Review: Lagrangian characterization of heat waves: The perspective matters

This paper quantifies the processes leading to two major heatwaves using two different methods. While both methods use Lagrangian information to decompose the temperature anomaly into different processes, the second method isolates the anomalous contribution of each process from its climatological value to the heat accumulation. Notably, the authors show that when considering the anomalous contributions, horizontal advection plays a larger role in near-surface heat accumulation than the total decomposition suggests. This paper presents a new framework for determining the relative importance of different processes to heatwave formation, and I would be happy to recommend it for publication with major revisions.

Main Comments:

I'm unclear as to what the θ_{pre} term represents physically. Does this capture the memory of the heatwave on daily timescales? Including some discussion of the physical meaning of this term in the methods would be useful.

The explanation of the Eulerian tracers was difficult to follow. I understand the full explanation is provided in a previous publication, but it would be helpful to show how θ enters into equations (9) and (11) to provide some physical intuition into how the Lagrangian information is obtained. Additionally, including an explanation why the Eulerian method was chosen instead of the direct trajectory calculations in the methods section would be useful.

I would appreciate more clarification on how the long-term averages of the decomposed potential temperature contributions are calculated in section 3.2 (i.e., the data shown in Figs. 7 & 8).

Minor Comments:

It would be helpful to explicitly state in the text which dates are considered part of the heatwave for both cases examined.

Line 91: There seems to be a $\bar{\theta}$ missing from the first term in R.

Line 138: Is there a reason only the years 2010 – 2022 were included in the time-averages?

Line 234: Stating the years included in the long-term averages would be helpful here. It is included in the caption of Fig. 7, but should also be stated in the text.

Line 255-260: It is stated that the contribution from diabatic heating in the UK differs from in the Pacific Northwest due to the UK is in closer proximity to the ocean. However, the Pacific Northwest is also close to the ocean, with the mean flow likely originating over the ocean. Is the difference that the UK is surrounded by ocean?

Line 335-336: Does the negative shift in diabatic heating indicate that surface fluxes are smaller than average on hot days, or that the air parcels are not close enough to the surface to be affected by surface fluxes for the majority of their trajectory?

Fig. 13: What is the bold black line on these figures? It seems to be the 50% contour, but it doesn't extend across the domain in every panel.

Fig. 15: Please make this figure bigger and perhaps bold the text on the pie charts. The percentages are difficult to read.

There are several misspellings in the text.