Review of: *Assessing the Material Coherence of Mesoscale Eddies as described from In Situ Data*, by Barabinot et al.

This manuscript uses in situ data from eight oceanographic cruises that cover seven different regions to assess the coherence of mesoscale ocean eddies. An impressive amount of work has been done, particularly insightful is the analysis on subsurface signatures of ocean eddies, and how these water masses of distinct thermohaline properties may propagate through the ocean without a surface signature. Below, I provide a list of major and particular concerns I have with the manuscript, as well as a list of minor typos/grammar. I don't think the manuscript, in its current form, is ready for acceptance.

Major comment - Material coherence, and materially coherent eddies:
The authors provide a broad overview of the literature for 'coherent eddies', with a nice introduction on the history of the field of understanding/quantifying eddy coherence. However, when it comes to material coherence, and materially coherent eddies, they've missed quite a number of relevant references from the Lagrangian coherent structures community.

Specifically, the discussion around how 'kinematic coherence' (KC) is reference frame dependent (and hence, not objective), is missing another key point, that eddies defined by Eulerian (frame-dependent) methods are not materially coherent. Quite a number of studies have shown that ocean eddies, identified from the Eulerian perspective (for e.g., Okubo-Weiss parameter, SSH anomaly), leak material across their identified boundaries relatively quickly (Andrade-Canto et al. 2020, Beron-Vera et al. 2013, Haller et al. 2013, Liu et al. 2019, Serra et al. 2017, Denes et al. 2022). This is a major drawback of using Eulerian approaches for quantifying material coherence, and mass transport by eddies.

I also found the definition of material coherence in the manuscript to be rather qualitative, that a 'vortex ceases to be coherent when it loses its trapped water mass'. How does one determine the point at which a vortex loses its trapped water mass? Recent Lagrangian analyses have considered the coherence of ocean eddies, specifically Abernathey et al. (2018), and Wang et al. (2015), found that only small 'coherent inner cores' of ocean eddies existed over long time-periods, while Denes et al. (2022) found ocean eddies may consists of coherent inner cores, and quasi/semi-coherent outer rings (and question the notion that ocean eddies have precise boundaries). This is briefly touched on by the authors on lines 70-71, but I think more is needed here.

A particular concept raised in 'material coherence' theory is that the coherence occurs over a finite window of time. As described in the manuscript, ocean eddies tend to have lifetimes of months up to years, however, with entrainment and flushing of water, an ocean eddy is not materially coherent over its entire lifetime. To assess the material coherence of an ocean eddy from a single snapshot in time seems rather suspect to me. As the authors note, it is 'impossible to perform temporal studies with in situ data', however material coherence has a time-evolution component that I think is quite important when assessing mesoscale ocean eddies.

I suggest the authors consider an alternate motivation for the paper. I can see the techniques described and analysis developed as nice ways to 'Reconstruct 3D ocean eddies from 2D in situ transects'.

Particular comments:

1. Methodology

I particularly liked the descriptive methodology of the paper, the authors explain and derive their methodology nicely. However, at times I was a little lost. I think the manuscript would benefit from an additional paragraph or two which may provide an overview of the methodology. For example, section 3 describes four identification methods, but no introductory sentence/paragraph which describes why these identification methods are used let alone useful. What alternative methods exist, and why are these four chosen?

2. 'Acronymisation'

I felt there were a few too many acronyms, but in cases some things could have been 'acronymised'. For example, at one point the authors use 'AC' and 'C', to describe (from what I understand) anticyclonic and cyclonic eddies (line 268, or 403, but not described anywhere in text). Especially when, in Table 3, 'C' is used to describe 'Complete' eddies. In contrast, to be in line with 'Kinematic coherence' and 'Material coherence' (KC and MC), I think the authors can use 'TC' for 'Thermohaline coherence', which would help with the flow of the discussion in section 5.2, where they describe 'thermohalinely coherent cores', for example.

3. Line 32, the term 'eddy boundary' needs to be placed in context.

Eddy boundaries are method dependent, and some literature argues that eddies do not have a sharp boundary. I think this sentence could be qualified further.

4. Line 64-66 on MC theory

Some material coherence theory is derived from 'minimal mixing' criteria, specifically, that a boundary of a materially coherent structure is undistorted/unfilamented over a finite-time window, such that diffusive mixing across the boundary is minimised. This sentence can be qualified further.

5. Line 67-68 on number of MC eddies vs. KC eddies.

I wasn't sure what the purpose of this sentence is. As the authors suggest, material coherence provides a rather rigorous definition for ocean eddies, and one should expect that more structures will appear from a less restrictive approach than a more restrictive approach. Is this considered a downside of MC approaches, or just a statement of fact?

6. Describing the trapped water as 'heterogeneous' (e.g. line 85, 306)

I found this a little confusing, as the trapped water mass is typically homogeneous within the eddy core, but distinct from the surrounding water mass, as it is 'characteristic of the region of formation of that eddy'. I would think of this water mass as 'distinct'.

7. Line 91 – providing a first answer to the question of the 3D material coherence of eddies

I'm not entirely sure what the question actually is. That 3D material coherence of eddies can be assessed using water masses and the PV approach? Certainly, 3D material coherence of eddies has been considered in the past, see Froyland et al. (2012).

8. Definition of the center of an eddy

The authors make contrasting statements '… is reached at the eddy center (region where the velocity tends to zero)' on line 100, and 'The center of the eddy is defined as the point where the mean tangential velocity is maximum', and '… cross the exact eddy center, defined as the location of the zero velocity' on lines 344-345. To me, these three statements are inconsistent.

Additionally, in the first instance the eddy center is a region, in the second it is either a point or a curve, and in the third it a point (possibly a region?).

9. Line 244 'These values are small compared to the first-order terms'

I would disagree, in the horizontal case they are of the same order of magnitude. They are smaller, but not negligibly small. Is this not important?

10. Lines 300-301 and lines 441-442 – Motivation

The motivation for the analysis described in both these sections would be great to see in the introduction. In both cases, I did not know this would form part of the analysis until I reached these sentences.

11. Line 361 – 'To reduce this uncertainty, …'

By computing volumes for a subset of eddies, you aren't exactly reducing uncertainty. Rather, you are only showing results for eddies where the uncertainty isn't too large. I would rephrase this.

12. Layout of Figures 4 and 5

Use of figure labelling would be really useful for both of these figures, using a), b) etc. Also, the change in orientation for the bottom panels of Figure 4 I found rather jarring, is there a reason to have these laid out vertically rather than horizontally like the top panel of Figure 4 and all of Figure 5? Lastly, I suggest a second diverging colorbar for the salinity anomaly (different to the thermal anomaly colorbar).

13. Lines 458-459 – 'Lagrangian studies suggest that the ability of eddies to trap a water mass is a consequence of closed trajectories'.

I'm not certain what you mean by this. What is a closed trajectory in this instance? A citation would be very useful.

14. Figure 7
    a. Plotted velocity vectors - I find these velocity vectors distract from the plots, especially in panel g), where I feel the figure is too busy to see what is really going on.
    b. Panel title (a) – 'fev', and panel title (i) – missing date (same on Figure 8)

15. Figure 9 colorbar and panel titles

I would change the colorbar to something that contrasts better with the isopycnal, EPV, and salinity curves (being black and blue), otherwise it is very hard to appreciate these curves. I also found the panel titles to be rather confusing and could do with an update.

16. Line 519-520 – 'The reconstructed volume associated with the thermal anomaly is the most convex of all shapes'.

I'm not sure what you mean by this, nor am I sure if I agree with the statement. In fact, some of the shapes plotted in Figure 10 aren't convex (panel (c) for example).

17. Figure 10 – panel titles

If you could add titles to each of the panels (not just in the figure caption), that would really help guide the reader. Also, given the circularity constraint, I think each of these plots should have equal x-y aspects, figure (c) looks out of shape due to the non-equal aspect ratio.

18. Lines 525 – 533 regarding Figures 11 and 12

The authors say 'This figure refers to the ellipses (E1) mentioned earlier' and later '(ellipses E2)', but the only mention I could find to E1 or E2 were in the caption of figure 9. Where do these numbers come from, and how were they computed? Secondly, what is the purpose of an elliptical reconstruction? That was not clear to me.

19. Line 569-570 – 'This paper presents an evaluation of the thermohaline coherence of mesoscale eddies based on in situ data collected during several cruises…'

While I agree with this statement, the title of the manuscript is on assessing 'material coherence'. While I don't think the manuscript has addressed the material coherence of ocean eddies, this statement in the conclusion is at odds with the title and abstract of the manuscript.

Typos/grammar/missing info:

Line 58 – there is an erroneous space between 'velocity fields' and the comma.
Line 128 "boundary current systems off Labrador" - Do you mean Labrador sea?
Line 196 – 'Localization' should be 'Localisation'.
Lines 105, 192, 438 – 'confront' could be 'compared'.
Line 195-197 – Could you provide a citation to the data described?
Line 233, Fig 4 caption, Fig 5 caption – you use 'gyre', do you mean eddy?
Line 280 – 'the EPV id the ocean at *rest*', what do you mean by 'rest'?
Equation 7 – f_0 is defined, but f is not defined.
Line 321 – what are \overbar{b} and b' ?
Line 327 – 'adimensionalizing' should be 'nondimensionalising'.
Line 364 – 'analytically', I wasn't sure what you meant by this.
Line 366 – 'descrided' should be 'described'.
Line 564 – 'this' should be 'thus'.
Line 582 – 'This result corroborates the findings of previous studies…' citations are needed for this statement.

References:

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