Referee 4

We would like to thank the referee for his/her comments. Please find our point-to-point response below.

Referee's Report on "Assessing the Material Coherence of Mesoscale Eddies as described from In Situ Data" by Barabinot et al. submitted to Ocean Science explored the material coherence of mesoscale eddies using in situ data.

From my point of view, this manuscript is not well organized. The key points/findings are not clearly shown in the abstract and conclusion sections. This introduction is very long, but the scientific questions are not well introduced/addressed, and I do not think the authors need some many short paragraphs in the introduction section. The authors also use two sections (Section 3 and section 4) to introduce the methods, it may be better to incorporate them into one section. The results shown here are not very convincing to me. There are no discussion section in this manuscript. Without discussions, the reader would not know the limitations and implications of the manuscript. Last but not the least, there are many typos and nonstandard writing. I suggest the authors carefully double check the writing, calculation and statements.

We would like to thank the reviewer for his/her thorough review. We are grateful for the effort invested in evaluating our work and for highlighting important points for consideration. While we value these insights, we respectfully disagree with some of the major concerns raised, as addressed in detail in our responses below.

Main concerns:
1. I do not think the authors need to get the climatology temperature and salinity profiles from two different datasets (ARGO and WOA2023). How are the temperature and salinity anomalies calculated? Are the seasonal cycle removed?

We thank the reviewer for their comment and appreciate the opportunity to clarify this point. The methodology for computing thermohaline anomalies on isopycnals is detailed in Section 3.1. For a given eddy, we aimed to construct a local climatological average. To achieve this, we applied the methodology described in Laxenaire et al. (2019), which utilizes ARGO float profiles and has been shown to be effective for studying eddy dynamics.

This approach calculates a climatological average centered on the position of the eddy during the specific month it was sampled. The climatological averages of temperature and salinity on geopotential levels were derived from ARGO float profiles collected over a 20-year period within a small area surrounding the sampled eddy. These averages are assumed to represent the baseline state of the ocean at the specific location and time of sampling, allowing for the construction of precise, localized, and temporally consistent climatological averages.

Once anomalies were computed, we imposed a threshold on the anomaly values using WOA2023, which provides robust standard deviation estimates. The use of WOA2023 was motivated by its tabulated standard deviations, which were essential for validating our climatology derived from Laxenaire et al. (2019). WOA2023 served as a reference for this validation, and its standard deviations values were used to compare against the computed anomalies.

To address the reviewer's final question, we did not remove the seasonal cycle. The climatological average of temperature and salinity on geopotential levels was calculated using ARGO float profiles collected over 20 years within a small area surrounding the sampled eddy. Importantly, this average incorporates profiles measured during the same month the eddy was sampled. Therefore, the seasonal cycle is inherently accounted for in this methodology and does not need to be explicitly removed.

2. In the vertical, eddies will tilt with depth. In horizontal, most eddies are not circular or elliptical in shape. The 3D reconstructions of eddies shown in figures 10-12 are not convincing.

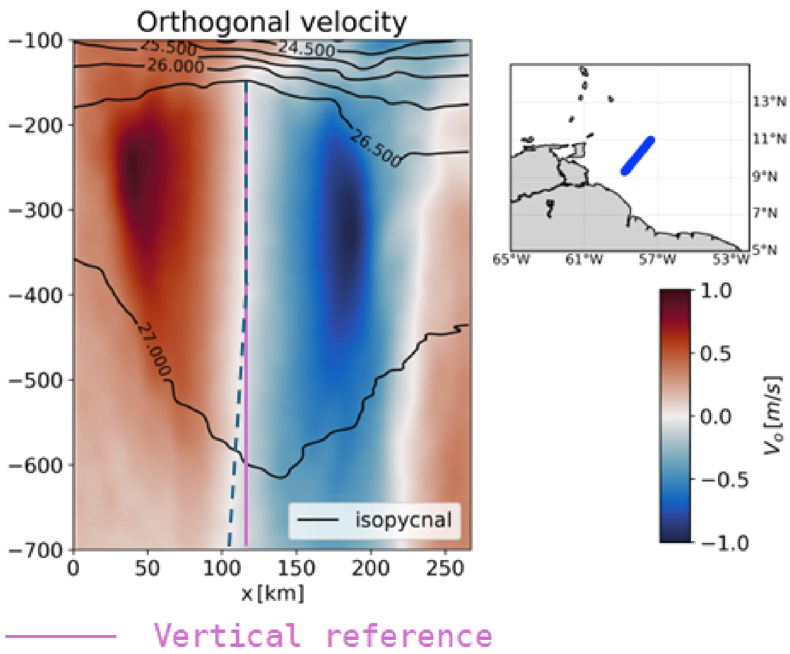
We thank the reviewer for his/her comment and appreciate the opportunity to clarify this point. As demonstrated in the supplementary materials, the centers of the eddies exhibit minimal tilting with depth, at least in 2D vertical sections. The tilting of eddy centers is typically limited to a few kilometers, which is negligible compared to their overall radius.

To support this observation, we have plotted the velocity fields of six representative eddies below. In each case, a vertical purple line has been added to highlight the vertical alignment of the eddy center (marked in dashed blue line). This vertical reference is built such that the shallower position of the eddy center belongs to this straight line. For each eddy, we provide the maximum deviation of the eddy center with respect to this vertical reference and express it as a percentage of the eddy maximum radius. Results are presented below. It is worth noting that the maximum deviation is an integer as data have been interpolated on a horizontal grid of 1 km resolution. Results show that the deviation of eddy centers from the vertical does not exceed 10% of the eddy maximum radius.

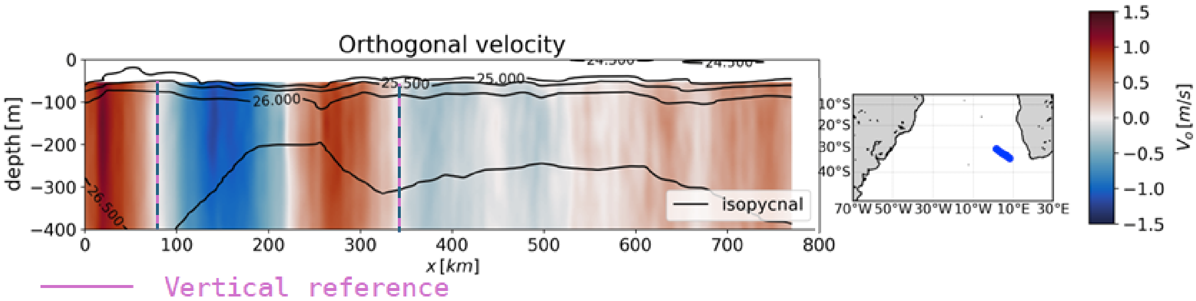
It is also worth noting that our methodology described in Figure 3 and in part 4.2 does take into account the eddy tilting. Here is our paragraph in the article: *"In summary, the approach consists of three steps. First, a criterion (the outermost closed contour of a given size) is chosen to delimit the materially coherent eddy core from its surroundings on the 2D vertical slice. Then, compute the position of the apparent eddy center as the location where the orthogonal velocity V_o is zero and the eddy radius $L(z)$ associated with the selected criterion. Finally, calculate the approximate volume as a sum of elementary cylinders."* As eddy centers are almost perfectly vertical in their thermohaline coherent core (see results below), the tilting appears negligible in Figure 10, 11 and 12. However, as mentioned, the tilting is well taken into account when reconstructing eddies.

Additionally, based on geophysical fluid dynamics and observational evidence discussed throughout Section 4, eddies are generally axially symmetric on average (Chaigneau et al., 2009; Chelton et al., 2011). They often exhibit mode-1 and mode-2 deformations. Furthermore, as written in the manuscript and corroborated by Chen et al. (2019), an elliptical shape best represents the geometry of eddies.

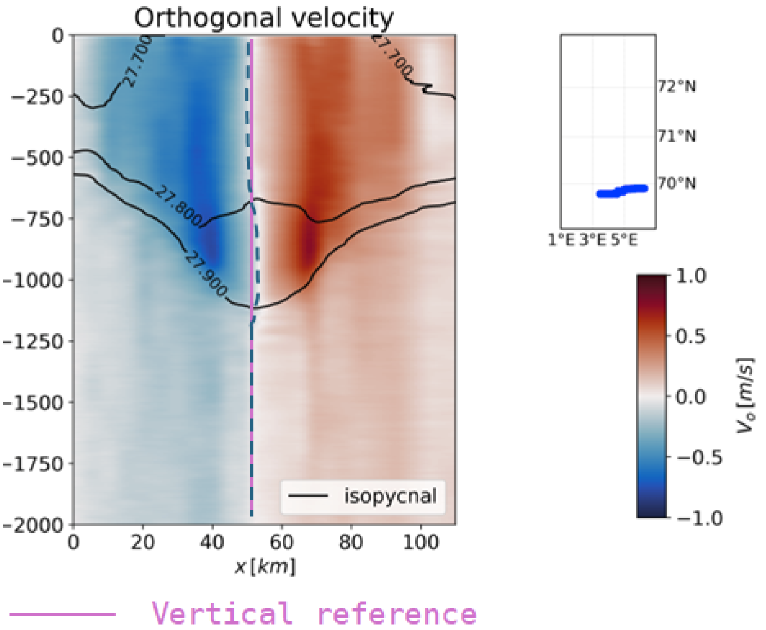
Subsurface AE sampled during EUREC4A-OA experiment: The maximum velocity radius is provided in Table 3 and equals 71 km. Above -400 m, the deviation of the center is inferior to 2 km which represents 2.8% of the eddy maximum radius. Below -400 m, the slope of the deviation is on average 0.04 km/m. Therefore, the maximum deviation of the center is reached at -700 m and its value is 12 km. Considering the full 2D vertical section, the maximum deviation corresponds to 16.9% considering the entire structure. However, what is important is the coherent core. Reducing our analysis to the thermohaline coherent core, the maximum deviation is reached at -630 m and its value is 7 km which represents 9.8% of the eddy maximum radius.



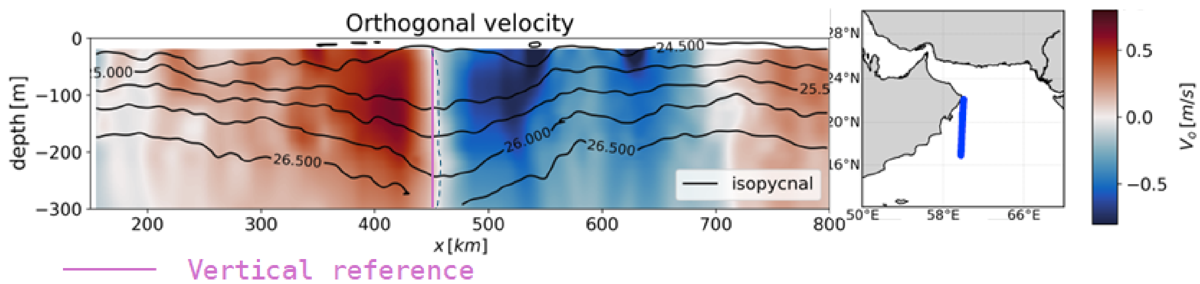
2 AEs sampled during M124 experiment: For these 3 eddies, as the resolution of ADCP is 32m on the vertical (see in Table 2.), eddies appear more barotropic. In these 3 cases, the maximum deviations of the center are respectively 1 and 2 km. These deviations are almost invisible and are reached around -60 m depth. The eddy maximum radii are respectively 58 and 55 km. The maximum deviations of center are respectively 1.7% and 3.6% of eddies maximum radius.



Subsurface AE sampled during KB2016 experiment: In that case, the maximum deviation of the center is 2 km. The maximum velocity radius is 15 km. The maximum deviation of the center represents 13.3% considering the entire section. In the thermohaline coherent core (here between isopycnals 27.8 kg/m^3 and 27.9 kg/m^3), the center is perfectly vertical (see the blue dashed line). Deviations of the center only occur when crossing isopycnals 27.8 kg/m^3 and 27.9 kg/m^3.



<u>AE sampled during Physindien 2011: </u>Here, the maximum deviation of the center is 6 km. The maximum velocity radius is 95 km. The maximum deviation of the center thus represents 6.3 % of the eddy maximum radius. Moreover, as shown in the following figure, the main deviation occurs at the surface. Below -50 m depth, the eddy center remains quite vertical and, taking this depth level as a new reference, the maximum deviation of the center is therefore 1 km and represents 1.1% of the eddy maximum velocity radius.



3. The authors argue that surface mesoscale eddies detected from satellite altimetry data do not match with subsurface eddies from in situ data. There are many factors that may cause this, such as the TOEddies algorithm, the methods and data used to extract the subsurface temperature/salinity anomalies here. The authors should double this before they draw any conclusions.

We thank the reviewer for their comment and appreciate the opportunity to clarify this point. Satellite altimetry measures the dynamic height of the ocean, a variable representing the integrated vertical thermohaline structure. Specifically, the absolute dynamic topography (ADT) on which the TOEddies algorithm is based is derived from satellite altimetry. ADT reflects the integral properties of the water column, which are influenced by the local vertical thermohaline structure (water masses). Intense mesoscale eddies imprint their signature on this property, whether they move at the ocean surface or below it.

When the upper ocean stratification is relatively weak (as is often the case at mid- and high-latitudes), deep subsurface eddies—such as Mediterranean Outflow Eddies (Meddies), which are typically centered at depths of 600–1000 m (Ienna et al., 2022; Ciani et al., 2017)—can still influence ADT. However, the surface geostrophic velocities derived from this ADT footprint do not necessarily represent the core velocities of the eddies creating the ADT anomaly. For example, the eddy core may begin at 200 m depth or deeper, as discussed by Laxenaire et al. (2019, 2020). Meddies are a good illustration of this: while their presence impacts the ADT, the ADT signal is also affected by the thermohaline stratification of the water column above them. It is also worth noting that ADT can be seen as a streamfunction and is thus not conserved contrary to the Potential Vorticity (PV) which is a Lagrangian invariant. This is the reason why the streamfunction is often not enough to locate an eddy.

In our article, the main conclusion is that the material core of eddies, identified by thermohaline anomalies on isopycnals, often lies below the ocean surface and thus cannot be fully captured by traditional altimetry. Specifically, applying Lagrangian methods to satellite data is insufficient to detect subsurface trapped waters. Regarding the velocity field, however, altimetry matches in situ measurements reasonably well at mid-latitudes (see Figure 9 in our manuscript).

We acknowledge the reviewer's suggestion to double-check our methodology and would like to respectfully emphasize the following points:

- The **TOEddies algorithm** has been demonstrated to effectively detect mesoscale eddies in comparison to traditional products, as shown in numerous studies (Laxenaire et al., 2018, 2019, 2020; Manta et al., 2021; Chen et al., 2022; Subirade et al., 2023). The TOEddies Atlas is now available as open source:
  Laxenaire, R., Guez, L., Chaigneau, A., Isic, M., Ioannou, A., and Speich, S.: TOEddies Global Mesoscale Eddy Atlas Colocated with Argo Float Profiles, https://doi.org/10.17882/102877 , 2024.

  and it is described (and compared with other atlases) in this new published manuscript:
  Ioannou, A.; Guez, L.; Laxenaire, R.; Speich, S. Global Assessment of Mesoscale Eddies with TOEddies: Comparison Between Multiple Datasets and Colocation with

In Situ Measurements. Remote Sens. ***2024****, 16, 4336.*
[*https://doi.org/10.3390/rs16224336*](https://doi.org/10.3390/rs16224336)

- The use of **thermohaline anomalies** has also proven highly effective for analyzing eddy dynamics and trapping, given that diffusion is negligible at the mesoscale (Robinson et al., 1983; Paillet et al., 2002; Carton et al., 2001, 2010).

We hope this clarifies the robustness of our approach and the validity of our conclusions.

Monir comments
1. Section 3.2 1.10-6°C.m-1 , 0.6.10-4°C.m-1 , 7.6.10-6°C.m-1 and 2.5.10-2°C.m-1 are incorrect.

We thank the reviewer for his/her comment. However, the values mentioned are not present in the current version of our article. Section 3.2 does not include these values. It is possible that the reviewer may have referred to a previous version of the manuscript.

2. Each panel should be labeled as in figures 4-5. The top right colorbar for the salinity seems incorrect to me.

We thank the reviewer for his/her comment. We respectfully request the reviewer to clarify their remark, as we are unsure which figure is being referred to. In the current version of the manuscript, all figure panels are labeled with letters.

Refs:

Carton, X. (2001). Hydrodynamical modeling of oceanic vortices. *Surveys in Geophysics*, *22*(3), 179-263.

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Chaigneau, A., Eldin, G., & Dewitte, B. (2009). Eddy activity in the four major upwelling systems from satellite altimetry (1992–2007). *Progress in Oceanography*, *83*(1-4), 117-123.

Chelton, D. B., Schlax, M. G., & Samelson, R. M. (2011). Global observations of nonlinear mesoscale eddies. *Progress in oceanography*, *91*(2), 167-216.

Chen, G., Han, G., & Yang, X. (2019). On the intrinsic shape of oceanic eddies derived from satellite altimetry. *Remote Sensing of Environment*, *228*, 75-89.

Chen, Y., Speich, S., & Laxenaire, R. (2022). Formation and transport of the South Atlantic subtropical mode water in Eddy‐Permitting observations. *Journal of Geophysical Research: Oceans*, *127*(1), e2021JC017767.

Ciani, D., Carton, X., Aguiar, A. B., Peliz, A., Bashmachnikov, I., Ienna, F., ... & Santoleri, R. (2017). Surface signature of Mediterranean water eddies in a long-term high-resolution simulation. *Deep Sea Research Part I: Oceanographic Research Papers*, *130*, 12-29.

Ienna, F., Bashmachnikov, I., & Dias, J. (2022). Meddies and their sea surface expressions: Observations and theory. *Journal of Physical Oceanography*, *52*(11), 2643-2656.

Ioannou, A.; Guez, L.; Laxenaire, R.; Speich, S. Global Assessment of Mesoscale Eddies with TOEddies: Comparison Between Multiple Datasets and Colocation with In Situ Measurements. Remote Sens. **2024**, 16, 4336. https://doi.org/10.3390/rs16224336

Laxenaire, R., Speich, S., Blanke, B., Chaigneau, A., Pegliasco, C., & Stegner, A. (2018). Anticyclonic eddies connecting the western boundaries of Indian and Atlantic Oceans. *Journal of Geophysical Research: Oceans*, *123*(11), 7651-7677.

Laxenaire, R., Speich, S., & Stegner, A. (2019). Evolution of the thermohaline structure of one Agulhas ring reconstructed from satellite altimetry and Argo floats. *Journal of Geophysical Research: Oceans*, *124*(12), 8969-9003.

Laxenaire, R., Speich, S., & Stegner, A. (2020). Agulhas ring heat content and transport in the South Atlantic estimated by combining satellite altimetry and Argo profiling floats data. *Journal of Geophysical Research: Oceans*, *125*(9), e2019JC015511.

Laxenaire, R., Guez, L., Chaigneau, A., Isic, M., Ioannou, A., and Speich, S.: TOEddies Global Mesoscale Eddy Atlas Colocated with Argo Float Profiles, https://doi.org/10.17882/102877 , 2024.

Manta, G., Speich, S., Karstensen, J., Hummels, R., Kersalé, M., Laxenaire, R., ... & Meinen, C. S. (2021). The South Atlantic meridional overturning circulation and mesoscale eddies in the first GO‐SHIP section at 34.5° S. *Journal of Geophysical Research: Oceans*, *126*(2), e2020JC016962.

Paillet, J., Le Cann, B., Carton, X., Morel, Y., & Serpette, A. (2002). Dynamics and evolution of a northern meddy. *Journal of Physical Oceanography*, *32*(1), 55-79.

Subirade, C., L'hégaret, P., Speich, S., Laxenaire, R., Karstensen, J., & Carton, X. (2023). Combining an Eddy Detection Algorithm with In-Situ Measurements to Study North Brazil Current Rings. *Remote Sensing*, *15*(7), 1897.

Referee 5

We would like to thank the referee for his/her comments. Please find our point-to-point response below.

Mesoscale ocean eddies are known to play a significant role in transporting heat, momentum, water mass, and biota large distances across ocean basins, thereby impacting productivity, biogeochemical properties, circulation, and climate. While there have been many studies that have identified and tracked mesoscale eddies as persistent signatures in surface fields such as sea surface height, temperature, or Okubo Weiss parameter, or as materially coherent regions of fluid derived from upper ocean velocity estimates, relatively few studies have studied the subsurface signature of eddies. This is important on two counts: not all eddies have a detectable surface expression, and estimates of transport by ocean eddies depends sensitively on the subsurface structure of the eddy.

This paper attempts to characterise the material coherence of mesoscale ocean eddies using in situ data gathered from a large number of oceanographic cruises in different ocean basins. This is a challenging task because very rarely do we have repeat observations of subsurface water mass properties in the same ocean feature over significant periods of time. Two notable exceptions (that I am aware of) are the classic study of Armi et al. (1989) -- cited in this article -- which tracked a Mediterranean salt lens (or "meddy") over a period of two years, and a more recent study by Rykova and Oke (2022) that analysed two Tasman Sea warm-core eddies using Argo float profiles co-located within the eddies.

There are many different -- sometimes contradictory -- definitions of eddy "coherence" in the oceanographic literature. The authors categorise previous analyses of mesoscale ocean eddies as identifying features that exhibit either "kinematic coherence" or "material coherence". The former features exhibit persistent signatures in a fixed (Eulerian) frame that can be tracked over long times/distance, while the latter are identified by either closed material boundaries that minimize mixing with the surrounding fluid or regions of the fluid that remain coherent and retain mass as they are advected by the flow. A similar categorisation is used by Denes et al. (2022), who distinguish between the "persistence" of eddy signals over long time periods (i.e. kinematic coherence) and the material coherence of these features, which can be over much shorter timescales. [Full disclosure: I am one of the co-authors of this paper.] As pointed out by the authors of the present article, these concepts are not mutually exclusive, but they are also not equivalent.

Here, the authors introduce a new approach which they call "thermohaline coherence", which seeks to describe the tendency of eddies to trap water mass with a distinctive signature of temperature and salinity and transport it, without modification, far from its region of formation. This idea is not without precedence: Robinson (1983) and Chelton et al. (2011) also defined eddies by identifying coherent anomalies of oceanographic variables. Unlike one of the reviewers, I don't see this approach as conceptually different from or inconsistent with material coherence --- the one implies the other. Rather, I view this as an alternative approach to identify material coherence of ocean eddies without a time history of observations. This is crucial. As described above, repeat subsurface observations of ocean eddy properties are vanishingly rare. Instead, the authors identify material coherence by looking for anomalous water mass properties in individual oceanographic sections. The *approach might be likened to a detective identifying a thief based on his possession of stolen goods, even though the detective did not follow the thief home from the scene of the crime.*

I found this to be a thoughtful, well-designed, and well-executed study of a large oceanographic data set. Some of the methods and results will be of great interest to both theorists and observers, and I am happy to recommend publication with (very) minor edits. My comments below are mostly cosmetic. I look forward to seeing this when it is published.

S. Keating, Sydney, Australia.

We would like to thank the reviewer for his thorough review and constructive feedback.We sincerely appreciate the effort and time the reviewer has invested in evaluating our work. We have carefully implemented all the suggested corrections and hope that the revised manuscript is now clearer and more reader-friendly.

L24: "Coherence" missing second quotation marks.

We thank the reviewer for bringing this to our attention. A second quotation mark has been added in the revised version of the manuscript.

L43: "persistence of water MASS properties"

We thank the reviewer for bringing this to our attention. The sentence has been corrected in the revised version of the manuscript.

L52: add hyphen in "frame-dependent". Add "and" before "imposed a vortex coherence criterion"

We thank the reviewer for his correction. The sentence has been corrected in the revised version of the manuscript.

L55: Vortices continually lose and entrain water mass, so there is no single "point" at which the vortex loses trapped water. Denes et al. (2022) define a median residence lifetime over which half the original fluid is lost, but there are likely other ways to define a coherence lifetime.

We thank the reviewer for this very interesting comment. While we cannot fully agree or disagree with this statement, we would like to provide additional context.

As far as we know, certain eddies do exhibit a distinct "point" where they lose their coherence rapidly, often within a few days, due to interactions with topography or through splitting and merging events. For instance, NBC rings interacting with the Lesser Antilles (Andrade-Canto et al., 2022; Subirade et al., 2023) and Gulf Stream rings interacting with the continental slope (Richardson et al., 1983) are notable examples. In these cases, eddies remain coherent but experience a sudden collapse, often driven by friction with topography.

To illustrate this, we reference Figure 3 from Andrade-Canto et al. (2022), which depicts an NBC ring losing its coherence between March 11th and March 20th, 2004. This example highlights a clear "point" of coherence loss in the dynamics of NBC rings.

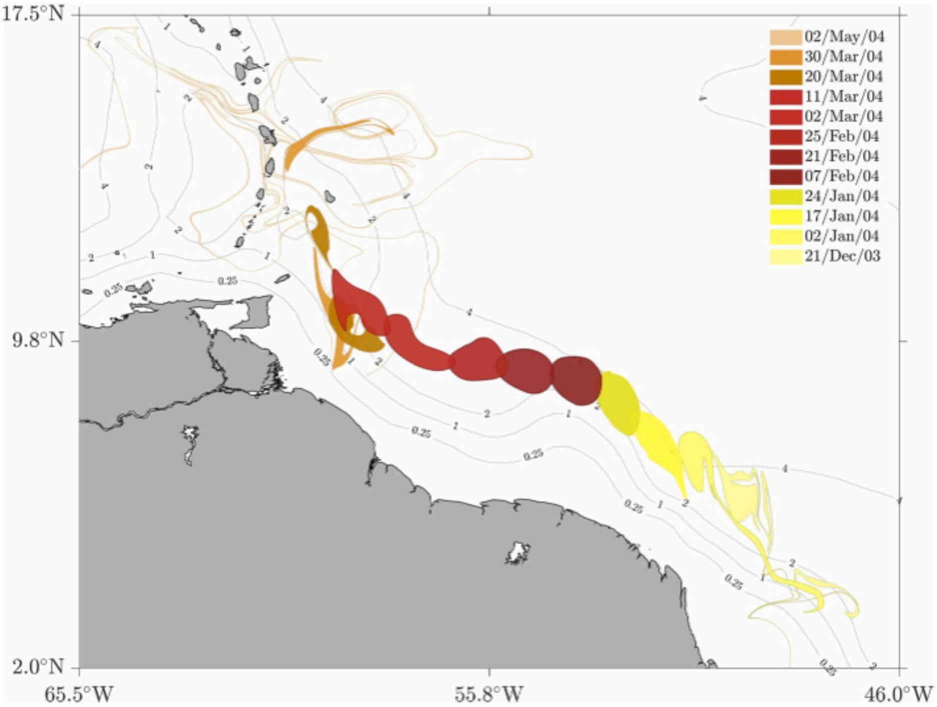

Figure 3. Genesis (yellow), evolution (red), and apocalypse (orange) of a coherent Lagrangian North Brazil Current Ring (NBCR), geodesically detected from altimetry-derived velocity data using the sea-surface heigh (SSH) eddy trajectory of Figure 2 as a reference. Selected isobaths (in km) are shown in gray.

We agree with the reviewer's comment regarding the continual entrainment of water masses by eddies, particularly along their boundaries. This is well illustrated by the anticyclonic eddy sampled during the Physindien 2011 experiment, for which thermohaline anomalies are plotted in Figure 4, panels c) and d). These panels show colder and fresher waters advected by the flow around the TC cores.

However, we respectfully do not fully agree with the assertion that vortices continually lose water mass. As long as the vortex is coherent, its loss rate is very small. For instance, Laxenaire et al. (2019) demonstrated that the thermohaline anomalies in the core of Agulhas rings can remain constant for at least up to a year. Similarly, the meddy studied by Armi et al. (1989) retained its trapped water for at least a year, as evidenced by the persistence of thermohaline anomalies.

If water is lost or exchanged, this typically occurs in the outer boundary region rather than within the eddy core itself. For this reason, in our previous work (Barabinot et al., 2024), we defined mesoscale eddy boundaries as the region where the trapped water transitions to and interacts with surrounding waters.

L56: Please be more specific about how you define "material coherence".

We thank the reviewer for their comment. Defining "material coherence" is indeed a challenging task, as multiple criteria have been proposed in the literature (e.g., Beron-Vera et al., 2013; Haller et al., 2016; Denes et al., 2022; Froyland et al., 2013, 2015; Bettencourt et al., 2012; El Aouni et al., 2020). These criteria are typically based on closed flow trajectories.

To address the reviewer's concern, we have added a sentence at line 55 to clarify our statement : *"This vortex ceases to be coherent when it loses its trapped water mass, that is when trajectories are no longer closed"*.

L67 and elsewhere. Parenthetical citations should not have the year in brackets. Use \citep or similar.

We thank the reviewer for bringing it to our attention. Citations have been corrected in the revised version.

L68: "do not consider... diffusion processes". Actually, several MC methods explicit include diffusion. See Refs 27, 28 and 35 in Denes et al. (2022).

We thank the reviewer for bringing this to our attention. We have revised the wording accordingly and added the suggested references at line 68 to support our statement in the updated manuscript. Here is the new formulation: "*Furthermore, MC theory is based on advection processes only and often does not consider the potential permeability of the eddy boundary due to diffusion processes or lateral intrusion (Joyce et al. 1977, 1984, Ruddick et al. 2010). Nevertheless, some criteria including diffusion can be found in the literature (Froyland et al. 2010, 2013, 2015).*"

L70: "meddy" is not defined.

We thank the reviewer for bringing this to our attention. The term "meddy" has been defined in line 70 in the revised version of the manuscript.

L79: Add comma before "thus challenging..."

We thank the reviewer for bringing this to our attention. A comma has been added on line 79 in the revised version of the manuscript.

L106: remove the word "vertically" from inside the brackets, i.e. "O(10m) vertically".

We thank the reviewer for bringing this to our attention. The word "vertically" has been removed from inside the brackets on line 79 in the revised version of the manuscript.

L110: "relies of the fact that thermohaline properties of the eddy are maintained throughout its lifetime". This is more of a hypothesis than a fact, albeit a reasonable one. As noted earlier, instabilities and mixing can lead to modifications of water mass properties near the eddy edge. So one might expect the T-S signature of trapped water masses to modulate as there is exchange and interaction with surrounding waters. The timescale over which this occurs is a good question, not addressed here.

We thank the reviewer for this very interesting comment. The advent of Argo floats has indeed made it easier to measure thermohaline anomalies in eddy cores across different regions, and several examples of persistent thermohaline anomalies maintained throughout eddy lifecycles can be found. Laxenaire et al. (2019, 2020) provide excellent examples of Agulhas rings where thermohaline anomalies remained stable and constant for over a year.

Similarly, Armi et al. (1989) and Paillet et al. (2002), both cited in our manuscript, describe meddies with persistent thermohaline anomalies on isopycnals. Additionally, Aguedjou et al. (2021) offers a comprehensive census of thermohaline anomalies in the eddy cores of the North Tropical Atlantic.

As noted, instabilities primarily affect the eddy boundary, and in the absence of significant mesoscale barotropic or baroclinic instabilities, the eddy core—and its thermohaline properties—remains largely unperturbed. Thermohaline anomalies dissipate when the eddy ceases to be materially coherent.

To strengthen our argument, we have added two sentences to the revised manuscript at line 110. We have revised the manuscript to include the following new sentence:

"Indeed, with the advent of Argo floats, measuring thermohaline anomalies in eddy cores across different regions has become easier, and several examples of thermohaline anomalies maintained throughout the eddy lifecycle can be found (Aguedjou et al., 2021; Laxenaire et al., 2019, 2020; Paillet et al., 2002; Armi et al., 1989)."

L113: Remove extra period.

We thank the reviewer for bringing this to our attention. We have removed the extra period in the revised version of the manuscript.

L118: Remove "To the best of our knowledge"

We thank the reviewer for his suggestion. We have removed the expression in the revised version of the manuscript.

L138: Fix name of EUREC4A-OA campaign.

We thank the reviewer for bringing this to our attention. We have fixed the name of EUREC4A-OA in the revised version of the manuscript.

L149: "TABLE 1 summarises..."

We thank the reviewer for bringing this to our attention. We have corrected the sentence in the revised version of the manuscript.

L150: "LOWERED and ship-mounted"

We thank the reviewer for bringing this to our attention. We have corrected the sentence in the revised version.

Table 1: XBT, xCTD and VM are listed in the table but not defined or used elsewhere in the article. Suggest you remove.

We thank the reviewer for their suggestion. However, we believe that mentioning the instruments is important, as they can be valuable references for future studies. The use of multiple instruments during cruises has facilitated the measurement of vertical sections with improved horizontal resolution. To address this point, we have defined the instruments in the title of Table 1 for clarity.

L207: "In the literature" --- provide refs.

We thank the reviewer for bringing this to our attention. In the revised manuscript, we have removed the sentence at line 207, as we felt the statement was too strong.

L215: Use in-line citation for Chaigneau et al. 2009.

We thank the reviewer for bringing this to our attention. We have used in-line citation in the revised version of the manuscript.

L257: "By recurrence" -- not sure what you mean here.

We thank the reviewer for bringing this to our attention. In the revised version of the manuscript, we have replaced this expression with the simpler term “therefore” at line 257.

L260: don't use bold text for x and z.

We thank the reviewer for bringing this to our attention. The expression has been corrected in the revised version of the manuscript.

L271 and elsewhere. Be consistent with the abbreviations used for anticyclonic and cyclonic eddies. I prefer AE and CE.

We thank the reviewer for bringing this to our attention. We have adopted AE and CE as notations for eddies in the revised version of the manuscript.

L279: "in the literature" -- provide refs.

We thank the reviewer for bringing this to our attention. In the revised version of the manuscript, this sentence has been removed as several references have already been provided between lines 278 and 285.

L282: "in studies" --- provide refs.

We thank the reviewer for bringing this to our attention. We have added three references in the revised version of the manuscript.

L287: "vertical component of the linear momentum EQUATION". Or simply "the vertical momentum equation"

We thank the reviewer for his suggestion. The wording has been corrected in the revised version of the manuscript.

L293: use italic for EPV for consistency.

We thank the reviewer for bringing this to our attention. We have corrected the expression in the revised version of the manuscript.

L341: not sure what lower case r means.

We thank the reviewer for bringing this to our attention. R refers to the eddy radius. We have modified the corresponding sentence in the revised version of the manuscript.

L351: "some altimetric studies" -- provide refs.

We thank the reviewer for bringing this to our attention. We have added three references in the revised version of the manuscript.

L378: the word "and" should not be italicised.

We thank the reviewer for bringing this to our attention. We have corrected the expression in the revised version of the manuscript.

L416: The two ellipses E1 and E1 provide inner and outer bounds on the best fitted ellipse. So why not take an average of the volumes calculated using each?

We thank the reviewer for their comment. Since eddies do not appear symmetrical on 2D vertical sections, the reconstructed 3D shapes of ellipses E1 and E2 differ significantly. The 3D structure of eddies remains an area of ongoing investigation within the scientific community, and our goal was to provide several examples of what this 3D shape might look like.

To address this point, we have added a sentence at line 423 in the revised manuscript to provide additional detail. Here is the sentence:

"As the vertical shape of eddies is not well understood, especially the shape of their thermohaline coherent core, in the literature, we present the two ellipses as examples of what an eddy core can look like in 3D."

L430: "Physindien" is capitalised in section 2.1 and called something different in Tables 1-3.

We thank the reviewer for bringing this to our attention. The wording has been revised throughout the updated version of the manuscript..

L460: Add space after period before "Our comparison"

We thank the reviewer for bringing this to our attention. A space has been added on line 467 in the revised version of the manuscript.

Figure 9 caption. Capitalise "velocity". Add space after period before "The symbols..."

We thank the reviewer for bringing this to our attention. Typos have been corrected in the revised version of the manuscript.

L512: Physindien again different from earlier in the manuscript.

We thank the reviewer for bringing this to our attention. The wording has been revised throughout the updated version of the manuscript.

Figure 10 caption. I think the green lines are actually "cyan".

We thank the reviewer for bringing this to our attention. The caption has been corrected in the revised version of the manuscript.

Equation A1 and L643: mixing lower case and upper case deltas here. Please be consistent.

We thank the reviewer for bringing this to our attention. Notations in equation (A1) have been corrected in the revised version of the manuscript.

L678: The units for alpha and beta are not consistent. I think alpha should be units kg m^{-3} / K and beta should be kg m^{-3} / g kg^{-1}, assuming that salinity is measured in g/kg.

We thank the reviewer for his correction. The units were indeed incorrect and now they have been corrected in the revised version of the manuscript.

L668: "f-PLANE"

We thank the reviewer for his correction. The expression has been corrected in the revised version of the manuscript.

L674: Not entirely sure, but I think there is an error here. I think the points P' and Q' need to be defined such that Q'C = C P and Q C = C P'.

We thank the reviewer for pointing out this oversight. It was an honest mistake, and we fully agree with the correction.

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