Dear Reviewer,

I attach in this document the answers to your comments. But first of all, I would like to thank you for spending time with the review of this manuscript. The answers are in blue and a new manuscript has been created to visualize the changes, with new contributions in red and deleted contributions in strikeout.

Major comments:

1. Study Design and Methodology

1. **Self-Consistent Scattering Model**:
    - The **description of the scattering model** (Section 3.1) lacks clarity, and the contribution of the authors versus existing models is unclear.
    - The equations used (1–3) need clearer documentation on their derivation. For example, the similarity of Equations (1) and (2) to Vidot et al. (2015) suggests that a citation or discussion is necessary.
    - **Equation (3)**: Clarify whether it has a unique physical solution under all conditions.

The description of self-consistent scattering model for cirrus clouds together with Figure 1 have been changed to make them clearer. The authors apologize because they were not aware of the (Vidot et al., 2015) study. Indeed, equations (1) and (2) correspond to (Vidot et al., 2015) equations (2) - (4). Therefore, (Vidot et al., 2015) has been quoted and these expressions have been omitted in this manuscript.

It has also been clarified that the Eq. 3 always has one unique physical solution and it is achieved with the addition operator. Therefore, the ± operator has been changed to +, in order to avoid confusion. The paragraph (177-179 lines) with the new changes is shown as follows:

"This formulation provides a unique physical solution and simplifies the IWC calculation based on extinction coefficient and cloud temperature, assuming no absorption, which is entirely reasonable because the working wavelength lies within the visible spectral range (Sun and Shine, 1994)."

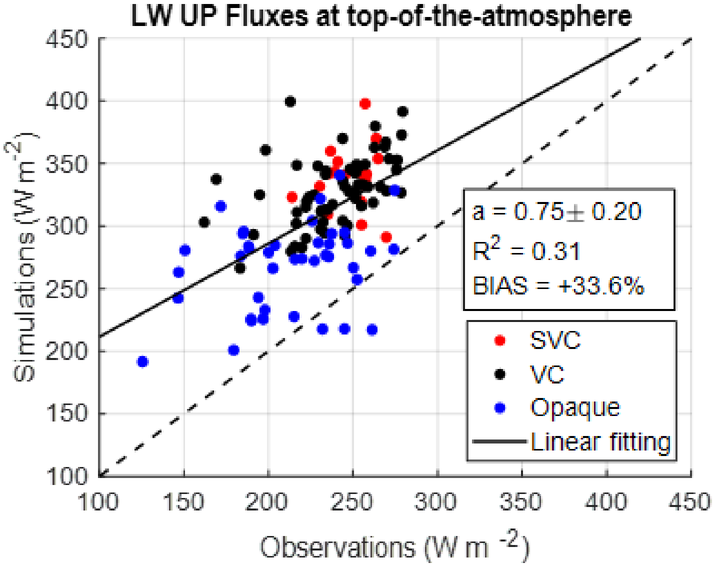
2. **Parameterization Choices**:
    - The use of **effective column extinction coefficient** to simplify radiative transfer modeling needs more justification. Explain how this affects IWC, single scattering albedo (SSA), and the asymmetry parameter.

To avoid confusion, the term effective column extinction coefficient has been changed to extinction coefficient in each vertical layer of the model. The resampling of the cloud extinction is necessary because after applying the two-way transmittance method, a vertical extinction profile with a vertical resolution of 75 m is obtained and needs to be rescaled to the model vertical resolution of 1 km.

The vertical resampling of the cirrus cloud extinction in the self-consistent scattering model could sometimes lead to a cloud layer having a low ice water content and consequently low optical scattering values, as mentioned in lines 298-305.

3. **Surface Properties**:
    - The use of **monthly averaged surface temperatures** from CERES could introduce biases in longwave flux calculations, particularly given daily variations in land temperatures. A discussion of this limitation and exploration of case-specific surface temperature/emissivity values are needed.

Yes, we used monthly surface temperatures to compare with CERES observations. Although the albedo and temperature variations are small, you are right that they induce a bias in the calculation of radiative fluxes in the longwave spectrum. To address this, we have downloaded new data on upward radiative fluxes at top-of-the-atmosphere, along with the surface emissivity, surface temperature and cloud mask from the NOAA-20 satellite to provide a temporal coincidence between satellite and ground-based observations over Barcelona. We then incorporated instantaneous values for surface albedo and temperature into the simulations, which were rerun accordingly. The results obtained from the comparison of the upward radiative fluxes in the longwave spectrum at top-of-the-atmosphere are shown in the figure below.



As you can see, the BIAS has decreased to +33.6% and the points exhibit a more linear trend. Despite the drop in BIAS, the current value is still considerable. As discussed in the manuscript, NOAA-20 may discern a different atmospheric scene, even covering part of the Mediterranean Sea. Further analysis of the cloud mask reveals that the 14% of the cases analyzed have more than 90% of clear sky footprint area, highlighting the complexity of the comparison between simulations and satellite observations at the top-of-the-atmosphere. The full discussion can be found in lines 326-341.

Given the significant improvement in validation with NOAA-20 data, we proceeded to rerun all the simulations to continue analyzing the direct radiative effect of cirrus clouds. The previous results in the manuscript have been replaced with those obtained with these new simulations.

4. **Limited Validation of IWC**:
    - While lidar extinction profiles are used to derive IWC, the validation of these IWC values against independent datasets is limited. For instance, thin cirrus clouds may have underestimated IWC, affecting radiative forcing results. More references and few lines of discussion are needed.

Figure 3 shows the mean IWC distribution for each cirrus cloud scene and lines 291-296 compare the values with the literature. Further references have been added based on your suggestion. The paragraph is shown as follows:

"In Fig. 3 one observes that cirrus clouds have an IWC between 0.03 and 30 mg m^{-3}, being characteristic of mid-latitude cirrus clouds (Korolev et al., 2001; Field et al., 2005, 2006; Schiller et al., 2008; Baran et al, 2011b; Sourdeval, 2012; Kramer et al.,2016, 2020). Where the average of IWC is ~5 mg m^{-3}, being a value close to 3 mg m^{-3}, which is the central value of the mid-latitude ice cloud distributions obtained by (Sourdeval, 2012) and the mean value of IWC for temperatures between 210 and 235K found in (Kramer et al., 2016). A slightly higher measured IWC value of 7 mg m^{-3} was found by (Korolev et al., 2001) for cirrus clouds whose temperature ranged from 233 to 243K."

2. Results and Data Analysis

1. **Large Bias in Longwave Fluxes**:
   - The **+51% bias** in simulated longwave fluxes at TOA compared to CERES observations is significant. While collocation issues are suggested as the cause, this explanation is not robust. Other potential causes, such as surface temperature/emissivity inaccuracies or errors in IWC parameterization, should be explored and stated in the text.

As mentioned in question 3, with the new NOAA-20 satellite instantaneous surface temperature and albedo data, it has been possible to reduce the BIAS of the longwave upward fluxes simulated and observed to +33.6%. In addition, as discussed in the lines 333-341, other possible causes of error may be involved in having such a BIAS, such as the satellite observing a different atmospheric scene or covering part of the Mediterranean Sea. The entire paragraph is shown below.

"In our case, the large BIAS = +33.6 % obtained could be due to the spatial resolution of the observed measurements taken, which may cover part of the Mediterranean Sea. In addition, the cloud mask associated with each observation indicates that in 14% of the cases it has more than 90% of clear sky footprint area. As demonstrated by (Gil-Díaz et al., 2024) most the cirrus clouds are visible and their horizontal extension is smaller than the cirrus clouds that form at higher altitudes (well-known as sub-visible cirrus clouds) (Kramer et al., 2020). This makes them more challenging to detect from top-of-the-atmosphere. Hence, the comparison of simulated radiative fluxes and NOAA-20 observations is not as trivial and conclusive as with SolRad-Net observations, since the NOAA-20 satellite can observe a slightly different atmospheric situation, as mentioned above. Not to mention the limitations of the 1-D radiative transfer model DISORT to represent an irregular composition of broken and/or overlapping clouds that the NOAA-20 satellite could observe."

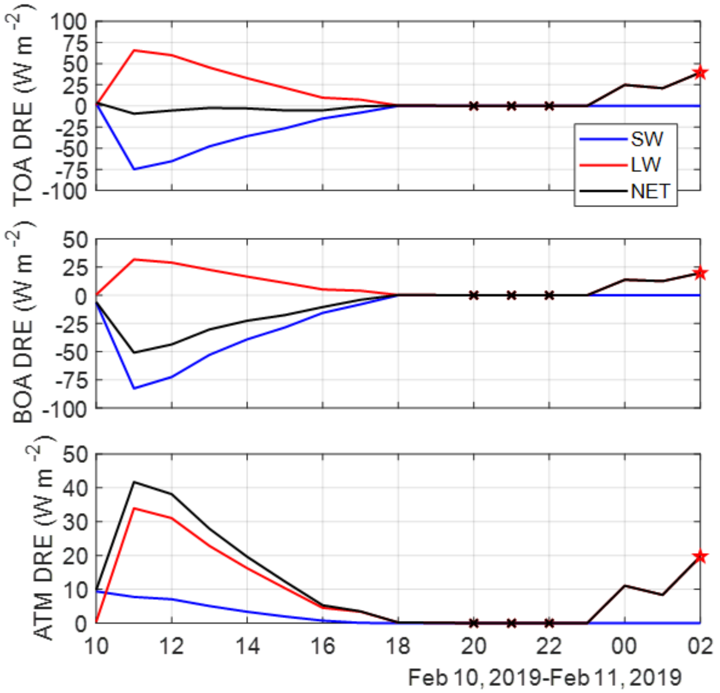
2. **Daytime TOA Net Radiative Forcing**:
   - The finding that **net daytime TOA forcing remains positive for COD < 1.2** (Section 4.4) contrasts with prior studies. For example, Campbell et al. (2016) report a transition to negative forcing for COD > 0.7 (for 30sr solution). A broader discussion on the discrepancies from Campbell et al. (2016) is required.

Right, this discrepancy has been highlighted in this paper and discussed in lines 381-389. The full paragraph is shown below:

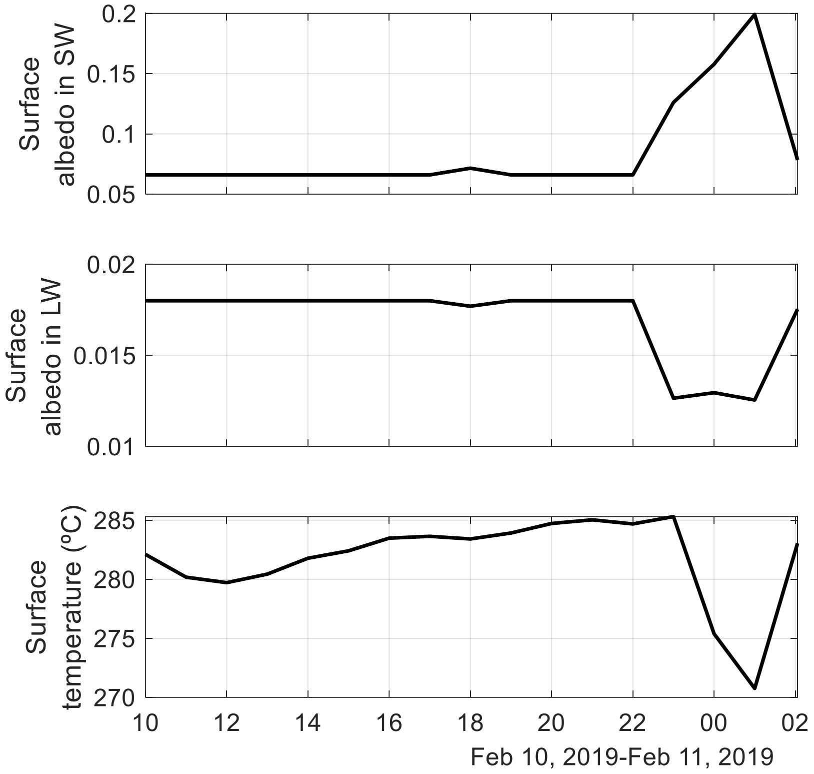
"At daytime, at TOA, the net direct radiative effect remains positive for all cirrus clouds, dominating the positive longwave component. This effect has been observed in (Campbell et al., 2016) for COD up to approximately 0.6. For higher COD values, (Campbell et al., 2016) reports a negative NET TOA DRE. In this study, a decreasing trend in NET TOA DRE is observed from COD values of 0.5, although no negative values are obtained. Additionally, the LW NET TOA DRE component grows faster than the one reported by (Campbell et al., 2016), suggesting that negative values of NET TOA DRE could occur for cirrus clouds with higher COD than those found in (Campbell et al., 2016). This discrepancy may be due to the higher surface emissivity and temperature values considered in the present work. Further measurements of NET TOA DRE for cirrus clouds with higher COD are needed to confirm the decreasing trend."

3. **Back-Trajectory Analysis**:
   - The investigation of cirrus cloud radiative properties using the CLaMS-Ice model and CALIPSO is interesting but underdeveloped. The daytime net TOA forcing close to zero over oceans raises questions about the surface parameters used.

Due to improvements in the evaluation of radiative flux in the longwave spectrum at top-of-the-atmosphere, new shortwave surface albedo, longwave emissivity and surface temperature data from the NOAA-20 satellite have been downloaded. The simulations were re-run using instantaneous surface property values. The figure below shows the new results.



As can be seen in the figure above, the direct radiative effect of the cirrus cloud along its back-trajectory is very similar but slightly lower. The NET TOA DRE over the Atlantic Ocean remains very small, practically negligible. The surface properties considered in the new simulations are shown in the figure below.



3. Writing Quality

1. **Copied Text**:
    - Sections 2 and 3 contain text copied from external sources. These sections should be rephrased in the authors' own words or explicitly quoted with citations.

Thank you for your feedback. We have carefully revised Sections 2 and 3, rewriting the relevant paragraphs to ensure they are properly paraphrased. We think these changes improve the overall clarity of the manuscript.

2. **Clarity and Conciseness**:
    - The discussion sections tend to be vague and require more **precise explanations**. For example, the reasoning behind discrepancies in longwave fluxes **(Fig. 4)** and **TOA net forcing (Fig. 6)** should be better supported by sensitivity studies or external validation.

Thank you for your feedback. We have carefully revised the explanations of the discrepancies in the evaluation of the longwave radiative fluxes observed and simulated at TOA and the positive direct radiative effect at TOA for all COD.

4. Literature Review and Comparison

1. **Novelty Discussion**:

    - The introduction of the inverse Baran model is claimed as novel. However, similar approaches have been explored in other studies. The manuscript should more clearly articulate its novelty compared to prior work.

In order to improve this part of the methodology, (Vidot et al., 2015) has been cited and the equations of the self-scattering model for cirrus clouds have been omitted. I have also tried to highlight the authors' contribution to the developed methodology based on the self-scattering model for cirrus clouds.