We would like to thank the referee for his/her comments. Please find our point-to-point response below.

The authors of this manuscript should conduct a more comprehensive review of the existing literature on material frame-indifferent (i.e., objective) coherent vortex detection. The manuscript contains several inaccurate or unsubstantiated statements, including the following:

The intention is not to offer a critique of the material frame-indifferent coherent (MC) vortex theory as we concur with its mathematical and physical formulation. The objective of our study is to examine the impact of material coherence on the thermohaline structure of eddies. It is accurate to note that our analysis does not explicitly demonstrate a temporal correspondence between the MC theory and the presence of anomalies in the core of eddies. However, the presence of anomalies is indicative of the existence of water that does not exhibit the same thermohaline properties as the surrounding water. As postulated by the MC theory, the material boundaries maintain the thermohaline characteristics of the transported water, from the eddy region of generation. Consequently, the trapped water exhibits disparities from the surrounding water as a consequence of the eddy motion in the ocean. Anomalies on isopycnals represent a primary indicator of heterogeneous water masses within eddy cores, enabling the computation of heat and salt content in these structures through the use of in situ data (Aguedjou et al., 2021; Yang et al., 2015; Dong et al., 2017; Dong et al., 2014; Chen et al., 2012).

The authors of this manuscript should conduct a more comprehensive review of the extant literature on material frame-indifferent (i.e., objective) coherent vortex detection. The manuscript contains several inaccurate or unsubstantiated statements, including the following:

1) 'Flierl (1981) showed that when the tangential velocity of the vortex is higher than its translational velocity, fluid particles are trapped in the vortex core.' This claim is more an expression of belief rather than a rigorous conclusion, as velocity is dependent on the observer.

We are grateful to the reviewer for bringing this point to our attention and for prompting us to address it. Nevertheless, we consider the assertion to be somewhat misleading in that it fails to acknowledge the role of mathematical Lagrangian arguments in the article by Flierl (1981). To clarify, the robust mathematical Lagrangian approach presented in the article by Flierl (1981) is fundamental to our framework. In particular, equations (3.6) and (3.7) are of significant relevance.

2) 'In particular, MC theory ignores the fact that water masses at the edge of eddies can change their properties due to various types of instabilities.' This statement is incorrect because, if a vortex is characterized as materially coherent, no fluid can traverse its boundary. More precisely, no material surface can be intersected by fluid flow since it is flow-invariant, regardless of its coherence.

From a theoretical standpoint, we concur with this assertion when considering the macroscopic and cinematic perspective. However, it should be noted that the actual boundaries of ocean eddies are not perfectly impermeable, and thus cannot be represented by line-sized walls. Rather, they are turbulent zones of a certain width where small-scale instabilities occur, such as centrifugal-symmetric instabilities, and where layering can appear, involving salt fingers and vertical recirculation. This is corroborated by the findings of Barabinot et al. (2024), Molodtsov et al. (2020), Bebieva et al. (2016), Hua et al. (2013) among others. A comparison between the boundary defined by the MC theory and the region where this small-scale turbulence occurs would undoubtedly prove insightful and contribute to the existing body of knowledge in this field.

To this end, we will include a paragraph in our manuscript to clarify this point.

3) 'Furthermore, MC theory is based only on fluid flow and does not consider the potential permeability of the eddy boundary due to diffusion processes or lateral intrusion (Joyce, 1977, 1984; Ruddick et al., 2010).' This is inaccurate since the boundaries of geodesically-detected coherent material vortices serve as minimizers of diffusion (refer to the Annual Review of Fluid Mechanics paper by Haller).

We appreciate the reviewer’s insightful comment. We agree that the boundaries of geodesically-detected coherent material vortices serve as minimizers of diffusion, as highlighted in Haller's work in the Annual Review of Fluid Mechanics. However, it is important to note that "minimizers of diffusion" does not equate to "no diffusion." Additionally, we must consider the impact of small-scale instabilities, which can still play a significant role in the permeability of the eddy boundary. These factors indicate that, while diffusion is minimized, it is not entirely eliminated, and small-scale processes can influence the overall dynamics of the eddy boundary.

4) 'We revisited coherence definitions and checked data accuracy.' This manuscript does not encompass an examination of coherent material vortex framing. Indeed, the manuscript lacks any explicit articulation or statement concerning this subject matter.

We appreciate the reviewer's comment. To clarify, we did review previous definitions of coherence and explained their relevance to our study in lines 18 to 83 of the manuscript. While our primary focus was not on framing coherent material vortices explicitly, we believe that our discussion on the definitions of coherence provides a necessary foundation for understanding the context and significance of our study.

A more detailed account of this point will be provided in the revised version of the manuscript.

5) 'Comparing the horizontal positions of these core anomalies with eddy surface signatures revealed that surface data alone are insufficient for characterizing the eddy material coherence.' This is plausible, but to assess it, time-dependent flow data must be analyzed using an objective method.

We appreciate the reviewer's comment and agree that analyzing time-dependent flow data using an objective method would provide a comprehensive assessment. However, we believe that the snapshots provided by *in situ* data are sufficient to analyze the position of the anomalies with respect to the sea surface. These snapshots allow us to capture the spatial relationship between core anomalies and the eddy surface signatures effectively, even without continuous time-dependent data. We will improve the wording on this point in the revised version of the manuscript.

Beyond these imprecise statements, the TC criterion remains nebulous and can only be regarded as qualitative in nature; temporal flow data are imperative to establish coherence. All criteria (gradients; potential vorticity — an observer-dependent quantity; pythagorean arguments) are applied to instantaneous snapshots of observed mass fields. A temporal history is requisite to ascertain if the 'gradient of a property of some kind' is conserved under advection by the flow. It is evident that transect data are unsuitable for this type of analysis.

In our article, we attempted to define a new concept that can be applied to *in situ* oceanographic data. The TC criterion has a physical sense and enables both to identify trapped water mass inside eddy cores and to compute heat and salt transport by these structures.

We acknowledge the reviewer's concern regarding the qualitative nature of the TC criterion and the necessity for temporal flow data to establish coherence. It is indeed a valid point that temporal assessment is crucial for a comprehensive understanding of whether the "gradient of a property of some kind" is conserved under advection by the flow. We agree that gradients, potential vorticity, and other criteria applied to instantaneous snapshots are limited in capturing the dynamic evolution of water masses.

However, we would like to address the reviewer's suggestion that *in situ* data obtained by hydrological transect are unsuitable for this type of analysis. While temporal history is ideal, it is not always feasible. Observations of ocean subsurface thermohaline and velocity properties are very scarce due to logistical and resource constraints. Our methodology aims to provide a practical approach to identifying heterogeneous water masses within eddy cores using available *in situ* data.

By applying the TC criterion to in situ observations along ship transects, we can still gain valuable insights into the structure and composition of eddies. Even without temporal data, the identification of distinct water masses within an eddy can help in understanding the spatial variability and potential transport processes. This approach can be seen as a first step, providing a foundation for future studies that may incorporate more comprehensive temporal assessments.

In conclusion, while we recognize the limitations of our dataset and the importance of temporal analysis, we believe that our methodology offers a meaningful contribution to the study of eddies using available data. It provides a practical tool for identifying and characterizing water masses within eddy cores, which can be further refined with more detailed temporal observations in future research.

In conclusion, I am unable to endorse the publication of this manuscript. I acknowledge the significant effort expended by the author in analyzing in-situ oceanographic campaign data, which holds intrinsic value. However, I urge the author to consider presenting their analysis within an alternative context, as the current application towards assessing material coherence is not appropriate for the available data.

We thank the reviewer for his/her thoughtful comments and for acknowledging the significant effort expended in analyzing the in-situ oceanographic campaign data, which indeed holds intrinsic value.

While we understand the reviewer's concerns regarding the application of our analysis towards assessing material coherence with the available data, we would like to address this point further. The notion of anomaly, as utilized in our study, is intrinsically linked to the concept of material coherence.

Anomalies in oceanographic data often refer to deviations from a mean state, indicating the presence of distinct water masses or features such as eddies. These anomalies are markers of coherent structures within the ocean that can have significant impacts on heat, salt, and carbon, oxygen and nutrient transport. Even though our data might not allow for a complete temporal assessment, it still provides valuable snapshots that reveal these coherent structures and their properties.

By identifying these anomalies, we can infer the presence and characteristics of coherent water masses. This approach, although limited by the lack of temporal data, is still valid and valuable for understanding the spatial distribution and potential impacts of these structures. It serves as a crucial first step, paving the way for future studies that can incorporate more extensive temporal datasets.

We appreciate the reviewer's suggestion to present our analysis within an alternative context. However, we believe that the current context of assessing material coherence through the detection of anomalies in in-situ observations is appropriate and provides significant insights. It aligns with the practical constraints of field oceanographic research, where temporal data may not always be available, and yet meaningful analysis can still be conducted.

In conclusion, while we acknowledge the limitations of our study, we maintain that our methodology and findings contribute valuable knowledge to the field. We are open to further refining our approach and incorporating additional data in future research to enhance the assessment of material coherence.

In the revised version of the manuscript, we will provide a more detailed conclusion in order to clarify this point.

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