Authors Response: Convex optimization of initial perturbations toward quantitative weather control egusphere-2024-1952

We would like to express our gratitude to the referees for the detailed comments and valuable questions. We should have highlighted the advantages of convex optimization in assessing the feasibility of weather control with small perturbations. Convex optimization can deal with various cost functions and constraints involving not only quadratic functions but also absolute values and maximum values. It is particularly suitable for evaluating the achievable performance in weather control and, therefore, essential for the future development of intervention technology. We need this kind of framework prior to a huge amount of social investment for the development of intervention technology. We will explicitly state the above motivations in the revised manuscript.

Below we respond to each of the referees' comments in the boxes.

Referee Comments 1

1. In this study, ℓ_2 and ℓ_1 norms were used. Figures 2 and 3 show that the results are significantly different for these two norms. Then, in the realistic weather control field experiments, which norm should be applied? Besides, why were these two norms used? How to choose a suitable norm?

Which: The norm to be applied depends on the desirable properties of the interventions. If the locations of interventions are strictly limited, the ℓ_1 norm should be used to obtain sparse solutions. If an arbitrary distribution of interventions can be realized, the ℓ_2 norm would be a natural choice to spread a modest magnitude of interventions over a region of interest. For example, atmospheric heating could focus on a limited number of points, for which the ℓ_1 norm might be suitable, while cloud seeding could spread over a region, for which the ℓ_2 norm might be suitable.

Why: In this study, the ℓ_2 norm and the ℓ_1 norm are used because minimization of these norms reduces to a quadratic programming problem (QP) or a linear programming problem (LP). QP and LP can be solved quite efficiently, and there are many reliable and easy-to-use solvers for QP and LP.

How: In general, a suitable norm can be chosen according to the achievable control performance and the required interventions. Although minimization of any norms is formulated as a convex optimization problem and can be solved efficiently, QP and LP are the most common and useful problems. Therefore, the ℓ_2 norm and the ℓ_1 norm are the most natural choices, and either one is chosen according to the desirable properties of the interventions as described above.

We will add these points in the revised manuscript, for example, in Subsection 2.2.

2. The optimization problem (4) seems to be solvable directly by solving a system of linear equations. If so, the optimization process may not be necessary.

The optimization problem (4) does not reduce to a linear equation in the case of the ℓ_1 norm. Even in the case of the ℓ_2 norm, the general form of the constrained norm minimization problem with an inequality constraint, presented in (7), does not reduce to a linear equation. The optimization problem (7) can represent a wide variety of problems, such as constraints on the magnitudes of inputs (state perturbations) and outputs (precipitations) and can be solved quite efficiently. These advantages of convex optimization motivate the formulation of the problem in this study. We will add some comments to Subsection 2.2 to clarify the above points.

3. In the proposed approach, the linearity assumption was made. The rationality of this assumption should be validated. For example, the results from the nonlinear simulation and linear simulation should be compared.

The linearity assumption was validated in the nonlinear simulation. Although the linearity assumption was made through the optimization of the initial perturbations, those perturbations were applied in a highly nonlinear NWP model, SCALE-RM, in the experiments. Additionally, the nominal response (Fig. R.1) was obtained with the highly nonlinear NWP model, although linear growth of state perturbations was assumed for the initial perturbations. We omit the results of linear simulation because they trivially satisfy all specifications and are less informative than the results of nonlinear simulation. The results of nonlinear simulation satisfy given specifications, i.e., constraints, reasonably well. These observations justify the linearity assumption.

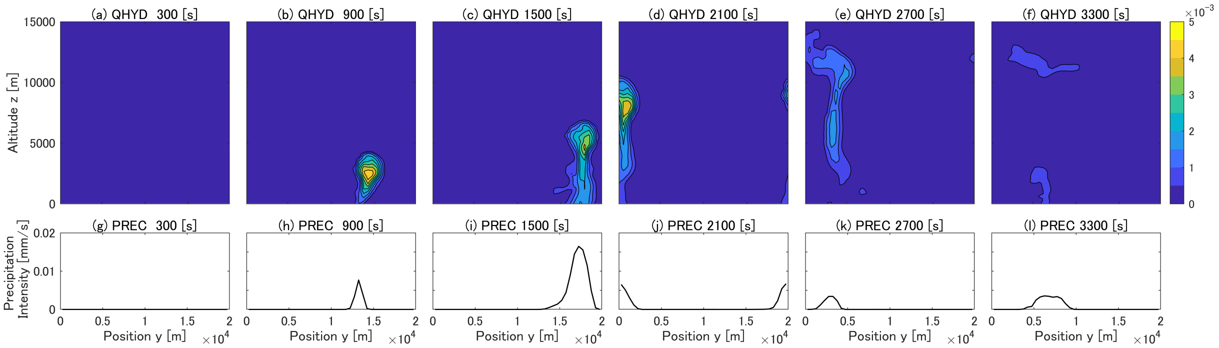

Figure R.1: Snapshots of (a)–(f) mass concentration of total hydrometeor (QHYD) and (g)–(l) precipitation intensity (PREC) in warm bubble experiment.

Assuming small rather than drastic perturbations in atmospheric states and precipitation is generally reasonable because the larger the perturbations, the more difficult their realization. Then, the proposed convex optimization approach is useful in assessing the feasibility of mitigating heavy rain disasters with small perturbations.

We will revise the manuscript to highlight the above points, possibly with an additional figure of the nonlinear simulation.

4. The sensitivity matrix S was obtained using the definition of partial derivative. For a relative simple model this study used, it can be obtained easily. But in a more realistic three-dimensional GCM model, calculating matrix S in this way is very difficult and time-consuming. How to overcome this problem in real application?

We can overcome the problem by 1) reducing the number of optimized variables and 2) exploring alternative computational methods or adopting different models to handle the sensitivity analysis more efficiently. Our main target is a regional climate model rather than a global climate model, and we can reduce the number of optimized variables by identifying effective grids or suitable bases for perturbations, which is one of our future work. The second possibility is to reduce the computational cost for sensitivity analysis by employing the adjoint method with ensemble approximation or AI-based weather prediction models that have been developed rapidly in recent years. In any case, we can freely choose the grids or bases for perturbations in control synthesis and make a trade-off between the computation cost and achievable control performance. Therefore, calculating the sensitivity matrix is not a fatal difficulty in the proposed approach. In this paper, we do not introduce any approximation or simplification, except for the finite difference approximation of the sensitivity matrix, to evaluate the best possible performance of convex optimization. We will add these points in the revised manuscript.

5. In the section 3.3, different control variables were examined. Could you please add a discussion about how to choose the control variables for weather control experiments in real application?

The control variables should be chosen according to the available intervention methods and their achievable performance. In general, different kinds of control variables can also be combined. The proposed approach can deal with various settings and therefore provides a useful tool to assess the achievable performance of various intervention methods. We will add these points in the revised manuscript.

6. Sections 3.2 and 3.3: the results of the numerical experiments show that the optimal perturbations can induce the atmosphere state to shift a desired state. But the physical processes about the state shift are unclear. Could you indicate the time development of the optimal perturbation and reveal the related physical processes?

We will include additional discussion on physical interpretations based on snapshots of changes in some atmospheric quantities in the NWP model before and after adding the perturbations (Fig. R.2). At $t = 1500$ [s] and $t = 2100$ [s] in Figs. R.1 and R.2, we observed increases in density ρ and decreases in the potential temperature θ at the top of the warm bubble. Such changes indicate that the perturbations suppressed the growth of the warm bubble and reduced precipitation. Furthermore, perturbations of the precipitation intensity in Fig. R.2 at different times and positions were successfully superposed to result in the distribution of accumulated precipitation in Fig. 2c in the manuscript, validating the proposed method.

7. Figure 3c displays that for ℓ_1 norm, the perturbed case is greater than the upper bound. This means that the perturbation does not satisfy the constraint condition. What is the reason? Does this reflect the important effects of nonlinear physical processes? If so, please make clear.

There is a mismatch between the linear model for optimization and the highly nonlinear NWP model for validation. This mismatch causes some discrepancies between the reference and actual distributions or the violation of constraints. However, the initial perturbations effectively reduce the accumulated precipitation, and therefore the proposed approach is validated. We will add an explanation about this in the revised manuscript.

8. Although optimization requires minimal perturbation, in practical applications, perturbation amplitude may still exceed the controllable range. How should we make decisions to control the weather in this situation?

We can easily impose a bound on the magnitude of the perturbations in both the initial conditions and the accumulated precipitation. In fact, we imposed an inequality constraint $q_v + \Delta q_v \geq 0$ in the experiment illustrated in Fig. 4 in the manuscript. We can also impose lower and upper bounds, such as $l_b \leq \Delta\rho\theta \leq u_b$ or $l_b \leq \Delta q_v \leq u_b$, to ensure that the optimized initial perturbations remain within realistic limits. Furthermore, we can also minimize the maximum precipitation with achievable ranges of initial perturbations to mitigate heavy rain disasters. This flexibility in control specifications is one of the advantages of the convex optimization approach. We will emphasize this point in the revised manuscript.

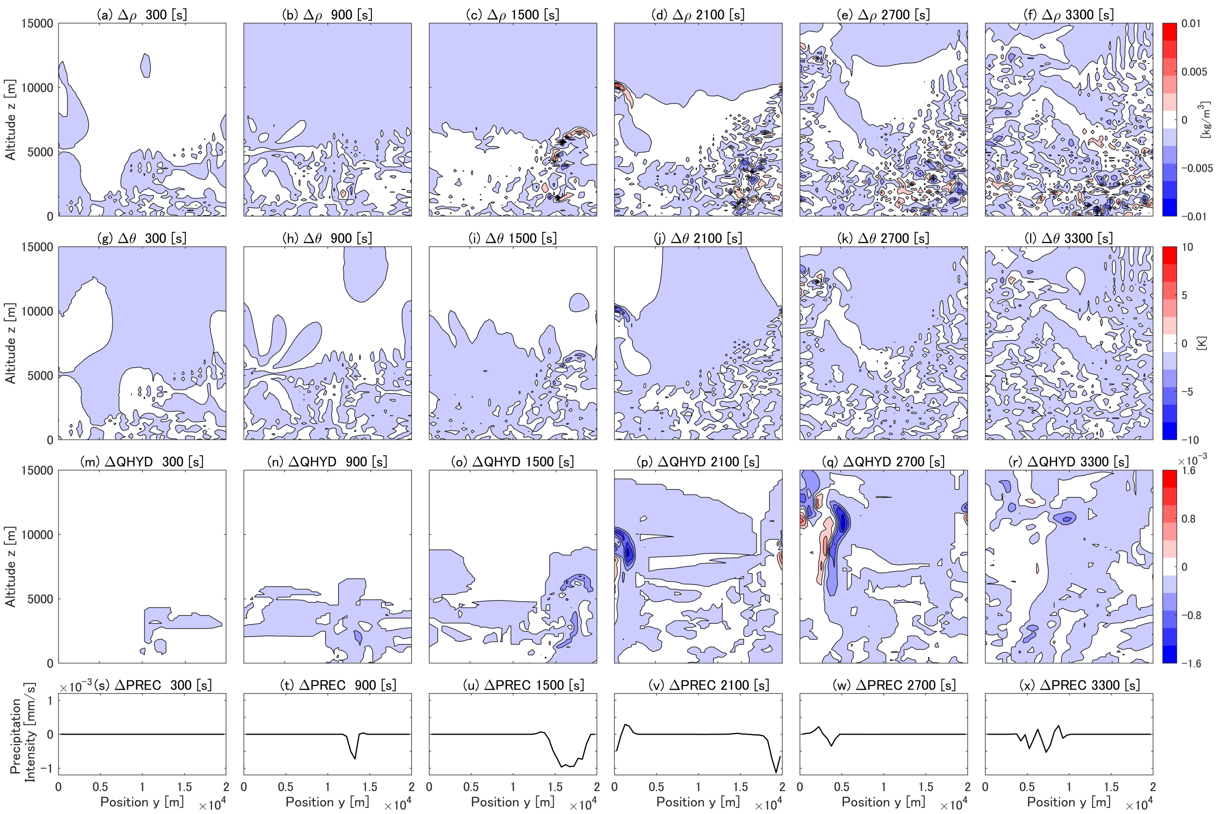

Figure R.2: Snapshots of perturbations in (a)–(f) density ρ, (g)–(l) potential temperature θ, (m)–(r) mass concentration of total hydrometeor (QHYD), and (s)–(x) precipitation intensity (PREC) with initial conditions of $\rho\theta$ perturbed according to the the minimum ℓ_1 norm solution for reducing accumulated precipitation to 90% of the nominal case.

Referee Comments 2

I regret I cannot recommend acceptance of the paper, even after a possible major revision. This is so because I consider that, in spite of the fundamental interest of the questions considered by the authors, the scientific content of the paper is not of sufficient novelty or originality to warrant publication.

The authors write that they have *demonstrated the possibility of controlling the real atmosphere by solving inverse problems and adding small perturbations to atmospheric states* (last sentence of abstract). They have not shown how those perturbations could in practice be added to atmospheric states. They consider in their numerical experiments perturbations of the product $\rho\theta$ of the density and the potential temperature, and of the specific humidity q_v, but they do not explain how these quantities could be perturbed in practice.

The authors are fully aware of the problem, and write (p. 2, l. 5) that the first of three remaining difficulties they have identified is that *there is no practical intervention technique for quantitative weather control at the present day.* They nevertheless write (same page, first sentence of third paragraph) *This paper overcomes the above three difficulties with a suitable problem setting.* There is no justification for that, at least certainly not as concerns the first of those difficulties.

There are similar statements at other places in the paper. Any claim by the authors that they have shown how to perturb the atmosphere for controlling weather is unfounded. The paper is only a numerical study of the sensitivity to initial conditions of the output of a Numerical Weather Prediction (NWP) model. If methods are found some day to actually perturb the atmosphere for controlling weather, this type of sensitivity study will of course be absolutely necessary. But I do not think anything more can be said at this stage.

We understand the concern of the referee about the lack of practical intervention techniques for quantitative weather control at present. Our study aims to address an important preliminary step: evaluating the potential performance of weather control using small perturbations before significant social investment is made in the development of such intervention technologies. The proposed convex optimization approach is particularly useful in evaluating achievable performance because it can deal with various cost functions and constraints that involve not only quadratic functions, but also absolute values and maximum values. Convex optimization is different from sensitivity analysis, although the problem in this study utilizes the sensitivity matrix.

Determining state perturbations rather than relying on particular methods of intervention is commonly seen in the literature on weather control (e.g., [HHLNG05, HHLGN06]). Although they deal with a realistic NWP model, these studies do not exploit convex optimization for flexible formulation and efficient optimization, which is different from this study. Moreover, some of the recent papers [MS22, SMR23, OTK23, KK24] in this journal propose methodologies to guide chaotic dynamics toward desirable behaviors, with the aim of future development in weather control. However, they deal with abstract and low-dimensional models such as Lorenz 63 and Lorenz 96 for theoretical research. There is still a large gap between those abstract models and realistic NWP models. In contrast, this study deals with a realistic NWP model and proposes a framework for quantitative control design.

We will clarify the above motivations, background, and contributions in the revised manuscript.

But even as a pure numerical sensitivity study, the present paper is not in my opinion of sufficient scientific novelty or interest for publication. The authors write (p. 2, ll. -6-5) *To the best of the authors' knowledge, this is the first method to compute control inputs that achieve quantitative specifications for weather control, all based on an NWP model.* That may be true as concerns specifically weather control, but controlling inputs of NWP models in order to achieve quantitative specifications for outputs has been done for a long time. I mention two examples.

- *Singular modes* are defined as initial perturbations that lead to the largest forecast error over a given forecast period. They are used, among other purposes, for defining the initial perturbations to be used in ensemble prediction [DL12]. In a linear setting, these modes are independent of their amplitude. They have been extended, by M. Mu and colleagues, under the name of Conditional Nonlinear Optimal Perturbations (CNOP), to nonlinear situations, in which a constraint on the initial amplitude of the perturbations is then included [MDWZ10].
- *Variational assimilation* is intended at defining the model state at a given time that leads to the model solution that fits most closely a given set of later observations. Variational assimilation has been used in operational NWP for a long time (see e.g., [RJKMS00]). Actually, determining initial conditions that fit later observations or that lead to specific conditions in the ensuing forecast is essentially the same problem.

The proposed method introduces a novel framework for weather control leveraging convex optimization, which sets it apart from the existing methods that the referee has mentioned. Specifically, unlike these existing methods, convex optimization provides greater flexibility in handling a wide range of cost functions and constraints, including not only quadratic functions but also absolute values, maximum values, and general ℓ_p norms ($1 \leq p \leq \infty$). The differences between the proposed method and existing methods are summarized in Table R.1. We will add Table R.1 and highlight the differences and advantages of convex optimization from the existing methods in the revised manuscript.

Table R.1: Comparison of methods based on cost function and constraint.

Method	Cost Function	Constraint
Singular Vector	Quadratic Function of Output	Tangent Linear Dynamics
Conditional Nonlinear Optimal Perturbation	Quadratic Function of Output	Nonlinear Dynamics
Variational Assimilation	Quadratic Functional of Input and Output	Nonlinear Dynamics
Convex Optimization	Convex Function of Input and Output (Linear Function, Quadratic Function, Absolute Value, Maximum Value, and ℓ_p Norm ($1 \leq p \leq \infty$))	Tangent Linear Dynamics (Linear Equality) and Non-positivity of Convex Function of Input and Output

I do not think that the possible purpose of weather control should lead to specific methods for the definition of initial perturbations. In any case, nothing in the present paper, as concerns the numerical method to be used, seems specific to weather control and any other method could *a priori* be used as well.

Convex optimization is not a specific method for numerical solutions, but a wide class of problems with favorable properties, particularly for large-scale problems. It is meaningful to demonstrate that convex optimization can deal with various specifications in weather control and results in a promising response of a realistic NWP model. We will emphasize this point in the revised manuscript.

But even concerning the purely numerical aspects, the present paper does not in my opinion bring anything that is instructive enough for publication. The authors use a strictly linear method for identifying the required initial perturbations (Eq. 7). From what I understand, if the initial perturbations achieved what they are meant to achieve, the deviations in, e.g., Figure 2d should be zero. They are not (and are large enough, in comparison with the results shown on Figure 2c, not to be due to round-off errors). The authors do not discuss that aspect. An obvious explanation could be that the basic model is nonlinear, and that the linear approximation of Eq. 7 is not sufficient to determine the optimal intended initial perturbations.

We agree with the referee that there is a mismatch between the linear model for optimization and the highly nonlinear NWP model for validation. This mismatch causes some discrepancies between the reference and actual distributions or the violation of constraints. However, the initial perturbations still effectively reduce the accumulated precipitation. Based on these observations, we conclude that the proposed convex optimization approach is promising as a synthesis tool for quantitative weather control to mitigate heavy rain disasters.

Assuming small rather than drastic perturbations in atmospheric states and precipitation is generally reasonable because the larger the perturbations, the more difficult their realization. Then, the proposed convex optimization approach is useful in assessing the feasibility of mitigating heavy rain disasters with small perturbations.

We will highlight the above points and add more instructive discussions on the physical meaning and limitation of the results in the revised manuscript with additional figures of nonlinear simulation (Figs. R.1 and R.2).

I mention that, in the two examples given above, the numerical optimization can be (and is often) exactly performed (at least to computer accuracy) with a full nonlinear model. That is made possible by the use of the *adjoint method*, which allows to compute economically the gradient of one scalar output of the model with respect to all inputs [CR97]. The gradient is introduced an iterative minimization process, which can actually be described as an example of what the authors mention in their conclusion as *successive linearization and optimization of perturbations*. My point is that this approach has been implemented for a long time.

As highlighted by the referee, the adjoint method and successive linearization and optimization have been widely used because they are common components of optimization methods. However, convex optimization is characterized by the convexity of cost functions and constraints rather than those common components. The framework of convex optimization with quite useful properties has not been fully exploited in weather control with performance criteria and constraints of reasonable generality. In the revised manuscript, we will highlight the difference and advantages of convex optimization from existing methods.

Sensitivity of future weather to initial conditions is a fundamental and very important question, and I do not want to look dismissive of the work of the authors. I encourage them in their interest in that question. But the present paper is not sufficiently instructive for publication. One point in the paper that has arisen particular interest for me is the comparison that the authors make between the ℓ_1 and ℓ_2 norms. The ℓ_2 norm is typically used in this kind of optimization problem, for the simple reason that quadratic functions are the easiest to minimize. Further study of the specific qualities and appropriateness of various norms, both from the physical and numerical points of view, may be instructive, but is not sufficiently developed in the present case.

Although the problem in this study utilizes the sensitivity matrix, convex optimization is different from sensitivity analysis. The purpose of this paper is to introduce the framework of convex optimization in weather control and to demonstrate its usefulness. Convex optimization can deal with various cost functions and constraints that involve convex functions such as absolute values, maximum values, and general ℓ_p norms ($1 \leq p \leq \infty$). The use of the ℓ_1 norm and more general norms other than the ℓ_2 norm has not been common in the literature on weather control. In particular, ℓ_1 norm minimization results in sparse solutions, which is potentially useful in weather control. We will add some comments to clarify the above points in the revised manuscript. In addition, we will incorporate additional figures (e.g., Figs. R.1 and R.2) and discussion from the physical and numerical points of view to enhance the instructional value of the paper.

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