Broad context and summary of the key results of the present study:

Warm and very dry air masses emanating from convective boundary layers over arid regions can be advected to remote regions, with a possibly profound impact on the weather there.
This phenomenon might be important in European summers, as the advection of such air masses may be associated with severe weather phenomena such as heatwaves and thunderstorms.
The authors base their study on previous work focusing on so-called elevated mixed layers (EML). By allowing for a gradual loss of the defining properties of EML during the advection towards Europe, the authors introduce a more generalized concept called atmospheric deserts (AD), which focuses mostly on the origin of the air mass. The robust detection of such air masses is based on a Lagrangian approach, for which a very high number forward trajectories are computed from the typical North African source region.
Using a case study of June 2022, the authors demonstrate the results of their Lagrangian detection algorithm. By applying k-means clustering, it is shown that only a certain fraction of the considered air masses retain their original well-mixed properties, while a majority is affected more or less strongly by diabatic processes along their way to Europe.
Finally, it is discussed in which ways AD exert an influence on the local weather in Europe. Within the centre of ADs, heatwave formation might be fostered due to convection suppression and build-up of heat due to the AD-induced capping inversion. In contrast, along the boundaries of ADs thunderstorm activity might be increased due to a combination of dynamical and thermodynamical mechanisms.

General comments:

The present study provides a well-motivated and intriguing investigation into the concept of atmospheric deserts. It builds upon existing literature about elevated mixed layers, but provides a novel perspective by focusing on Europe and using a Lagrangian framework. While the emphasis of this paper is on introducing the methods of AD detection, the inclusion of a case study also provides a first glimpse at the potential importance of such air masses for European summer weather.

Overall, I am pleased with the quality of this work. The paper is well-written and mostly easy to understand. For the most part, the methods used are sound and well-suited for the objectives.
Both the presentation and the interpretation of the results easily meet the standard of this journal. I am particularly happy about how the potential impacts of ADs on the local weather during the case study are really communicated carefully, always making clear that other processes than the presence of ADs might have been more important for heat build-up or thunderstorm initiation.

In conclusion, I am therefore positive that this work is suitable for publication in WCD.
Below I have a few comments and questions mostly regarding the methods, which can hopefully be addressed easily. If that is the case, this publication can be accepted after some minor or possibly major revision.

Specific comments:

L58-60:
I was wondering how strongly your results depend on the choice that a grid cell is already defined as an AD grid cell when only a single forward trajectory finds its way to the given grid point. I would assume

that the fact that the end point of one forward trajectory lies in a grid cell probably does not necessarily mean that the local air mass is then strongly characterized by the AD air mass properties.
Maybe this could be tested by initiating a certain number of backward trajectories from an AD grid point? If a majority of these backward trajectories can be traced back to the African source region, then it is safe to say that the local air mass is indeed strongly composed of the AD air.
In the end it is also simply a matter of perspective. In your article, you are strongly focused on the investigation of the air masses emanating from dry convective boundary layers, their properties and the changes they undergo during the transport. From that perspective it makes perfect sense to use forward trajectories. So I do not view this as a potential major problem for your article.

L68-75:
As far as I understand you want to include the nocturnal residual layer air masses for your Lagrangian approach. However, it is in my opinion not fully appropriate to use this kind of a smoothing algorithm because the residual layer might also be of similar height than the daytime convective boundary layer; it might also develop in a certain way over night depending on environmental parameters. The smoothing is unlikely to fully reflect this behavior. At the same time, you maybe would also want to exclude the lowermost layers, which will feature a stable nocturnal boundary layer forming under strong radiative cooling.
I would suggest to test whether your results would substantially change if you would only use day-time data for which the use of ERA5's boundary layer height might be suitable to estimate the height of the convective boundary layer.

L80-83:
Although I think that I understand how the clustering is performed, it would be beneficial to state a bit more clearly that you are using a multivariate clustering approach. I assume that the data points are clustered within a 11-dimensional space, in which each dimension reflects one of the 11 standardized variables? (such that any of the variables have the same weight)

L245-248:
At least in my opinion, this short paragraph appears to be of minor importance compared to earlier paragraphs. Looking at the plots, I also had problems to identify the air mass that is found between the near-surface air and the overlying AD air mass. Therefore I would suggest to rephrase or remove this short paragraph.

L275-280:
I would suggest rewording these three sentences. In their current form, they confused me somewhat because I had difficulty understanding which kind of diabatic processes predominate over other diabatic processes.

Minor comments:

L19:
In the parenthesis, the listing "economic, ecologic, …" should be removed or replaced by a formulation such as "can cause severe economic and ecologic damage"

L46 and elsewhere:
I would replace “thunderstorm eruption” and “thunderstorms erupt” by some other formulations such as “thunderstorm outbreak” or “thunderstorm initiation” and “thunderstorms are initiated..” or “thunderstorms occur..”

L144:
typo: 18 June 2002 should be 2022

L313:
I think it should say “western edge” instead of “eastern edge” if I am not mistaken

L346:
“Diabatic process**es**”