Response to Referee #02

Overview:

This manuscript presents results of two gravity wave events at 36.31W, 07.40S using photometer, all-sky imager, and meteor radar data. The authors use this data to determine characteristics about the present waves and associated momentum and energy fluxes. The technique presented is interesting, and could provide beneficial scientific information. It is a useful idea from the authors to use multiple airglow layers to better understand gravity wave propagation in the MLT region. Nevertheless, there are several issues with the manuscript that are concerning. Importantly, the calculated values need to be better explained in the context of what assumptions were made, and what specific measurements were used. The "reconstructed" waves need more discussion and justification for how the wave parameters were chosen.

It is concerning that the kinetic energy calculation does not seem to be for the wave itself, rather it is based off of >1hr wind perturbation measurements (1hr resolution) that do not have the resolution to capture the waves being studied in this manuscript. Additionally, there is lacking information on exactly how the momentum flux was calculated. A temperature perturbation value is used, but it is not clear how this was obtained. It should be the average perturbation value over the wave packet, or the actual temperature perturbation amplitude. Instead, it appears that the raw residual temperatures were applied directly to the MF calculation.

Given these aspects, and the other concerns listed below, I am suggesting the paper be rejected. If these analysis issues can be mitigated, and the techniques more properly explained, there is potential that the manuscript could be resubmitted.

Detailed concerns are listed below:

1. **Lines 67-68 "The temporal resolution of the observation is 2 minutes, thus GWs with periods greater than 2 minutes can be observed." Lines 67-68**

The airglow photometer that measures OI, O2, NaD, and OH is said to have a 2 minute resolution. That would mean that GWs with periods of greater than 4 minutes can be observed (at best). More importantly, the authors need to provide explanation as to how phase shifts can be determined with a 2-minute resolution. Also, a 3-point running mean has been applied to the data (discussed on lines 125-126), which would further reduce the resolution. Furthermore, the data itself will have associated noise. The fit (equation 2) would also have some errors associated with it. So, how does this affect the calculation of phase differences between the different airglow layers?

Response:

As you have rightly said, the 3-point running mean further reduced the resolution. However, considering your comments, a 5-point running mean has rather been applied to obtain a resolution of 10 minutes. With his resolutions, GWs with periods of ~20 minutes and above are detected.

Due to the resolution being used, the phase shift is determined from the 10-minute resolution time series data.

Explanation of determination of Phase Shift

The phase shifts were determined from the phases determined from each emission layer time series. The phase (ϕ) is estimated from the Equation (2). The phase is given in hours. However, the phase needs to be estimated considering the start time of the data been used. This is to give the phase in relation to the start time of the time series. This is done by adding the individual dominant period obtained to the time series until it corresponds to the first hour that corresponds to the start time of the time series. For instance, the phase of the 04/12/204 OI GW event is 0.0744475 hours for GWs with a period of 0.424451 hours (~25.47 minutes). The period is added to the phase until 23.419253 (23:25:00 UT) was attained. A similar procedure was applied to the other emissions. In table 1, the result of the phases of the two waves selected are presented. From the phases obtained for the individual emission layers, the phase shifts can be estimated between each two consecutive layers as well as the first and the fourth layers.

Table 1: Estimated phase of 04/12/2004 GW event.

Layers	$\phi_{\tau=25.24}$	$\phi_{\tau=38.00}$
OI	23.419253	23.259999
O_2	23.417403	23.312784
Na D-Line	23.829630	23.293561
OH	23.419026	23.683039

Error Analysis

As commented by the referee, error analysis has been performed in order to evaluate the errors associated with our calculations.

The error associated with each error layer has been performed to evaluate the impact on the result obtained. The error was assessed by estimating the standard error in the original data, the smoothed data and the harmonics. It is important to mention that the estimated standard error of the mean (σ_M) for OI 5577, O2, Na D-Line and OH intensities, temperature and time are presented in Table 2.

Table 2: Associated errors in the time series of the 04/12/2004 GW event.

Layers	OI	O_2	Na D-Line	OH
		Original Data		
$\sigma_{M_{Time}}$(s)	± 09.87097	± 09.87104	± 09.87093	± 09.87096
σ_{M_I}(R)	± 07.33892	± 14.74540	± 04.07605	± 23.24270
σ_{M_T} (K)		± 01.72822		± 03.13196
		Usable Data Range		
$\sigma_{M_{Time}}$(s)	± 05.60431	± 05.60443	± 05.60471	± 05.60436

σ_{M_I}(R)	± 06.95750	± 10.91080	± 01.05222	± 07.99789
σ_{M_T} (K)		± 00.67040		± 00.41251
Residual				
$\sigma_{M_{Time}}$(s)	±05.60432	± 05.60443	± 05.60471	± 05.60436
σ_{M_I}(R)	±00.96403	± 02.53697	± 00.20267	± 05.02443
σ_{M_T} (K)		± 00.23636		± 00.25339

From Table 2, the estimated errors in the time, intensities and temperature are presented. In general, the errors associated with the original data is higher than that of the usable data and the residual. These values are less than the measurement errors of the intensities which is of the order of 5% whereas for intensities and 2-3K for O2 and 4-5K for OH (Wrasse et al., 2004).

The error associated with the fit was evaluated by estimating the cross correlation between the time series of the residual of the intensities and temperature and their respective harmonics are indicated in Figure 6a. The cross correlation of the intensities, that is, their residuals and harmonics ranged are 0.76, 0.79, 0.65 and 0.77 for IO, O2, NaD and OH intensities, respectively. For the temperature residuals and harmonics for O2 and OH are 0.52 and 0.67, respectively.

2. **In the processing section, "a Lomb-Scargle periodogram and Wavelet analysis were used to determine the dominant periods in the time series of each emission layer. At least a dominant peak is chosen and used to reconstruct new harmonics and over plotted on the residual." Lines 135-136.**

In this case, if there is any error in the harmonics chosen, how would this influence the result? The waves present appear to have a spectrum of associated periods. Is one chosen harmonic effectively characterizing the waves present?

Response:

In Table 2, the errors associated with the harmonics are presented. From these values, no significant errors were associated. Also, to assess the degree of deviation of the harmonics from the residual. As shown in the main text and in the response to item #1, except the intensity of the NaD, the remaining intensities have cross correlation above 0.75. This, however, shows that no significant differences exist between the residual and the harmonics. Therefore, there would be no significant influence on the results.

Yes, the one chosen characterized the wave event of interest. It is for this reason the reconstructed harmonics are over plotted over the residuals.

3. **Lines 146-147: "After the residual time series was determined, the periodicities were calculated. For these residuals, the dominant period are 25.47 min and 33.47 min."**

What process was used to determine these were the periods? Was a peak finding routine used with the PSD shown in figure 3d? Was this the period at all times? Was a particular time period chosen for each emission line? Are they all the same?

Response:

Lomb-Scargle periodogram and Wavelet analysis were used to determine the periods. Peak selection procedure considering the power spectral densities (PSD) were used to detect the significant periods. To determine whether the periods were present all time, the Wavelet analysis was implemented. As shown in Figure 3d (now 6d), it can be seen that the peak periods range between ~25 to 50 minutes and extend almost across the entire interval of the time series considered for all emission layers. In the case whether the periods are the same or not, we only considered the same period in each emission layer, or periods with deviations with ±15%. These periods are observed. The deviations were included in the choice of period because they are observed period. The result on the estimated intrinsic parameters are discussed later.

Furthermore, looking at figure 3a, the "reconstructed" signal appears to fit well for some portions of the night, but not for others. Was there a particular time used to determine the phase differences between the emission layers? Just a slight offset between the fit and actual data could result in significant differences for the vertical wavelength calculation.

Response:

As mentioned in the response to item #1, the start time of the time series are all the same. This is done to avoid offsets that could result in significant differences.

4. **Lines 152-153: "From the reconstructed time series, it is clear that all the emission layers are similar, indicating that the same GWs propagate through these layers."**

How was this determined? There are certainly similarities in all of the layers. This could also be expected for a ducted wave as well. It also appears that the wavelets in Figure 3d are not the same for all of the layers, so more explanation should be provided here. While it is mentioned that the wavelet shows the presence of waves from 30-90 minutes, there is still variability between the layers.

Response:

The reconstructed time series (harmonic) was constructed using equation (2), using the time of observation (x), amplitudes, observed period(s) of the GWs and phases of each emission layer. Similarities between layers indeed exist, however as mentioned, this could be due to ducted waves. As a result, the propagation conditions are investigated in the discussion section in order to verify whether or not they are ducted waves.

Explanation on the variations of the periods in all layers

In Figure 3d, even though similarities exist in the periodicities, some degree of variations can be seen. These variations can be attributed to the variations of the background wind, since the result obtained in the Lomb Scargle periodogram, and the Wavelet are observed periods.

5. **Section 3.4 needs more discussion about how the parameters were calculated, this is detailed in the following comments:**

-How was the Brunt Vaisala Frequency in Figure 4d calculated? What assumptions were made to calculate the potential temperature? How was dTpot/dz, the change in potential temperature with altitude calculated, and were there any assumptions made? Figure 4c shows a variable potential temperature, but the Brunt Vaisala frequency is plotted as a constant at each altitude.

Response:

The Brunt Väisäla frequency is estimated using Equation (5). During this wave event, SABER (Sounding of the Atmosphere using Broadband Emission Radiometry) instrument onboard the TIMED (Thermosphere Ionosphere Mesosphere Energetics Dynamics) satellite, made a passage ~735km away from the observation site. The temperature and pressure were used in the determination of the potential temperature, where $K/c_p = 0.286$. Using a first order derivative procedure in idl (interactive data language), the $d\theta/dz$, profile were determined. The potential temperature is the peak altitude of OH and O_2, which are ~87 and ~92 km where chosen. However, since, the temperatures of the OH and O_2 layers varies with time at fix altitude, so $d\theta/dz$ is estimated considering the value of dz at the respective emission layers. The time series of θ is then estimated considering a constant dz.

After the estimation of Brunt Väisäla frequency from the profile determined from using SABER temperature, the Brunt Väisäla frequency at the respective emission layer peak altitude was chosen for the entire time. Due to your comment, the result of Figure 4c (now 7c) has been corrected. The profile has been estimated using the OH and O_2 temperature.

6. **What measured parameters are used for the energies? How is T'/T being calculated? Is the amplitude of the residual temperatures being used? How is the amplitude being calculated?**

Response:

The measured parameters used in the estimations of the energies are k,l, m, k_H^2, $\frac{T'}{\bar{T}}$, and N^4. Basically, it can be said that except g, all other parameters are determined from observations. To determine the relative amplitude of the temperature time series, the T' was first determined by subtracting the background from the from the temperature time series $\bar{T}$ (Narayanan et al., 2024). The amplitude of the gravity wave temperature perturbation was estimated by dividing the temperature perturbation by the background temperature. In the previous version of the manuscript (and mentioned in item #6), the vertical flux of the horizontal momentum was assumed to be estimated over the wave packet. Also, only the temperature fluctuations, T', due to the gravity wave was used in the estimation of the potential energy.

However, due to your comments and questions, the entire calculations on the momentum flux and energies have been done all over again.

7. **Where are u' and v' being calculated from equation 4 for kinetic energy? What assumptions were made for the calculations of Ek shown in figure 4g?**

Response:

The u' and v' were calculated from the observed wind from meteor radar. No assumptions were made in the estimation of the E_k. Considering your comment regarding the temporal resolution of the meteor radar wind and the observed GW periods in item #12, the estimation of the E_k for such waves have been removed.

8. **For horizontal momentum flux, as you show in equation 6 (now 7), this is typically assumed to be the average vertical flux of horizontal momentum over a wave packet. Over what time period is the wave packet defined? What is being used at the T' calculation?**

Response:

As mentioned earlier, it is the average vertical flux of horizontal momentum over a wave packet, thus, no period over the time series was defined. Here, the estimated $T'$was used.

9. **For equations 6/7, on line 175 it says "where rho_o is the density at the emission layers" but it is not clear where or how this density was obtained to make the calculation shown in Figure 4e.**

Response:

The density, ρ_0, used in equations 6 and 7 was obtained from SABER sounding close to the observation site during each GW event. The density information has been included in the text.

10. **For equation 6/7, how was kh, the horizontal wavenumber, calculated from the data? This was not discussed elsewhere.**

Response:

The horizontal wavenumber, k_H, was estimated from the horizontal wavelength estimated in the Equation A5 in Appendix A2, using the relation

$$k_H = \frac{2\pi}{\lambda_H},$$

where, λ_H is the horizontal wavelength. This has been defined in the main text.

11. **Similarly, how was the intrinsic frequency calculated or measured for the MF calculation?**

Response:

The intrinsic frequency, ω, can be estimated from the expression, $\omega = \omega_0 - kU - lV$, where $\omega_0 = 2\pi/\tau$=. U and V are the zonal and meridional wind speed, respectively, at each peak emission altitude. This has been defined in the main text.

12. **Line 187-188: "Since the meteor radar wind has a temporal resolution of one (1) hour, Ek at each hour was determined and presented in a contour plot"**

The meteor radar is giving you the background wind. However, with a 1 hour resolution, the meteor radar is unable to give the perturbations u' and v' associated with the gravity wave that would be necessary to calculate Ek. The gravity wave periods present are all less than 1 hour. These Ek calculations are not correct.

Response:

Well, noted. As a result, the estimation of E_k and it discussion has been removed. This aspect will be left for a companion paper.

13. **Lines 193-194: "The spectral analysis technique described in Wrasse et al 2024 was used to determine the horizontal wavelength, period, phase speed, and propagation direction"**

More details need to be given here about what exactly was done. This is a very cursory explanation for a significant calculation within the manuscript.

In the referenced 2024 paper, it appears that Fig 1 is a flow chart and Figure 2 shows individual images and keograms. This case presented there is a little different because the waves were very clear both in the individual images and in the keograms. In the manuscript here, and the data presented in A2, the waves are not necessarily clear in the keogram. It is also a little strange that individual images are not shown. Looking at data in Tabe 1, the determined kh was between 20-35km, which should be within the field of view of the imager. Why were spatial images not included? The horizontal wavelengths can easily be obtained from the images themselves and not a keogram. There needs to be more discussion about how all of these parameters were obtained.

Response:

Thanks for your comments! Based on your comments, the means by which the wave parameters are estimated are discussed below. This discussion has been included in Appendix A1.

In order to extract the parameters of gravity waves, a discrete Fourier transform based spectral analysis was used. First, a region containing GW oscillations were selected in both zonal and meridional component of the keogram components, as shown in Figure **xy**. Note that the same area in each of the components were considered for analysis. Next, a discrete Fourier transform (equation **A1**) is applied to the selected areas.

$$\mathrm{F}(\omega) = \sum_{n=0}^{N-1} f(t) e^{\frac{-2\pi\omega n_i}{N}}$$

in which $F(\omega)$ is the transform of the Fourier function $f(t), \omega = 0, \ldots, N-1$ is the frequency index, and N is the number of points in time series within the selected regions. Then, the cross spectrum defined by

$$C(x) = F_s(\omega) F_{s+1}^{*}(\omega),$$

in which $C(\omega)$ is the cross spectrum between two time series and $F_s(\omega)$ and $F_{s+1}^{*}(\omega)$ represent the Fourier transform of the series $f_S(t)$ and $f_{s+1}(t)$, respectively. $F_{s+1}^{*}(\omega)$ is the complex conjugate of $F_{s+1}(\omega)$. The one-dimensional cross power spectrum is defined by the quadratic modulus, |C2|.

The amplitude of the cross power spectrum is then determined using $\sqrt[2]{|C^2|}$, with the phase of the cross spectrum defined by

$$\Delta\psi = tan^{-1}\left\{\frac{Im(C(\omega))}{Re(C(\omega))}\right\}, -\pi \leq \psi \geq \pi.$$

The phase difference between these time series caused by the wave propagation is considered to be the frequency ω, corresponding to the maximum amplitude. From the above estimations, the wave parameters are determined as follows:

Period (min):

$$\tau = \frac{1}{|f(\omega)|};$$

Horizontal wavelength (km):

$$\lambda_H = \frac{\lambda_{NS}\lambda_{EW}}{\sqrt{\lambda_{NS} + \lambda_{EW}}},$$

Where, wavelength (km) for the zonal and meridional components $(\lambda_{NS}, \lambda_{EW})$ is $\lambda_{NS,EW} = \frac{\Delta d}{\Delta\psi/_{360°}}$, in which Δd is the distance between the time series.

The horizontal phase velocity $C_H(\mathrm{m/s})$, and phase propagation direction $\phi(°)$, are determined by

$$C_H = \frac{\lambda_H}{\tau} \; and \; \phi = cos^{-1}\left(\frac{\lambda_H}{\lambda_{NS}}\right)$$

14. **Lines 200-207: For event 1 "the dominant periods used in the reconstruction of the waves of these events are 00.42 hr (25.47 min) for all emission layers, and 00.50 hr (30.29 mins) for IO 557.7, O2 (0-1) and NaD. However, the period of the OH (6 - 2) was 0.55 hr (33.47min)."**

This is a bit contradictory to read. It sounds like there were three different periods used here. Why are there so many different periods? Shouldn't they all be the same? These are also very close periods, close enough that a 2-minute measurement resolution would suggest a slight error in the fit could describe differences in presumed periods. Can it be demonstrated that this is not a fitting error and these periods are real? From Figure 5b, it looks like there is a lot of variability in periods over the dataset.

Response:

These lines have been re-written for clarity as below and in the main text. Also, regarding obtaining the same observe period in all emission layers depends on the background wind variations. Meteor radar wind showed some variations with altitude, and this can change in the observed period. In order to demonstrate that the obtained periods are real, the Lomb Scargle periodogram and the Wavelet analysis was applied to the original data to see the periods present before the application of the fit.

15. **Figure 5a makes it appear that there is little to no phase change between the different layers/altitudes. This is usually indicative of a ducted wave. Yet, the vertical wavelength listed in Table 1 is only 10km. Wouldn't there would be more variation in phase over the layers if the vertical wavelength were only 10km?**

Response:

That is right. In Figure 5a (now 7a) there is little to no phase change, especially at the beginning of the data used. As pointed out that this kind is an indicative of ducted waves, background studies has been conduct to investigate the propagation characteristics of this waves within the altitude range of the emission layers.

The vertical wavelength had been checked and verified, but the estimation still yielded 10 km.

16. **Lines 210-215: descriptions of the wavelet plots are given, but it really is not clear how the peaks/wave periods were determined. The plots show a broad spectrum. How was one particular peak chosen to represent a wave across the entire dataset?**

Response:

In the current version of the manuscript, the procedure of determination of the peaks had been described.

Two periods were chosen for the reconstruction. For the GW event of 25 May 2006, the wavelet analysis showed a period (of 70 mins) with strong amplitude was observed in Figure 5d(i), thus for the OI emission layer. However, this period only appeared in the OH wavelet (in Figure 5d(vi). Because of this, only two periods were observed and all these two can be seen with a peak ~30 minutes, which extend to about ~50 minutes. For the case of event of 04 December 2004, the period are quite consistent in all the four emission layers. As mentioned earlier, the procedure of the period selection has been discussed in Section 4 (Result).

17. **Table 1: Are there any errors associated with these measurements/calculations?**

Response:

Yes. There are errors associated with the measurements and calculations. These errors are included in the current version of the manuscript.

18. **Table 1: It is still not clear how the vertical wavelength was calculated from the measurements.**

Response:

In the processing section (i.e., 3.3), the procedure to the estimation of the vertical wavelength has been elaborated in detail.

19. **Lines 218-291: "Only the potential energy for Event #02 could be determined due to unavailability of observed winds. Hence, no estimated values for kinetic energy and subsequently total energy were presented in Table 1."**

It would appear based on what has been presented in this paper that Ek cannot be calculated for any of the events.

Response:

This will exactly be the case. As mentioned in your previous comment in in the previous comments regarding the estimation of the Ek, the temporal resolution of the meteor radar wind will not capture the spectrum of GWs under study. Due to this, the estimation and discussion of the kinetic energies have been removed.

20. **Lines 220-225 and Figure 5c and d: The wavelet analysis shows periods that are all over the place. The final fitted waves based on “dominant periods” in the wavelet are shown in Figure 5c. However, the original data plotted with the fit are never shown like they were for event 1 in Figure 3. This needs to be included.**

Response:

Thanks for the comment. The original data has been plotted with fit in the current version of the manuscript.

21. **Lines 237-238: “two events with similar periods were selected. For Event #01, two dominant periods were detected, however, the first period present no phase change, implying it is possibly a ducted wave.”**

It is still not clear how the “dominant periods” were chosen. It would also appear that there is little to no phase change between the different layers.

Response:

In item 15, a response regarding the investigation of the propagation characteristics of GW event of 04 December 2004 if the waves during this event are ducted or not. A follow up discussion on the potential and momentum flux was made to investigate the dynamics of the momentum flux in such conditions. The selection of the dominant period was based on the PSD using a procedure in IDL.

22. **Line 229: “For Event #02, the two dominant periods are within the gravity wave spectrum.”**

This is not at all clear from the wavelet.

Response:

Event two has been re-analyzed and plot. However, further analysis showed that this case cannot be used, hence another case was selected.

23. **23.Line 233-235: "From the phases of the GWs of Event #01, OH leads NaD by 08.60 min, whereas NaD leads O2 by 01.21 min. O2 lags OI by 03.25 min. A consistent phase lead can be observed from OH through NaD to O2 except between O2 and OI, where a phase lag is observed."**

Where was this demonstrated in the data? Figure 5a shows nearly identical phases over each layer. Furthermore, the resolution of the measurements would not allow for these sorts of phase differences to be measured. 1.21 minutes is less than the resolution of the measurement.

Response:

It is true that the resolution of the measurement will not permit the determination of a phase less than 2 minutes. However, considering the altitude difference between the two emission layers and if it is the same wave propagating through the two layers with a high velocity, it is possible there will be near no phase difference. We can also consider the possibility that the overlapping of the two emission layers might possibly contribute as well as reflection.

24. **Line 235:** ***"The phase lag observed between the emission layers of O2 and OI was induced by the background wind due to a shear."***

Where? How was this proven mathematically in any of the previous data / measurements / calculations presented?

Response:

Phase change can possibly occur when wave reflection occurs, and wave reflection are induced by wind. This implies possible formation of regions of $m^2 < 0$. The characteristics of ducts are observed during the two events considered in the may possible cause this kind of behavior.

25. **Line 236-238:** ***"Despite this phase lag, the mean phase propagation of these GWs shows that OH leads OI by ~06.58 min. Using this phase information and the period, Figure 6(a) is produced. Clearly, it is observed that the similar GW oscillation in the OH (red line) emission layer leads to the OI (green line) emission."***

None of these phase differences have clearly been shown. The calculations to obtain them have not been clearly demonstrated.

A sinusoid can be fit to anything. There needs to be a determination of how good the fit actually is. Figure 6 shows "reconstructed waves" but it is not clear where this is obtained from. Is this from the data shown in Figure 3? The original data should be plotted with the fits.

The discussion of "leading" and "lagging waves" needs to be tied back to the data more clearly, and this also needs to be put into context of the actual resolution of the measurements. Ultimately, instrument resolution is going limit the ability to determine phase differences.

Response:

Thanks very much for the comments and suggestions. They items mentioned above have been addressed in the new version of the manuscript

26. **The section "Momentum Flux and Wave Energy" will likely need to be redone with more explanation regarding how the different parameters were calculated and what assumptions were made for the calculations. Most importantly, how was the average temperature perturbation determined for the wave packets present in the measurements?**

Additionally, it does not seem that the kinetic energy calculations for a wave are not correct.

Response:

Due to your earlier comments, the calculation of the kinetic energy was removed. For the momentum flux estimation, more clarification has been given as to how the individual parameters have been determined.

27. **The conclusions that there are upward and downward propagating gravity waves need to be better supported. It seems like arbitrary wave periods were chosen from the wavelet analysis, and sinusoids were plotted based off of this. There needs to be more quantitative analysis performed and a justification for the reconstructed waves provided.**

Response:

As mentioned earlier, the wave periods were not chosen arbitrarily. They were chosen based on the intensity of the PSD, thus, the peak PSD corresponding to the time and period (for Wavelet analysis) and for the Lomb-Scargle (the period corresponding to the peak PSD).