

Although the authors have improved the quality of the manuscript with this revision, there remain some significant errors in the formulation and presentation of the mathematical model (Section 3.3) that I believe must be corrected before the manuscript can be considered for publication. Fortunately, the specific equations that have been used to calculate the time evolution of sulphate peaks appear to be correct, even if their mathematical derivations were flawed. My hope is that the derivations will be corrected so that mathematically-inclined readers will accept the model (and its results) as valid, and I will therefore focus on these corrections here. I believe that the results of this study are scientifically significant will prove to be valuable to ice core scientists following correction of the following errors in the model and/or its presentation:

- (1) Eq. 8 is, as defined, physically nonsensical: a “layer thickness” cannot simply be assigned units of “m yr⁻¹”, any more than one can accurately claim that “the distance between New York and Chicago is 790 miles per hour.”

The correct form of this equation was derived by Nye [1963] and has been used by many others since (e.g., Cuffey and Paterson [2010]):

$$\frac{\lambda}{\lambda_0} = \frac{z}{H}$$

Here λ_0 is the original (ice-equivalent) layer thickness when precipitated on the surface of the ice sheet (located at constant height H above the bed) and λ is the thickness of the same layer when located at height z above the bed some time later. In this expression, λ , λ_0 , z , and H all represent clearly defined spatial dimensions, and therefore all have units of length.

- (2) Reading between the lines, it appears that the motivation for assigning units of “m yr⁻¹” to the layer thickness is to allow derivation of Eq. 9. through a simple algebraic manipulation of Eq. 8. This manipulation requires equating $v_z(z)$ to z – a mathematical impossibility, given their different dimensions. Additionally, the origin of the negative sign in Eq. 9 is not immediately apparent, as it does not appear in Eq. 8.

Eq. 9 can be derived by starting with Nye [1963]’s assumption that “the vertical plastic strain-rate along any vertical line in the ice is uniform at any given instant,” which can be expressed mathematically as

$$\dot{\epsilon}_{zz} = -\frac{a_{ice}}{H}.$$

Here the negative strain rate indicates vertical compression of the ice, which is necessary to maintain a constant ice thickness H despite the continuous addition of new ice to the surface (at rate a_{ice}).

To obtain the vertical ice velocity v_z at height z above the bed, we integrate the strain rate upwards from the base of the ice (where $v_x = v_y = v_z = 0$; this represents the second of the two assumptions that form the basis of Nye [1963]'s model):

$$v_z(z) = \int_0^z \dot{\epsilon}_{zz} dz = \int_0^z -\frac{a_{ice}}{H} dz = -\frac{a_{ice}z}{H}$$

This method arrives at the correct expression for $v_z(z)$ without assigning physically nonsensical units to ice layers.

- (3) In the revised manuscript, x is defined as the “distance from the centre of the reference frame” (p.8, l.189-190). This is ambiguous: it defines neither specifically what “the reference frame” is, nor whether x increases upwards (towards the ice surface) or downwards (towards the bed). From context, it can be inferred that the “centre of the reference frame” is “the center [or location of maximum concentration] of a given sulphate concentration peak” and that x increases upwards, but these details should be stated clearly.

*As an aside, the “ \times ” in Eq. 12 is unnecessary and can be omitted.

- (4) Relatedly, in my review of the first version of this manuscript, I asked whether the model was framed in a Eulerian coordinate system (i.e., one that is spatially fixed) or a Lagrangian coordinate system (which follows an individual material parcel as it moves through time), as the distinction between x and z was not made in the initial manuscript. The authors responded that the model employed a Eulerian coordinate system.

This is not correct: the coordinate system tracks a given sulphate peak as it advects downward due to ice deformation and layer thinning, maintaining position $x(t) = 0$ even though the peak is continuously moving towards the base of the ice as time progresses. *The model is therefore cast in a Lagrangian coordinate system.*

This can also be seen by comparing the expression for $v_z(z)$ given in my point (2) above with the revised manuscript's Eq. 11. As z -space is defined (with $z = 0$ at the bed and $z = H$ at the ice surface), z is positive wherever ice is present. So framed, vertical ice velocities are negative everywhere within the ice column, with all ice parcels moving downward towards the bed through decreasing z values. Mathematically, ice layers thin in z -space because ice in the upper portions of a given layer have greater negative velocities than does ice in the lower portions of the layer.

In contrast, x -space is defined to have value $x = 0$ at the location of the sulphate peak, with negative x values below the centre of the peak and positive x values above the peak centre. As a result of this, the sign of the vertical velocity $v_x(x)$ differs for points above and below the peak's centre: it is negative for values $x > 0$ (i.e., downward for ice

above the peak), but positive for values $x < 0$ (i.e., upward for ice below the peak) – *even though the entire sulphate signal is being advected downwards in z-space.*

This isn't a problem, mathematically speaking: the flip in the velocity's sign is necessary to maintain thinning of the sulphate peak in x -space. Rather, I bring this up to again demonstrate that *the model is cast in a Lagrangian coordinate system.* Stating this explicitly would benefit the reader.

- (5) It's not entirely accurate to state that "The Nye model ignores density changes" (p.9, l.203). Rather, the model requires that "ice-equivalents [be] used so as not to include snow and firn compaction in the strain-rate" (Nye [1963]).

Ice is commonly treated as incompressible in models, as this makes the deformation problem much more tractable. In the case of Nye [1963]'s model, this assumption allows the flow field to be uniquely defined with only the two simple assumptions given in points (1) and (2) above.

- (6) There are some minor formatting errors related to Eq. 2, 5, 6, and 8 (for which text is included on the same line as the numbered equations), 9, and 12 (for which equation numbers appear on separate lines from their respective equations).