify the modeling section, we have merged and modified this part based on your suggestion. Please refer to the section of 3.2 Inertial MKF.

7. There is a discrepancy between the conclusion and the results presented. Specifically, the conclusion states that the method "improves the precision and dependability of differential equation models," but this claim is not clearly demonstrated or quantified in the results section. The authors need to either provide evidence for this claim in the results or remove it from the conclusion.

Response: Thanks for the reviewer's insightful comments. We added relevant statements and provide evidence for this claim. Our method is more focused on predicting nonlinear changes and trends in ENSO and is able to predict the trend change in ENSO up to 26 months in advance. Please refer to:

Lines 407-419: Our results capture the system's nonlinear characteristics and reflect its trend changes. From January 1951 to July 2022, we successfully identified the SST change trend and forecasted ENSO trend changes up to 26 months in advance. For comparison, Petrova et al. (2024) employed a statistical unobserved dynamic components model to predict ENSO events 11 to 17 months in advance. Additionally, the extended nonlinear recharge oscillator model demonstrates proficient ENSO forecasts with lead times of up to 16–18 months (Zhao et al., 2024). Ham et al. (2019)

used a CNN model to achieve an effective 18-month advance prediction. Hu et al. (2021) enhanced model performance and stability by incorporating dropout technology and residual connection modules, leading to the development of the Res-CNN model, which refines prediction capability up to 20 months in advance. Overall, our results demonstrate a superior capability to capture ENSO trend changes, allowing us to predict ENSO trends up to 26 months in advance. However, the prediction accuracy for some anomalously high and low SST values needs improvement, which we aim to address in future work.

Minor comments:

1. Figure 1 is difficult to interpret. Consider adding more detailed captions and explanations in the text.

Response: Thank you for the detailed and thoughtful feedback. We have revised Figure 2 (Figure 1 of previous versions) and added the analysis. The most frequently used function of wavelet analysis in the field of climate change is to see if the time series has a certain periodicity. Please refer in the revised version. As follows:

Figure 2 (Figure 1 of previous versions):

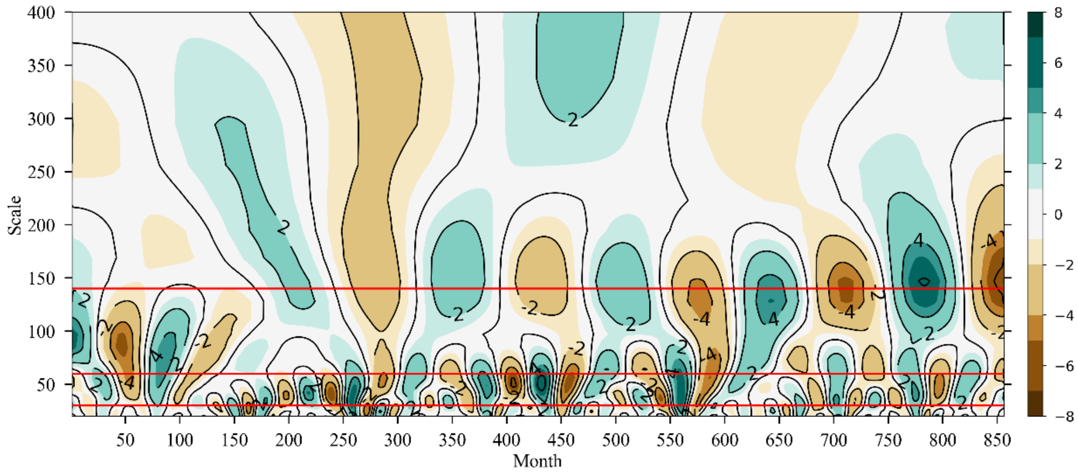

Figure 2: Time-frequency plot of wavelet transform for SST anomaly sequence. The red lines in the figure indicate scales of 140, 60, and 30 months.

The Figure 2 shows the changes of SST anomalies over time and frequency, revealing the characteristics of changes over different periods. By looking at color changes on different time and frequency scales, it is possible to understand the trend of SST changes over these time periods.

2. The methodology section could benefit from a flow chart or schematic diagram to illustrate the overall modeling approach.

Response: We appreciate the reviewer's valuable insights. We add Schematic diagram of inverse differential equation modeling of MKFs in the method section of this paper. Please refer in the revised version. As follows:

Figure 1:

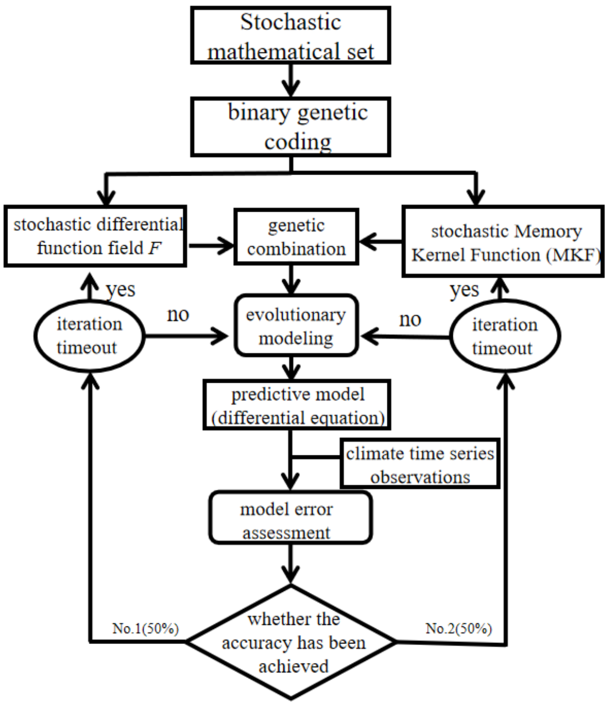

Figure 1: Schematic diagram of inverse differential equation modeling of MKFs

3. Some technical terms (e.g., "inertial MKF", and "ODE-MKF") are not well defined upon first use. Providing clear definitions of all key terms will be better.

Response: Thanks for the reviewer's insightful comments. "inertial MKF" should be understand in two part, inertial and MKF. Typically, complex systems exhibit memory characteristics through two main approaches: periodic oscillations, referred to as "periodic memory" and inertial motion, termed "inertial memory". The "inertial MKF" means the MKF which involves the memory and periodicity of the system. "ODE-MKF" stands for ordinary differential equation – memory kernel function, which combining with higher-frequency component and kernel function. Please refer in the revised version. As follows:

Line 246: which named as ordinary differential equation-MKF (ODE-MKF).

4. In line 151, there should be a space between "Nino 3.4" and "was".

Response: Your valuable feedback is greatly appreciated. We have revised and checked the full text for syntax problems. Please refer in the revised version. As follows:

Line 196: “Niño 3.4 was”

5. After Equation 4, a comma should be added, and there should not be a space before "where".

Response: Thanks for the reviewer’s insightful comments. We have modified the format of all the formulas in the paper. Please refer to the all the equations.

6. Equation 4 requires more explanation. The time unit of t and the time step should be specified.

Response: Thank you for your insightful comments and suggestions. Equation 4, derived from evolutionary modeling, describes a memory signal with a period of 60 months and incorporates memory information from the SST from 3 months prior. Based on the data time scale, SST [-3] refers to the SST information from 3 months prior to the current point SST [0]. Please refer in the revised version. As follows:

Lines 233-234: based on the data time scale, SST [-3] refers to the SST information from 3 months prior to the current point SST [0].

7. The labeling and presentation of Figures 2-3 are unclear and confusing, particularly the x-axis labels showing "Year/Month". This lack of clarity in crucial

figures hinders the reader's understanding of the results and makes it difficult to interpret the data presented.

Response: Thank you for your helpful and perceptive remarks. We have corrected the labels of the X-Y axes in the graph. Please refer in the revised version. As follows:

Figure 3 (Figure 2 of previous versions)

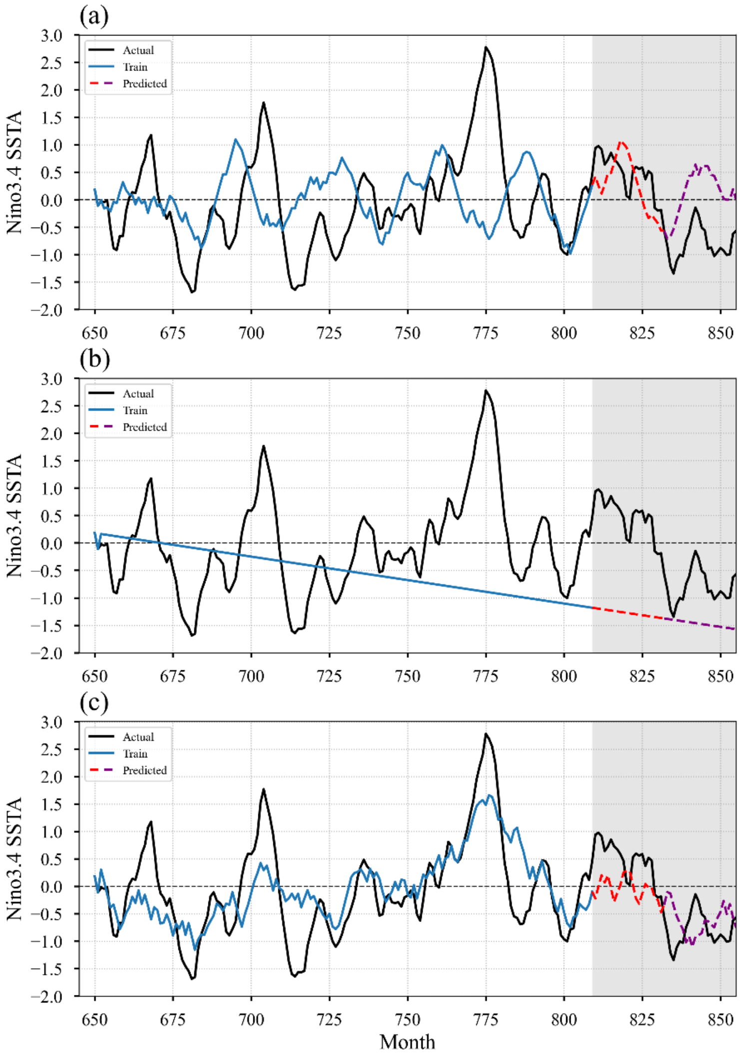

Figure 3: (a) Integral curve of the memory kernel function (MKF) (t, SST), (b) high-frequency component, and (c) integral curve of the SST modeling based on MKFs. The black line represents observed values, the blue line indicates fitting, and other colors represent the testing interval. The red curve in the figure represents the model selection process, while the purple curve shows the predicted Niño 3.4 SSTA based on the optimal model.

Figure 4 (Figure 3 of previous versions)

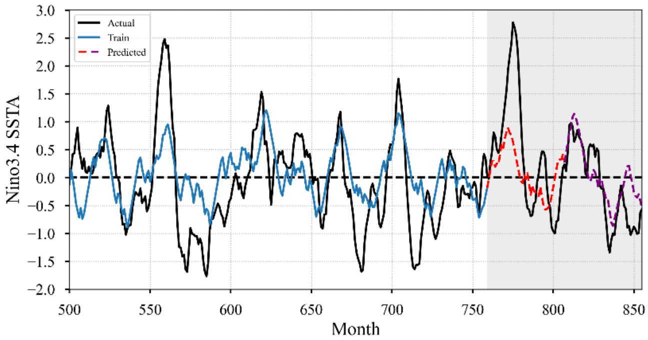

Figure 4: Same as figure 3c, but for the integral curve of ODE-MKF for the long sequence.

8. In Section 3.2, the units of time-related values are unclear throughout. It is not apparent what Δt represents or what time units are being used. This lack of clarity makes it difficult to interpret the results and understand the temporal scales of the analysis.

Response: Your thorough review and valuable suggestions are much appreciated. Since the data we use is a monthly scale Niño 3.4 index, while Δt represents the amount of change in time, Since the data we are using are exponents on a monthly scale, Δt represents the change in time, and Δt=1 represents the change in time in 1 month. Please refer in the revised version. As follows:

Line 301: For the third sequence group, the sampling interval was △t=t3-t2=t2-t1=1–20 (month).

9. "ff" in line 288 is not explained, is it a typo?

Response: Thanks for the reviewer's insightful comments. We are sorry for this, here is a typing error, we have corrected.

Reference:

Alexander, M. A., Shin, S.-I., and Battisti, D. S.: The Influence of the Trend, Basin Interactions, and Ocean Dynamics on Tropical Ocean Prediction, Geophys. Res. Lett., 49, e2021GL096120, https://doi.org/10.1029/2021GL096120, 2022.

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