

# 1 Internal tides off the Amazon shelf in the western tropical Atlantic: Analysis of 2 SWOT Cal/Val Mission Data

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## 12 Abstract

13 This study focuses on the internal tides (IT) off the Amazon shelf in the tropical Atlantic. It is based  
14 on 2 km horizontally gridded observations along the swaths of SWOT (Surface Water and Ocean  
15 Topography) track 20 during the calibration/validation phase (Cal/Val, 1-day orbit) from late March to  
16 early July 2023. We evaluate the amplitude of M<sub>2</sub>, N<sub>2</sub> and S<sub>2</sub> frequencies and use the M<sub>2</sub> atlas as an  
17 internal tide correction model for SWOT observations. Internal tide amplitudes (models or atlases) are  
18 first derived by harmonic analysis of the SWOT sea level anomaly (SLA). The estimation is improved by  
19 performing a principal component analysis before the harmonic analysis. The results compare very  
20 well with the high-resolution empirical tide (HRET) internal tide model, the reference product for  
21 internal tide corrections in altimetry observations. The coherent mode 1 and mode 2 M<sub>2</sub> can be  
22 distinguished in the internal tide model derived from SWOT, while the higher modes with their strong  
23 SLA signature are seen mostly in the incoherent part. In comparison to HRET, the correction of SWOT  
24 observations with SWOT-based atlases may be more relevant for this track.

## 25 Introduction

26 The launch of the SWOT (Surface Water and Ocean Topography) mission at the end of 2022  
27 certainly marks a new phase in spatial altimetry. SWOT is equipped with the KaRIn instrument, a Ka-  
28 band radar interferometer capable of measuring the sea surface topography with unprecedented  
29 resolution in two-dimensional swaths. KaRIn consists of two antennae that take 2D measurements in  
30 two 50 km-wide swaths separated by a 20 km gap covered by the conventional nadir radar altimeter  
31 also carried by the mission. The accuracy of SWOT's instruments is such that SWOT should be able to  
32 observe the ocean down to a spatial scale of 15-30 km (Morrow et al., 2019; Dufau et al., 2016; Wang  
33 et al., 2019), thus, complementing our 2D view of the ocean with Topex/Poseidon class nadir altimetry,  
34 which is limited to scales larger than 150 km (Chelton et al., 2011; Ballarotta et al., 2019) along one-  
35 dimensional tracks rather than two-dimensional swaths. The main oceanographic objective of the  
36 SWOT mission is to characterize mesoscale and sub-mesoscale ocean circulation (Fu et al., 2012; Fu  
37 and Ubelmann, 2014). However, ocean processes at the scales targeted by SWOT (150-15 km)  
38 encompass both "balanced" geostrophic motions, as well as surface and internal inertia-gravity waves  
39 at tidal frequencies. The correction of internal tides (IT) surface signatures presents a significant

40 challenge to the useability of SWOT data, considering that the spatial scales of these waves overlap  
41 with those of balanced motions. Conversely, the exploitation of SWOT data to study IT is an  
42 opportunity for learning more about these waves and quantifying their impacts in the ocean.

43 Efforts have been made in recent years to map internal tides using conventional altimetry  
44 observations. This was made possible by the fact that the internal tide has a SSH (Sea Surface height)  
45 signature of the order of one to several centimeters (Ray and Mitchum, 1997). However, the coarse  
46 sampling in both space and time of conventional altimetry is a hindrance. To derive spatially  
47 continuous high-resolution maps of the internal tide SSH from the sparse altimeter sampling, Dushaw  
48 (2015), Zhao et al. (2019) and Zaron (2019) used least-squares techniques to fit kinematic wave  
49 solutions to nadir altimetry. Ubelmann et al., (2022) proposed jointly estimating internal tides and  
50 mesoscale eddies to produce 2D maps of internal tides from conventional altimetry observations. The  
51 advent of SWOT presents an opportunity to validate these internal tide maps using direct 2D  
52 observations of the ocean. However, there is still some debate about the extraction of the internal  
53 tidal signal along SWOT swaths. The first objective of our study is thus to estimate the internal tidal  
54 signal along the SWOT swaths. Le Guillou et al., (2021) propose a data assimilation method coupled  
55 with a simple dynamical model to separate internal tides and balanced motion in SWOT data. The  
56 possibility of using deep learning to access internal tide signals is raised by Wang et al. (2022). Without  
57 questioning these methods, we will show that classical methods of harmonic analysis and principal  
58 component analysis (PCA) can be used to obtain internal tide maps from SWOT data.

59 Following the linear theory of ocean vertical modes, internal tides can be decomposed as a sum of  
60 orthogonal baroclinic modes (Gill, 1982; Kelly et al., 2016). The first modes (mode 1 and mode 2)  
61 propagate over hundreds or even thousands of kilometers. Higher modes have much shorter  
62 wavelengths and are likely to dissipate close to the internal tide generation site, due to their low group  
63 velocity and high shear (St Laurent and Garrett, 2002; Vic et al., 2019) and therefore could barely be  
64 observed in classical nadir altimetry observations. In practice, the internal tide is separated into the  
65 so-called coherent and incoherent internal tides. The coherent internal tide is the part of the internal  
66 tide which remains phase-locked with the generating barotropic tide over an arbitrary period and is  
67 easily obtained by harmonic analysis over the targeted period. Consequently, the residual that escapes  
68 harmonic analysis constitutes the incoherent internal tide. The amplitude, phase, and trajectory of  
69 incoherent internal tide results from refraction, reflection, and advection of internal tide by the ocean  
70 background circulation including eddies, currents, and stratification (Ponte and Klein, 2015; Nelson et  
71 al., 2019; Buijsman et al., 2017; Dunphy et al., 2017; Dunphy and Lamb, 2014; Duda et al., 2018; Savage  
72 et al., 2020; Barbot et al., 2021). The incoherency of the internal tide makes it difficult to correct for in  
73 altimetry observations. In the SWOT data processing protocol (Dibarboure et al., 2024), the coherent  
74 part of the internal tide is corrected using the HRET (High-Resolution Empirical Tide) model of Zaron  
75 (2019). The second objective of this study thus concerns the correction of the coherent internal tide in  
76 the SWOT data: between HRET (the reference model) and the internal tide estimates directly on SWOT,  
77 which is most relevant for correcting the internal tide on SWOT data?

78 Like the barotropic tides, the internal tides are a mixture of long- and short-period waves, among  
79 which are the main astronomical tides, such as the diurnal waves (O1, K1, P1) and the semi-diurnal  
80 waves (M2, S2, N2, K2). Due to the long repeat cycles of altimetry satellites, short tidal periods are  
81 aliased to longer periods (Le Provost, 2001). The M2 tide, for example, is aliased to 62.11 days for the  
82 TOPEX/Jason 10-day orbit (9.92 days precisely). With SWOT sampling, M2 is aliased to 66.02 days or  
83 12.35 days (Table 1), depending on whether we consider the 21-day final science orbit or the 1-day  
84 calibration/validation (Cal/Val) orbit (0.99343 days exactly). Table 1 gives an overview of the aliasing  
85 periods of the main diurnal and semi-diurnal tidal frequencies on the SWOT Cal/Val orbit. Table 1 is

86 completed by the Rayleigh criterion values which provide information on the duration of the records  
 87 needed to separate the different waves. SWOT was maintained in its Cal/Val orbit for about 6 months,  
 88 providing slightly more than 3 months of usable data from March to early July 2023. Our study is based  
 89 on this unprecedented 1-day orbit database and concerns observations along a single SWOT track in  
 90 the Atlantic Ocean.

91 **Table 1:** Period of aliasing (in days, second line) and separability following the Rayleigh criterion (in  
 92 days, from the third line to the end) of main tidal waves for SWOT's 1-day orbit.

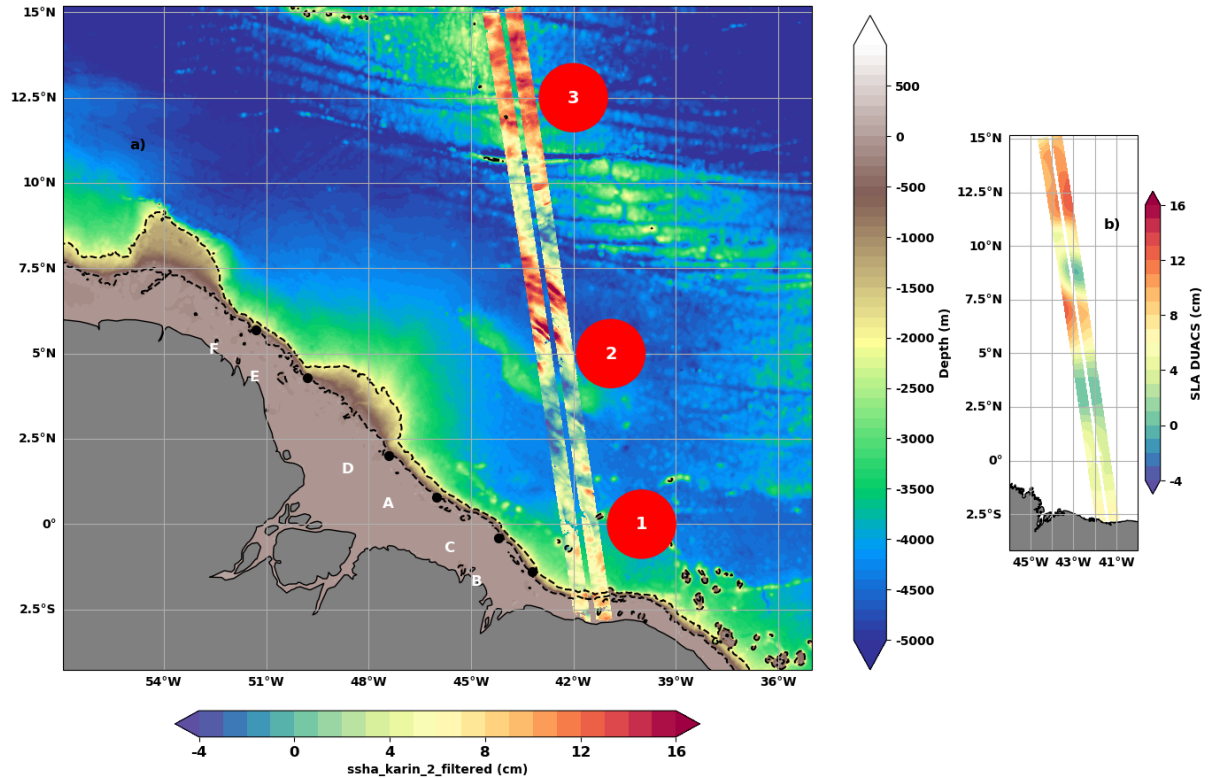
	M2	S2	N2	K2	O1	P1	K1	Sa	Ssa
Periods	12.35	75.60	8.53	129.01	12.97	106.94	258.03	365.26	182.62
M2	-----	14.77	27.55	13.66	258.03	13.97	12.97	12.79	13.25
S2	-----	-----	9.61	182.62	15.66	258.03	106.94	95.34	129.01
N2	-----	-----	-----	9.13	24.9	9.27	8.82	8.73	8.95
K2	-----	-----	-----	-----	14.42	624.89	258.03	199.47	439.51
O1	-----	-----	-----	-----	-----	14.77	13.66	13.45	13.97
P1	-----	-----	-----	-----	-----	-----	182.62	151.20	258.03
K1	-----	-----	-----	-----	-----	-----	-----	878.92	624.89
Sa	-----	-----	-----	-----	-----	-----	-----	-----	365.22

93

94 This study focuses on the Cal/Val track 20 off the Amazon shelf in the western tropical Atlantic  
 95 between 2°S and 15°N (Figure 1). The track has been chosen because the Amazon shelf is one of the  
 96 hot spots for internal tide generation in the ocean (Arbic et al., 2012; Solano et al. 2023; Niwa and  
 97 Hibiya et al., 2011). The region is marked by strong seasonal cycles of stratification, circulation and  
 98 eddies that regulate the generation and propagation of internal tides (Barbot 2021, Tchilibou et al.,  
 99 2022). The stratification is modulated by freshwater inflows from precipitation (under the inter-  
 100 tropical convergence zone) and rivers (Amazon and Para rivers). The strong western boundary current,  
 101 the North Brazil Current (NBC), controls the extension of the Amazon's plume and develops a double  
 102 retroflexion into the Equatorial UnderCurrent (EUC, around 2°S-2°N) and the North Equatorial  
 103 CounterCurrent (NECC, around 5°N-8°N). The barotropic and baroclinic instabilities of these currents  
 104 generate some of the eddies present in the region (Aguedjou et al., 2019). Internal tides generated  
 105 between the isobaths 100 and 2000 m along the shelf break propagate mainly from the six sites  
 106 indicated in Figure 1 (Tchilibou et al., 2022; Assene et al., 2024). Between March and July, the  
 107 pycnocline is shallow, the mesoscale activity and currents are low, consequently, internal tides tend to  
 108 keep more coherent (Tchilibou et al., 2022). During the rest of the year, the pycnocline is deeper,  
 109 mesoscale and currents are strong, and, consequently, the incoherence of internal tides increases as  
 110 their reflection and advection by the circulation intensifies. As they evolve, internal tides disintegrate  
 111 into nonlinear internal solitary waves (Jackson et al., 2012; Alford et al., 2015; Egbert and Erofeeva  
 112 2021). Packets of nonlinear internal solitary waves (ISWs) have been reported along the Amazon  
 113 continental shelf and offshore (Lentini et al., 2016; Bai et al., 2021, Brandt et al., 2002; Magalhães et  
 114 al., 2016). They are highly active in the area (4-8°N /40-45°W, see Figure 2 of de Macedo et al., 2023)  
 115 of concentration of internal tides rays emanating from sites A and D, and they have a seasonal cycle of  
 116 occurrence and wavelengths in agreement with those of internal waves (de Macedo et al., 2023).  
 117

118 The orientation of SWOT track 20 in this part of the ocean is such that it intersects three areas with  
 119 potentially different dynamics (Tchilibou et al., 2022). Between 2.5°S and 2.5°N (area 1, Figure 1), the  
 120 track is in the path of internal tides generated at points B, C and, to a lesser extent, A. In area 2,  
 121 between 2.5°N and 8°N (Figure 1), the track crosses the zone of interaction between internal tides and  
 122 mesoscale. Finally, area 3, north of 10°N (Figure 1), lies on the mid-Atlantic Ridge, where some IT can  
 123 likely be generated also. We will keep all this in mind when interpreting our results.

124 The paper is structured as follows: The data used, the evidence for the presence of internal tides in  
 125 the SWOT data and the variability of the SLA at different scales are presented in section 1. The  
 126 amplitude of the internal tides is first estimated from the SWOT data in section 2. In section 3, the  
 127 estimation of internal tides is improved by introducing PCA. The SWOT based internal tide models and  
 128 HRET are further compared in section 4. The paper ends with a conclusion and discussion.



129  
 130 **Figure 1:** a) Bathymetry (m) off the Amazon shelf in the western tropical Atlantic. SLA KaRin (cm) on  
 131 08 April, 2023, along track 20 of SWOT's 1-day cycle. The main internal tide generation sites are marked  
 132 by the letters A to F. The 200 and 2000 m isobaths are dotted. The circles locate area 1 (2.5°S to 2.5°N),  
 133 area 2 (2.5°N to 8°N) and area 3 (north of 10°N) along the track. b) large scale structure from DUACS  
 134 at the same date.

135 **1- Data and Variability: Evidence of IT propagation at different scales**

136 **1.1- Description of the database:**

137 We use the V0.3 version of the Level 3 (L3) SWOT products, published in December 2023 on the  
 138 AVISO website (<https://www.aviso.altimetry.fr/en/missions/current-missions/swot/access-to-data.html>, last accessed on 06/11/2024) and the cluster of the CNES (Centre National d'Etudes Spatiales). The data, made up of several variables, are provided on regular horizontal grids of 2 km by 2 km. Using the naming convention used in the CNES cluster dataset, we have defined the SLA by equation 1 below:

143 
$$\text{SLA} = \text{ssha\_karin\_2\_filtered} + \text{internal\_tide\_hret} - \text{duacs\_ssha\_karin\_2\_oi}, \quad (1)$$

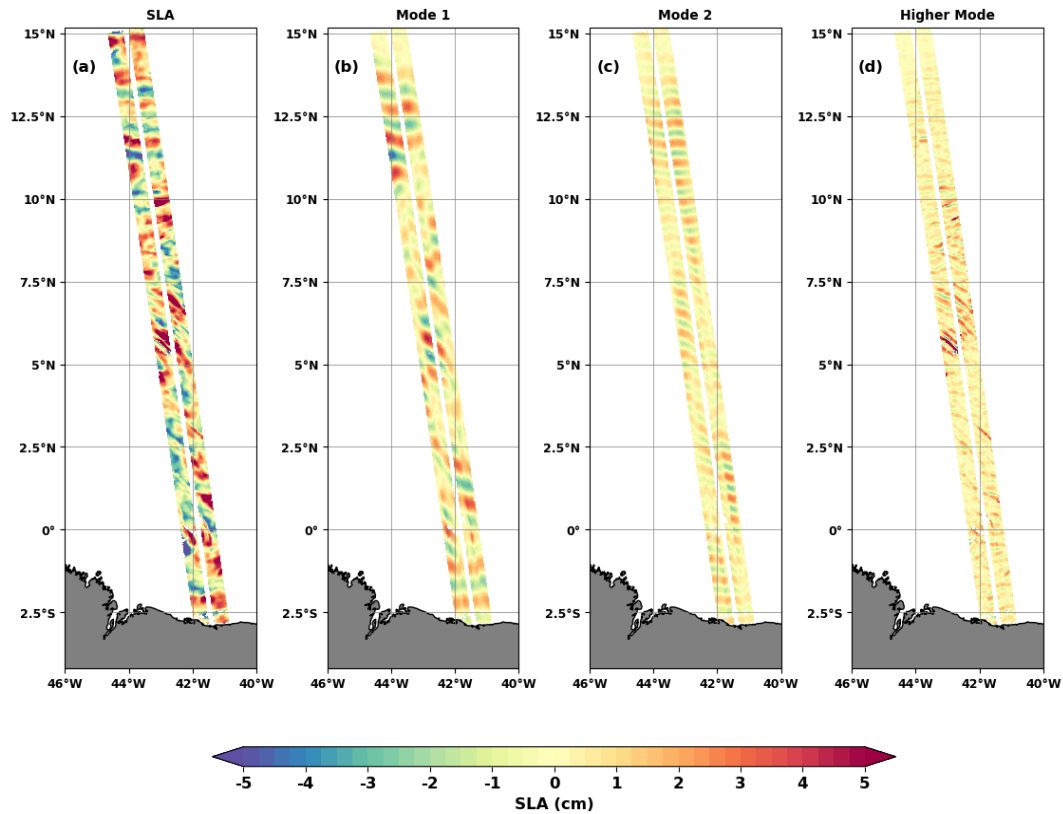
144 The first term on the right, 'ssha\_karin\_2\_filtered' (the same as ssha\_noiseless on AVISO), is the SWOT  
 145 observation at the two KaRin swaths only. We exclude SWOT nadir observations, to focus on the  
 146 SWOT's potential to observe directly 2D maps of the ocean. The ssha\_karin\_2\_filtered has been

147 denoised using data-driven machine-learning noise reduction and corrected from all the classic  
148 physical, instrumental and environmental corrections applied in altimetry (Dibarboure et al. 2024). The  
149 tidal corrections applied are FES2022 model (Lyard et al., personal communication; Lyard et al., 2021)  
150 for the barotropic tide and HRET for the internal tide (Zaron, 2019). We reintroduced HRET's internal  
151 tide SSH (internal\_tide\_hret), so that our final SLA contains the total internal tide signal. The last term  
152 'duacs\_ssha\_karin\_2\_oi' corresponds to the DUACS Maps of Sea Level Anomaly (MSLA) interpolated  
153 on SWOT swaths (Ballarotta et al., 2023; Ubelmann et al. 2015, 2021). It removes the large-scale ocean  
154 signals and particularly the mesoscale eddies that can mask internal waves at these latitudes. On track  
155 20, we have the SLA from March 29 to July 10, 2023, i.e. 104 cycles with completely or partially filled  
156 swaths. We have removed the mean SLA from the entire Cal/Val mission.

157 We recall that HRET is an empirical estimate of the internal tides at the M2, S2, K1 and O1  
158 frequencies. The variable internal\_tide\_hret in the SWOT data and HRET model in this paper refers to  
159 the HRETv8.1 version (Zaron, 2019). This version was developed by analyzing 25 years (1993-2017) of  
160 exact-repeat mission altimetry including the TOPEX/Poseidon-Jason missions, the ERS-Envisat-AltiKa  
161 missions and the GEOSAT Follow-On mission. The implementation of HRET involves a local two-  
162 dimensional Fourier analysis of the along-track data, and the determination of the coefficients of a  
163 spatial model by weighted least-squares fitting (second order polynomials fitting). The estimated tidal  
164 fields are gridded on a regular latitude-longitude grid by weighted averaging, and a mask is used to set  
165 the values to zero in regions where the estimate is too noisy. HRET includes mode 1 and mode 2  
166 internal tides, but mode 2 is very weak in the present study area.

## 167 **1.2- Evidence of IT propagation at different scales:**

168 The snapshots in Figure 2a show very fine-scale crest-like structures superimposed on positive and  
169 negative SLA spaced tens and hundreds of kilometers apart. The scenario repeats itself on the other  
170 cycles (see movie in the supplementary material), indicating that SWOT likely sees internal waves of  
171 different spatial scales. We submitted the SLA to spectral analysis to learn more about its frequency  
172 and wavelength. The 2D FFT (Fast Fourier Transform) spectra are computed in the time (cycles) and  
173 along-track (latitude) dimensions and then averaged over the cross-track (longitude) dimension. A  
174 25 % cosine taper window or Tukey 0.25 window is used for windowing. The wavenumber-frequency  
175 (Figure 3a) was integrated to derive the wavenumber spectrum (Figure 3b) and the frequency  
176 spectrum (Figure 3c).



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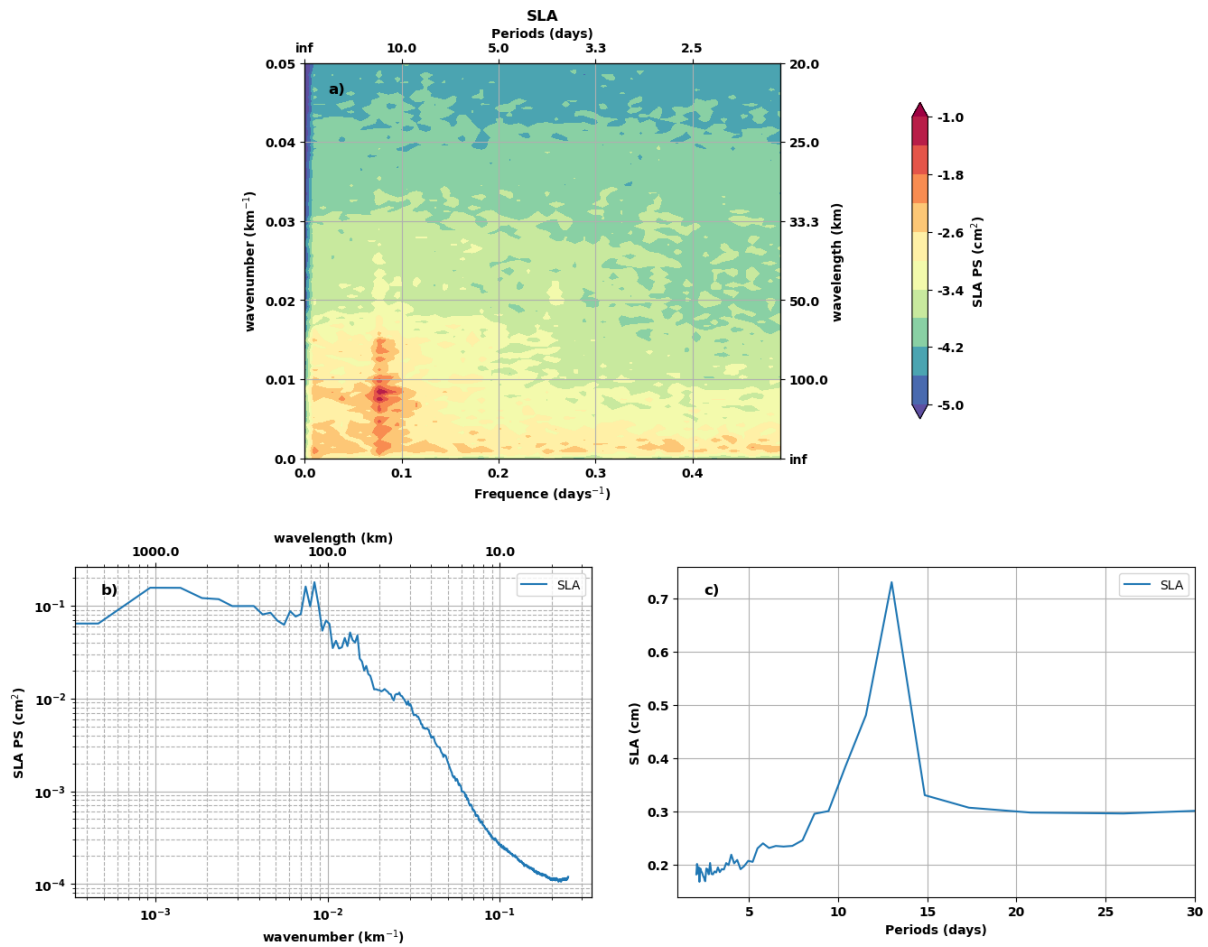
178 **Figure 2:** Snapshot of SWOT SLA on April 8, 2023. a) Total SLA, b) Mode 1 FFT-filtered SLA (180-90 km),  
 179 c) Mode 2 FFT-filtered SLA (80-60 km) and d) Higher mode FFT-filtered SLA (50-2 km).

180

181 The wavenumber-frequency (Figure 3a), the wavenumber (Figure 3b) and the frequency (Figure 3c)  
 182 spectra of SWOT SLA indicate that the dominant signal is M2 aliased to 12.22 days (see Table 1). At  
 183 the M2 aliased frequency, the energy is greatest between 180-90 km and between 80-60 km (Figure  
 184 3a), leading to the spectral peaks in Figure 3b. These two wavelength bands correspond well to the  
 185 theoretical baroclinic mode 1 and 2 scales expected for the internal tide in this region (Zhao, 2021).  
 186 We isolated the SLA for these two wavelength bands using FFT filtering. When filtering, the FFT is  
 187 calculated on the along-track dimension. Snapshots of the Mode 1 and Mode 2 SLA are shown in  
 188 Figures 2b and 2c for the same day as Figure 2a, revealing more of the SLA's wave-like behavior.

189 Figure 2d shows the FFT-filtered SLA between 50-2km. This band contains all the small-scale  
 190 structures, including the very remarkable and intense one that appears as wave crests on the SLA. On  
 191 the wavenumber-frequency spectrum (Figure 3a), the energy maximum at frequency M2 extends to  
 192 scales smaller than 50 km. According to Barbot et al., (2021), this could be associated to internal tide  
 193 of mode 3, mode 4 and mode 5. We therefore consider the 50-2 km band as consisting of higher  
 194 modes.

195



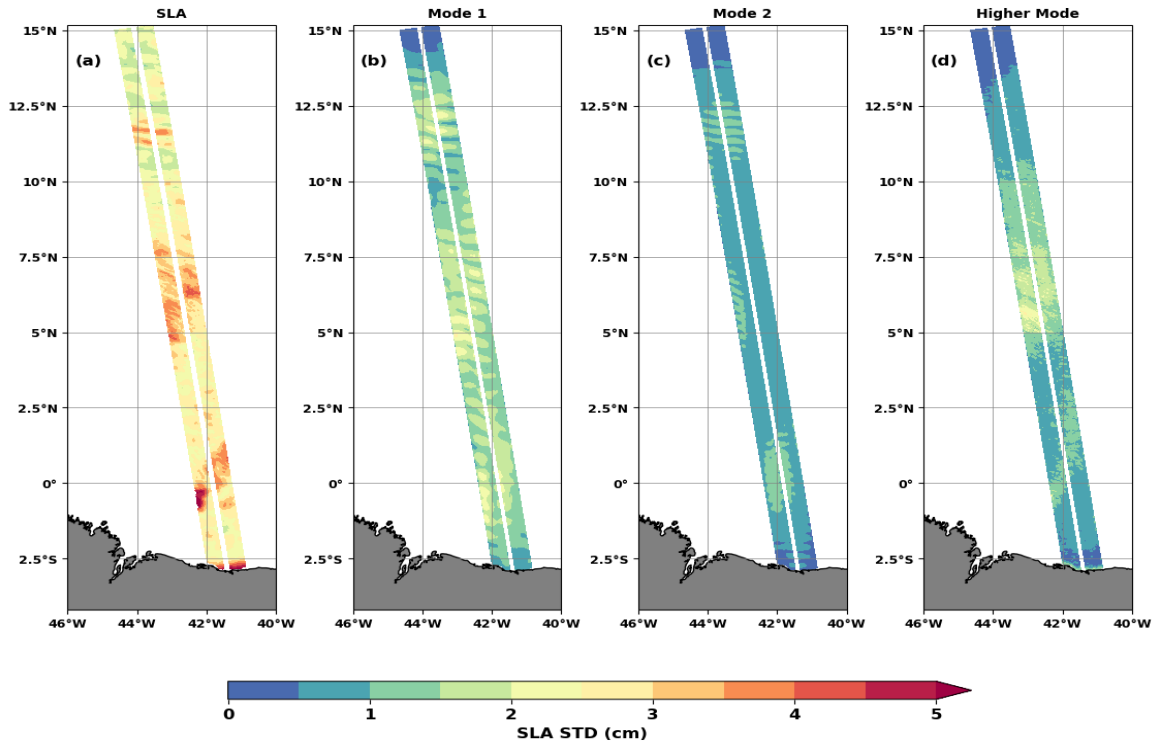
196

197 **Figure 3:** a) Wavenumber-frequency , b) wavenumber and c) frequency spectra of the total SLA.

198 **1.3- Variability analysis of IT observations:**

199

200 Analyses of SLA variability are completed by calculating the standard deviations of the total and the  
 201 spatially FFT-filtered SLAs in the wavelength bands defined above. Over the Cal/Val period, SLA varies  
 202 between 1 and 5 cm under track 20 (Figure 4a). Apart from the area very close to the coastline, there  
 203 are three main patches of maximum variability, each located in one of the dynamic areas highlighted  
 204 in the introduction. The maximum variability of the SLA in area 1 (2.5°S-2.5°N) is mainly due to the  
 205 regular mode 1 internal tide flux likely coming from sites A, B and C (Figure 4b). Mode 2 and higher  
 206 modes contributions are secondary (Figures 4c and 4d) in area 1. Higher modes have a major impact  
 207 on the variability in area 2 where they make the SLA vary by 2 to 3 cm (Figure 4d), i.e. almost of the  
 208 same order as mode 1 in the same area. As area 2 is far from the generation sites of the Amazonian  
 209 shelf-break, the higher modes here are likely to originate from desintegration of mode 1 and mode 2.  
 210 In area 3, SLA variability is driven mainly by mode 1 and mode 2.



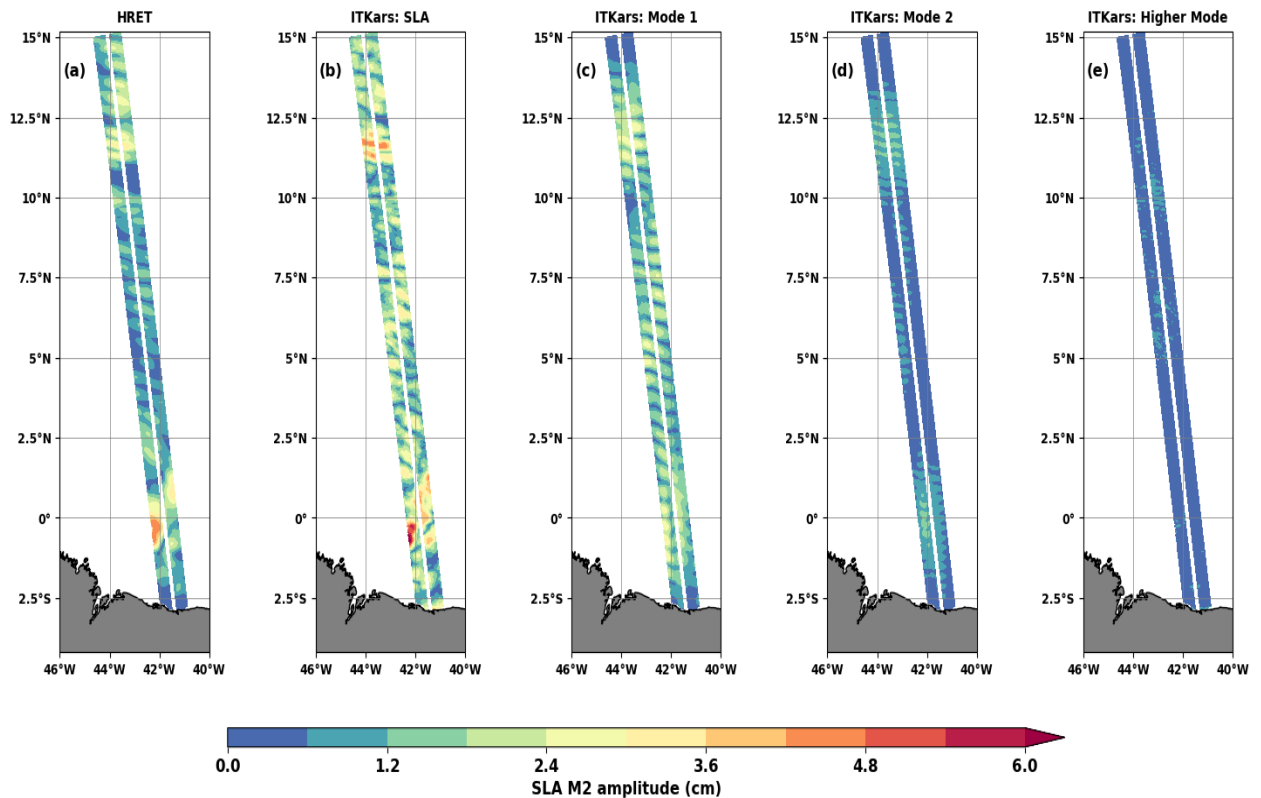
211  
 212 **Figure 4:** Standard deviation (in cm) of the total (a) SLA, and mode 1(b), mode 2 (c) and higher mode  
 213 (d) FFT-filtered SLA.

214 **2- The M2, S2 and N2 coherent internal tides from SWOT: ITkars model**

215 In Table 1, 4 waves (M2, N2, S2, and O1) have aliasing periods shorter than the 104 days  
 216 corresponding to the total length of our SWOT SLA series and are a priori of interest for our analysis.  
 217 But given the Rayleigh criterion between them in Table 1, it is reasonable to restrict ourselves to the  
 218 three semi-diurnal waves. Harmonic analysis based on least-squares fitting is used to extract the  
 219 coherent internal tide at each band point with at least 80 valid cycles over the entire SWOT Cal/Val  
 220 observation period. In the following, ITkars (IT from KARin Swot) refer to SWOT estimation of IT.

221





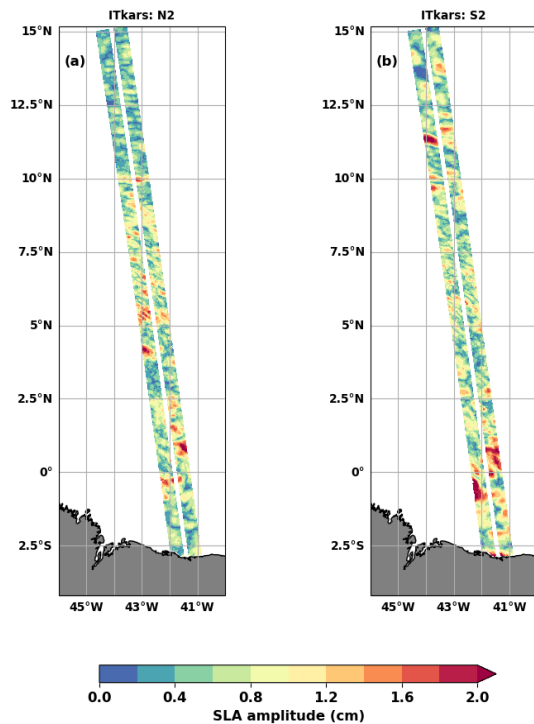
222

223 **Figure 5:** The amplitude (in cm) of the M2 internal tide from the HRET model (a) and the ITkars (b to  
 224 e) model over the cal/val period. ITkars is derived by harmonic analysis of the total SWOT SLA (b) and  
 225 FFT-filtered SWOT SLA for mode 1 (c), mode 2 (d) and higher mode (e) SLA. Only swath points with at  
 226 least 80 valid cycles were analyzed.

227 The amplitudes of the coherent internal tide at the M2 frequency are presented in Figure 5 for  
 228 both HRET (which include mostly mode 1 on this area) and ITkars models (which include mode 1 and  
 229 mode 2). We first performed the harmonic analysis of the total SLA (Figure 5b) and repeated the  
 230 harmonic analyses for each of the FFT-filtered SLAs (Figure 5c to e). The HRET model (Figure 5a) and  
 231 the ITkars model based on the total SLA (Figure 5b) are similar in terms of spatial distribution, but as  
 232 expected HRET has smoother and lower amplitudes because it represents a mean on many years of  
 233 altimetry data. In areas 1 and 3, ITkars shows spatial features identical to those already observed on  
 234 the standard deviation in Figure 4a. So, the maximum variability for these two parts of the SWOT track  
 235 is indeed due to the M2 coherent internal tide. The discrepancies between standard deviations (Figure  
 236 4) and internal tide amplitudes (Figure 5) are best seen by directly comparing the maps for the different  
 237 modes or wavelength bands. In area 2, the amplitude of the coherent internal tide is less than 1.5 cm  
 238 for the higher modes (Figure 5e), whereas at these scales the standard deviation is maximal (Figure  
 239 4d). The high variability of the SLA found in area 2 is evidently related to internal tide incoherency.

240 As S2 from HRET shows unexpected patterns (not shown) and, and N2 is not available in HRET, we  
 241 show only ITkars results in Figure 6. Both waves have smaller amplitudes than M2 and do not have the  
 242 same structure as the latter. As the semi-diurnal S2 and N2 IT should have similar patterns to M2, those  
 243 results indicate that these frequencies are certainly contaminated by other tidal waves due to poor  
 244 separability on the available period (see Table 1) and are likely also contaminated by the mesoscale.  
 245 Can we hope to improve our estimate of the coherent internal tide from SWOT observations?

246



247

248 **Figure 6:** The amplitude (in cm) of the N2 (a) and S2 (b) internal tide of the ITkars model derived by  
 249 harmonic analysis of the total SLA over the Cal/Val period. Only swath points with at least 80 valid  
 250 cycles were analyzed.

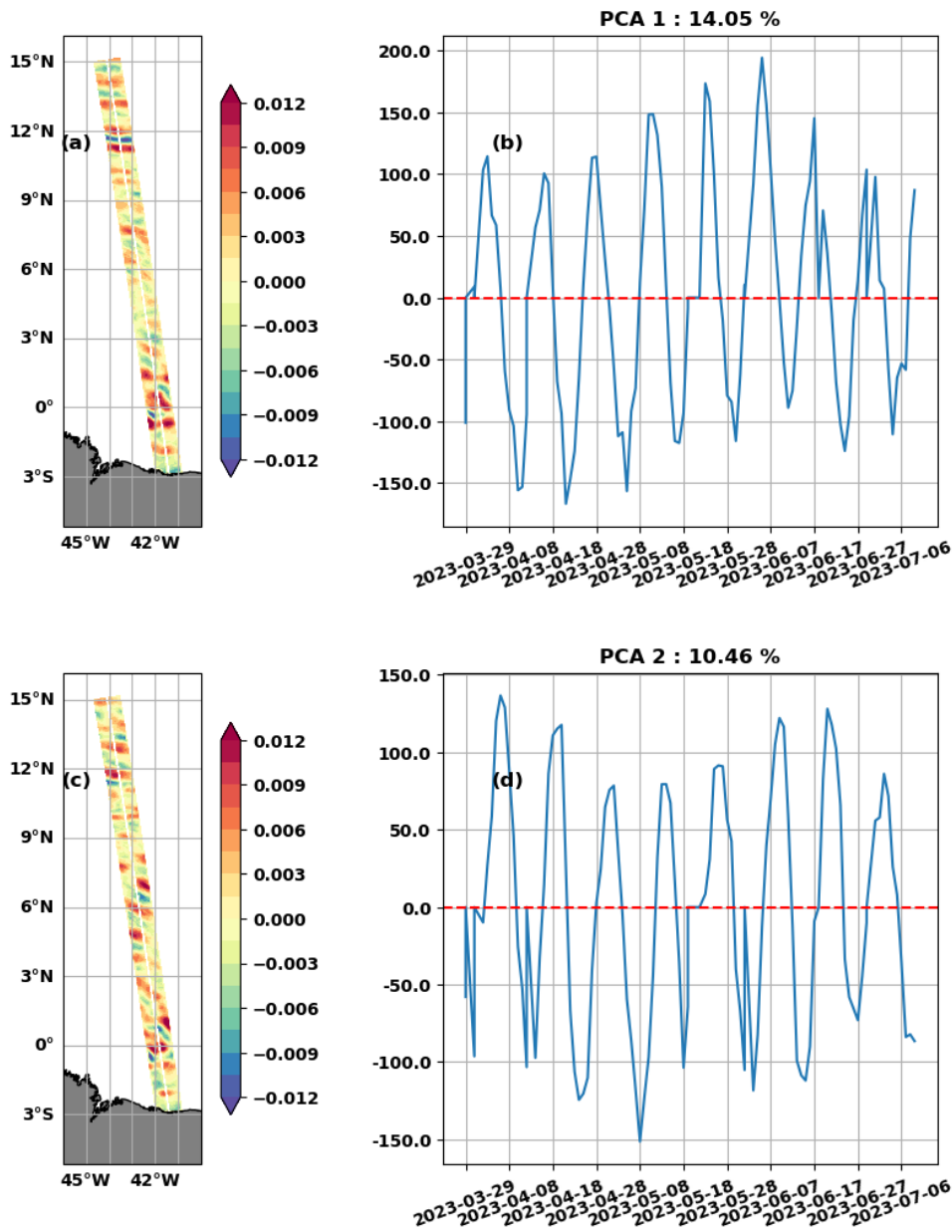
251 **3- An attempt to improve the estimation of coherent internal tide from SWOT Cal/Val data: Using**  
 252 **principal component analysis (PCA) to separate SLA content**

253 **3.1- Separation using PCA**

254 PCA, also known as EOF (Empirical Orthogonal Function), is a statistical analysis technique for  
 255 reducing the dimensionality of a data set (Jolliffe, 1986). Applied to geophysical data, PCA separates  
 256 the total signal into independent spatial patterns associated with independent temporal components  
 257 (Principal Component) and gives a measure of the relative importance of each pattern (a percentage  
 258 of the total variance). The first principal components (PC) capture most of the variance in the data and  
 259 generally have a repetitive and persistent structure, thus behaving approximately like the stationary  
 260 component of the signal. In particular, coherent internal tides have significant spatial correlations that  
 261 PCA could identify and isolate. On this basis, we believe that PCA applied to our total SLA can help  
 262 better isolate the coherent internal tide (which is stationary) from the remaining residual tidal  
 263 (incoherent internal tides) and non-tidal signals observed by SWOT. Egbert and Erofeeva (2021) have  
 264 successfully used PCA to determine the characteristics of the incoherent internal tide around the  
 265 Amazon shelf. In this paper we focus on the coherent internal tide. Therefore, we performed the PCA  
 266 on the 104 cycles of the SWOT KaRin total SLA as defined in Equation 1. At each point in the swath,  
 267 we filled in the missing value with the local time mean, then normalized the SLA to ensure that the  
 268 global mean and standard deviation become zero and one respectively. The covariance matrix is  
 269 calculated on the normalized SLA, the PCA focuses on eigenvalues and not absolute values.

270 The two leading PCA modes shown in Figure 7 account for 14.05% (PCA1, Figure 7a and c) and  
 271 10.46% (PCA2, Figure 7b and d) of the total variance. Their spatial patterns correspond to IT structures:  
 272 on PCA1 (Figure 7a) the IT is intensified in area 1 and area 3, while PCA2 (Figure 7c) is characterized by  
 273 an increase of the IT intensity in area 2. PCs show 12–13 days oscillations with modulations around 70

274 days (Figure 7b and 7d), therefore recalling the aliasing periods of M2 and S2 waves (see table 1). To  
 275 get a more precise idea of the wavelengths and frequencies contained in PCA1 and PCA2, we  
 276 reconstructed the SLA for both components (SLA\_pca1 and SLA\_pca2) and calculated the spectra  
 277 shown in Figure 8 (blue line for PCA1 and orange line for PCA2).

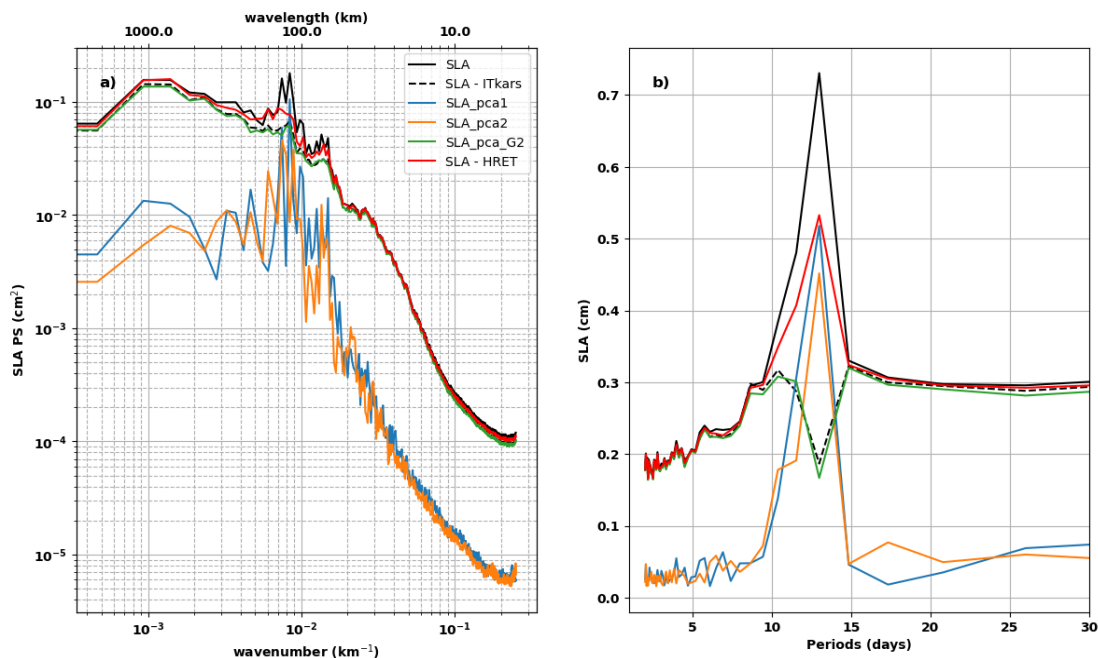


278  
 279 **Figure 7:** Spatial (left) and principal (right) components of PCA1 (top) and PCA2 (bottom) of the SLA  
 280 along SWOT swaths over the Cal/Val period.

281 The wavenumber spectra (Figure 8a) indicate that PCA1 and PCA2 consist mainly of mode 1 (180-90  
 282 km) and mode 2 (80-60 km) IT. A peak that could be associated with mode 3 stands out on the PCA2  
 283 spectrum, but overall, the energy levels of both spectra remain low for higher modes (50-2 km). The  
 284 frequency spectra (Figure 8b) confirm that M2 is the dominant signal. At this frequency, the mean SLA  
 285 amplitudes are 0.52 cm for PCA1 and 0.45 cm for PCA2, respectively 71% and 61% of the 0.73 cm  
 286 associated with the peak of the total SLA reported by the solid black line in Figure 8b. Amplitudes are  
 287 low for other frequencies. The 104 available cycles are not enough to observe 70-days modulation on  
 288 the frequency spectra. Given the wavenumber and frequency spectra, we can say that PCA1 and PCA2

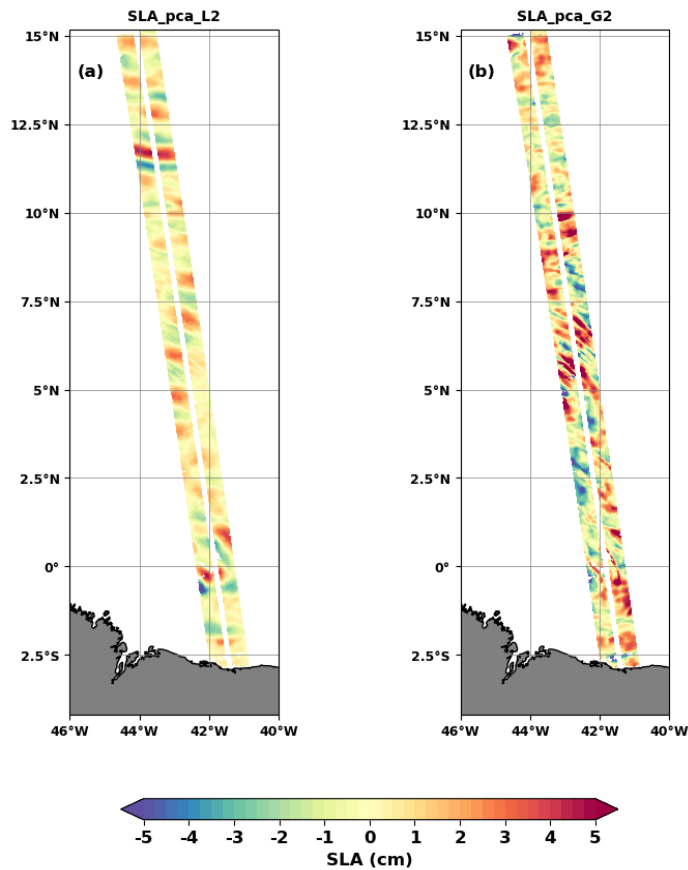
289 are two complementary representations of the propagation and evolution of the M2 dominant internal  
 290 tide, so they can be merged to form a single signal. We have summed SLA\_pca1 and SLA\_pca2 into  
 291 SLA\_pca\_L2 (L2 refers to lower or equal to 2). A snapshot of SLA\_pca\_L2 is shown in Figure 9a for the  
 292 same cycle as in Figure 2. Interestingly, the SLA reconstructed with PCA1 and PCA2 have similar  
 293 patterns to the mode 1 and mode 2 FFT-filtered SLAs (Figures 2b and 2c).

294 Between PCA3 and PCA10 the variance explained is less than 3.58% per PCA, from PCA11 onwards,  
 295 the variance becomes less than 2% (not shown). The PCs are a mixture of several wave frequencies,  
 296 with M2 of lower intensity than in PCA1 and PCA2, high frequency (faster than 10 days) and low  
 297 frequency (15, 17 or even 25 days). It is difficult to associate the spatial patterns of these PCAs with  
 298 the propagation of a persistent IT in time and along the track, or even with a mode of ocean variability  
 299 to our knowledge. The last 3 PCA patterns resemble residual noise from the processing of raw SWOT  
 300 data. We grouped PCA3 to PCA104 into SLA\_pca\_G2 (G2 for greater than 2). The small-scale structures  
 301 that were identified in Figure 2a are clearly visible in the snapshot of the SLA\_pca\_G2 in Figure 9b.  
 302 Figures 9a and 9b are complementary, as the PCA acted as a filter. The total SLA is now split into  
 303 SLA\_pca\_L2 and SLA\_pca\_G2. The spectra of SLA\_pca\_G2 are shown in green in Figure 8. Nearly all  
 304 energies for scales above 180 km (seen as large scale) and below 50 km (for higher modes) are found  
 305 in the wave number spectrum of SLA\_pca\_G2. Around the aliased frequency of M2, the mean  
 306 amplitude is 0.19 cm for SLA\_pca\_G2. This is about a quarter of the mean amplitude of the total SLA  
 307 (0.73 cm). With such a drop in the energy of the spectrum, it's tempting to say that separation by the  
 308 PCA has acted in the same way as a classical detiding. To verify this, we calculated the spectra of the  
 309 total SLA detided with M2 from ITkars and plotted them as a dashed black line in Figure 8. At all  
 310 frequencies and wavelengths, they overlap well with the spectra of SLA\_pca\_G2. Therefore,  
 311 SLA\_pca\_G2 is more representative of the incoherent internal tide and SLA\_pca\_L2 is more suitable  
 312 for improving the estimation of the coherent internal tide.



313

314 **Figure 8:** Wavenumber (a) and frequency (b) spectra of SLA\_pca1 ( in blue), SLA\_pca2 (in orange),  
 315 SLA\_pca\_G2 (in green), SLA – HRET (in red) and SLA - ITkars (black dashed line). SLA\_pca\_G2 is the  
 316 sum of the SLAs of PCAs greater than 2. The spectra of total SLA (black solid line) from Figure 3 are  
 317 reported here. ITKars is the internal tide model derived from SWOT KaRIn data (cf in section 2).

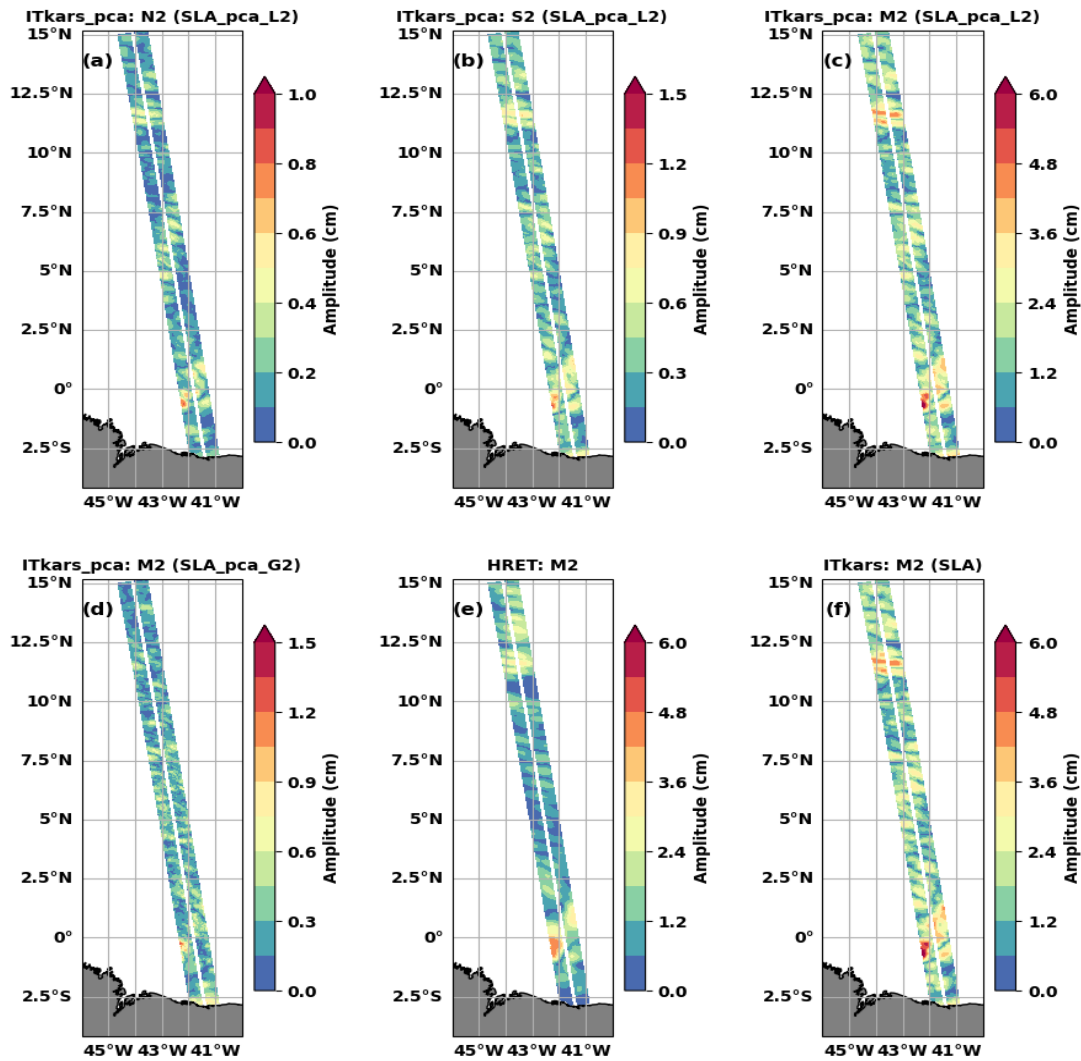


318

319 **Figure 9:** Snapshot of SWOT SLA\_pca\_L2 (a) and SLA\_pca\_G2 (b) on 08 April, 2023 (as in Figure 2).  
 320 SLA\_pca\_L2 is the sum of the SLAs of PCs less than or equal to 2 (PC1 and PC2).  
 321

322 **3.2- ITkars\_pca internal tide model**

323 We have performed the harmonic analysis of SLA\_pca\_L2 at the semi-diurnal frequencies M2, N2  
 324 and S2 (Figure 10). The resulting internal tide amplitude (model) is referred to as ITkars\_pca to  
 325 distinguish it from ITkars based solely on harmonic analysis of SWOT Karin data. Compared to Figure  
 326 6 corresponding to ITkars, the ITkars\_pca internal tide maps for N2 (Figure 10a) and S2 (Figure 10b)  
 327 are cleared of small scales, and the patterns for both waves are now close to that of M2 as expected  
 328 (Figure 10c and 5b reported in 10f). At first glance, there seems to be no difference between ITkars  
 329 (Figure 5b or 10f) and ITkars\_pca (Figure 10c) for M2. By making the complex difference between the  
 330 two signals (Figure 10c and 10f), we deduce the amplitude shown in Figure 10d, which is equivalent to  
 331 the amplitude of the harmonic analysis of SLA\_pca\_G2 at M2. As with N2 and S2, Figure 10d shows  
 332 that M2 ITkars (Figure 5b or 10f) also contains an additional signal dominated by small scales, and  
 333 which does not resemble the classic internal tide.



334

335 **Figure 10:** The amplitude (in cm) of the internal tides N2 (a), S2 (b) and M2 (c and d) of the ITkars\_pca  
 336 model derived by harmonic analysis of SLA\_pca\_L2 (a to c) and SLA\_pca\_G2 (d) over the Cal/Val period.  
 337 SLA\_pca\_L2 is the SLA based on PCA1 and PCA2, SLA\_pca\_G2 is compiled from PCA3 to PCA104. Only  
 338 swath points with at least 80 valid cycles were analyzed. Figure 5a and 5b are reported here, M2 HRET  
 339 (e) and M2 ITkars (f).

340 The origin of the extra signal contaminating ITkars could be dynamic or numerical. Dynamically, these  
 341 could be very intense non-linear waves, solitons, or incoherent internal tides, which are retained in the  
 342 harmonic analysis of section 2 due to the short length of the time series. On the numerical side, noise  
 343 linked to the pre-processing of SWOT data cannot be ruled out. Another source of contamination could  
 344 also be the DUACS correction we apply beforehand to distinguish internal tides.

#### 345 4 - Complementary comparison between SWOT and HRET IT models: predictability of internal tides

346 The comparison between HRET and SWOT based internal tides models (ITkars and ITkars\_pca) is  
 347 taken a step further in this final section. Each of the atlases (amplitude and phase) will be used as an  
 348 internal tide correction model for the SWOT data. This involves making internal tide predictions over  
 349 a given period and then subtracting these predictions from raw observations. Various metrics are used  
 350 to quantify the capability of each model to reduce the variance.

351 In Figure 8, the ITkars M2 atlas has been used as a correction model to detide the total SLA over the  
352 entire Cal/Val observation period (black dashed line). We have done the same with M2 from HRET and  
353 the corresponding spectra is shown in red in Figure 8. As in figure 3, the 1D spectra are integrations of  
354 the 2D wavenumber-frequency spectrum. On the wavenumber spectra, the peaks of modes 1 and 2  
355 are reduced but remain visible whatever the detiding applied (Figure 8a). We have integrated the  
356 wavenumber spectra of the total SLA (Black line in Figure 8) and the detided SLA (Black dashed and red  
357 line in Figure 8) over all wavelengths, between 180 and 90 km for mode 1, between 80 and 60 km for  
358 mode 2, from 50 to 2 km for the higher modes, and finally over wavelengths greater than 180 km for  
359 the large scale. The derived standard deviations are presented in Table 2 for the total SLA and the SLA  
360 detided with ITkars or HRET, as well as the percentages expressing the residual variance rate (ratio of  
361 the variances of the detided SLA on the total SLA). The higher is the standard deviation of the detided  
362 SLA or the percentage in Table 2, the less efficient is the correction used for detiding. According to  
363 Table 2, the application of the M2 internal tide prediction of each of the models removes very little  
364 variance from the SLA, nevertheless ITkars is more efficient than HRET especially at mode 1 and mode  
365 2 scales. For these scales, the residual variance reaches 76% and 84% of the SLA after correction by  
366 ITkars and is likely to be incoherent internal tide. For the higher modes, Table 2 agrees with Figure 5e:  
367 the M2 correction has no effect on these scales. ITkars has a greater impact on large scale SLAs than  
368 HRET: HRET has almost no signal at large scale by construction, while ITkars can capture some  
369 variability at large scales due to short time-series and no fitting approximation. When detiding with  
370 ITkars, the energy spectrum (Figure 8b) decreases strongly around the aliased frequency of M2 (around  
371 13 days, see Table 1). The mean of SLA amplitude along the SWOT swaths drops by 74% (from 0.73 cm  
372 to 0.19 cm) after ITkars detiding at M2 frequency. With the HRET correction (Figure 8), the peak  
373 amplitude at the M2 frequency is 0.53 cm, i.e. a 27% reduction of the peak of the total SLA, which is  
374 more than twice lower than with the ITkars correction. It can also be seen that periods of more than  
375 15 days and less than 5 days are not affected by the correction, since we have limited ourselves to M2  
376 frequency.

377 **Table 2:** Comparative table of the standard deviations of total SLA and SLA detided with HRET or ITkars.  
378 Standard deviations are obtained by integrating the spectra of Figure 3c on different wavelength bands  
379 (in cm). The ratio between detided SLA and total SLA, computed as a percentage, is given in  
380 parentheses.

	All wavelengths	Large scales >180km	Mode 1 180 - 90km	Mode 2 80 - 60km	Higher modes 50 - 2km
Total SLA	1.82	1.07	1.03	0.58	0.74
Detided ITkars	1.6 (88%)	0.99 (93%)	0.78 (76%)	0.49 (84%)	0.71 (96%)
Detided Hret	1.71 (94%)	1.04 (97%)	0.89 (86%)	0.54 (93%)	0.73 (99%)

381

382 The better performance of ITkars correction in comparison to HRET is not surprising, since ITkars is  
383 derived from the same database to which the detiding is applied. The result is almost identical to ITkars  
384 when the SLA is detided with M2 from ITkars\_pca. This is not surprising as the amplitude of the M2  
385 residual in SLA\_pca\_G2 is small (Figure 10d). To obtain the best possible comparison between ITkars  
386 and ITkars\_pca still focusing on M2 wave, we propose to apply the detiding to data that are  
387 independent of those used to derive the internal tide atlas. Thus, the SWOT data were divided into  
388 two periods: period 1, comprising the first 70 cycles (from late March to early June), and period 2 (from  
389 early June to early July), comprising the last 34 cycles. We repeated the M2 harmonic analysis of the  
390 total SLA over period 1 and derived the atlas "ITkars\_p1" (p1 indicates period 1). We did the same with

391 SLA\_pca\_L2 and derived the atlas "ITkars\_pca\_p1". The atlases ITkars\_p1, ITkars\_pca\_p1 and HRET are  
 392 then used to detide the SLA and SLA\_pca\_L2, first in period 1 and secondly in period 2, which is  
 393 independent of period 1. Since period 2 is short for frequency spectral analysis, we're going to look at  
 394 the standard deviation (Table 3) and the variance reduction (Figure 11). Variance reduction is  
 395 calculated from equation 2 as the difference between the variance of corrected SLA and the variance  
 396 of the uncorrected SLA (or SLA\_pca\_L2). A negative variance reduction indicates that the internal tide  
 397 correction reduces the SLA variance.

$$398 \quad \text{Variance reduction} = \text{var}(\text{SLA} - \text{M2 atlas}) - \text{var}(\text{SLA}) \quad (2)$$

399 The spatial mean of the standard deviation is summarized in Table 3. On period 1, the standard  
 400 deviation of the total SLA is 2.56 cm. After correction with M2 of the HRET model, the standard  
 401 deviation drops by 7% (to 2.39 cm). The SLA standard deviation decreases by about 15% and 14% with  
 402 ITkars\_p1 and ITkars\_pca\_p1, respectively. The difference between the two ITkars models is not  
 403 significant as we detide the same data that are used to derive the models (ITkarsp1 in particular). The  
 404 application of the corrections to SLA\_pca\_L2 reduces the residual standard deviation from 1.33 cm to  
 405 0.99 cm for HRET, to 0.53 cm for ITkars\_p1 and to 0.42 cm for ITkars\_pca\_p1. This shows that even in  
 406 an SLA dominated by coherent internal tides, HRET removes only 15% of the variance, whereas SWOT-  
 407 based internal tide models remove over 60%. ITkars\_pca\_p1 is obviously the best correction for  
 408 SLA\_pca\_L2, with ITkars\_p1 being slightly less efficient.

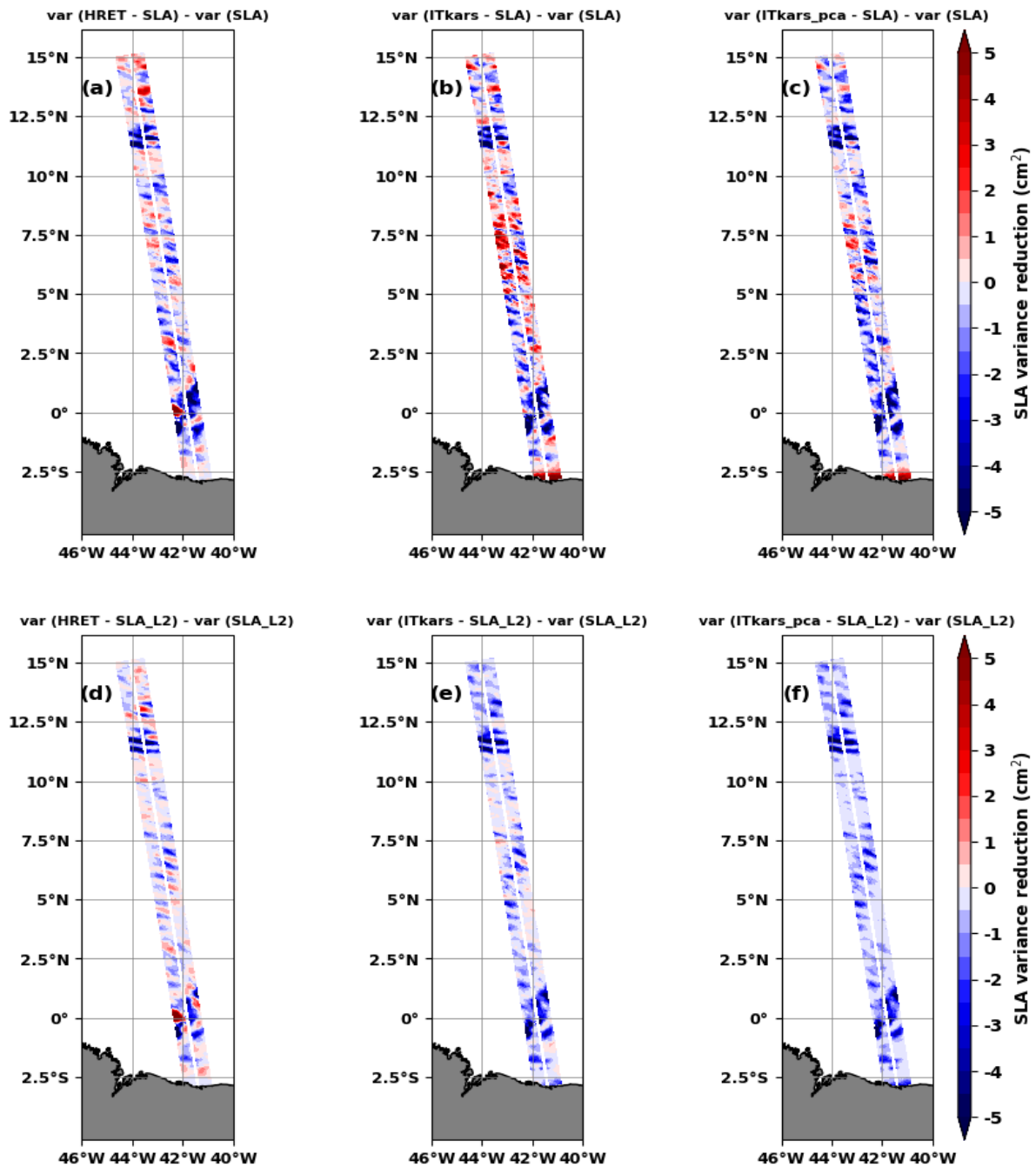
409 On period 2, the corrections of the SLA with the three internal tide models are less efficient. Table  
 410 3 shows that, on average, between 5% and 9% of the SLA variance is suppressed, with ITkars\_pca\_p1  
 411 being the best corrective internal tide model. As can be seen from the variance reduction maps (Figure  
 412 11a to c), the three models reduce variance (blue color) in the parts of the swath where the coherent  
 413 internal tide signal is strong enough (Figure 10). Outside these areas, the internal tide models tend to  
 414 add variance, and the variance reductions become positive. This undesirable effect of the correction  
 415 models is mainly observed in the central part of the swath (area 2), where the higher modes contribute  
 416 significantly to the SLA variability (see Figure 4). The effect in area 2 is pronounced for ITkars\_p1 (Figure  
 417 11b), indicating a prediction failure. When considering the independent SLA\_pca\_L2 data (period 2,  
 418 Table 3), HRET removes 21% of the variance. For ITkars\_p1 and ITkars\_pca\_p1, the variance is reduced  
 419 by 45% and 55% respectively. The impact of PCA on the derivation of the M2 internal tide atlas is  
 420 highlighted here by the 10% gap between the percentages of variance reduction when applying the  
 421 two models based on SWOT observations. The variance reduction figures confirm that the correction  
 422 works better on SLA\_pca\_L2 (Figure 11 d to e). Although there are still swath locations with positive  
 423 variance reduction, this is no longer concentrated in the central part. Variance reductions are  
 424 dominated by negative values for ITkars\_p1 (Figure 11e) and ITkars\_pca\_p1 (Figure 11f).

425 **Table3:** Comparative table of standard deviations (cm) of SLA and SLA\_pca\_L2 detided with either M2  
 426 HRET, M2 ITkars or M2 ITkars\_pca models over period 1 (from late March to early June 2023, the first  
 427 70 cycles) and period 2 (from early June to early July 2023, the last 34 cycles). ITkars\_p1 and  
 428 ITkar\_pca\_p1 models were built on period 1 and validated on period 2. The ratio between detided SLA  
 429 and total SLA is indicated in the parentheses (in percent).

	Period 1		Period 2	
	SLA	SLA_pca_L2	SLA	SLA_pca_L2
no IT correction	2.56	1.33	2.79	1.15
HRET	2.39 (93%)	0.99 (74%)	2.64 (95%)	0.91 (79%)
ITkars_p1	2.18 (85%)	0.53 (40%)	2.65 (95%)	0.63 (55%)
ITkars_pca_p1	2.21 (86%)	0.42 (32%)	2.55 (91%)	0.52 (45%)



430  
431  
432



433

434 **Figure 11:** Variance reduction of SWOT SLA (in  $\text{cm}^2$ ) on period 2 from early June to early July (last 34  
435 cycles) when using either M2 HRET (left column) or M2 ITkars\_p1 (middle column) or M2  
436 ITkars\_pca\_p1 (right column) to correct the total SLA (top ) or SLA\_pca\_L2 (SLA\_L2 in the title, bottom  
437 ), and compared to the ZERO IT correction case.

#### 438 4- Discussion and perspectives

439 In this study, we explored and characterized the internal tide signal in SWOT KaRIn observations over  
440 the Cal/Val period (1-day orbit) between late March and early July 2023 (104 cycles) and along the

441 track 20 located off the Amazon shelf in the tropical Atlantic between 2°S and 15°N. The internal tide  
442 as seen by SWOT is a mixture of several spatial scales, including baroclinic modes 1 and 2 defined by  
443 wavelengths between 180-90 km and 80-60 km respectively. SWOT also sees very intense fine-scale  
444 structures (wavelengths between 50-2 km) that we have associated with higher baroclinic modes,  
445 including modes 3, 4 and 5 according to Barbot et al., (2021). As a result, SWOT seems to live up to  
446 expectations, providing a direct 2D view of the internal tide sea surface signatures and even access to  
447 smaller scales.

448 Our approach to extract the internal tide signal through the 1-day SWOT data consisted firstly of  
449 filtering the large scale (including the mesoscale) by subtracting the DUACS MSLA from the SWOT  
450 observations; then we reintroduced the internal tide correction HRET from Zaron (2019) to obtain a  
451 SLA consisting of the total internal tide signal. We either performed the harmonic analysis (as in section  
452 2) or proceeded upstream to the PCA before the harmonic analysis (as in section 3). The internal tide  
453 model based on harmonic analysis of SWOT KaRin data was referenced ITkars (Internal Tide from KaRin  
454 Swot), the one obtained by combining PCA and harmonic analysis ITkars\_pca. We focused on the semi-  
455 diurnal frequencies M2, S2 and N2.

456 Spatial patterns of M2 internal tides from ITkars and ITkars\_pca models agreed with the M2 HRET  
457 model based on nearly 25 years of conventional altimeter (nadir) observations. The similarities  
458 between models based on SWOT Karin and model with conventional altimeter are partly linked to the  
459 fact that SWOT data are analyzed over March to July during which the internal tide is most stable and  
460 coherent off the Amazon shelf (Tchilibou et al., 2022). One consequence of analyzing SWOT data over  
461 this short 104-day window is that the amplitude of the internal tide is stronger with SWOT estimation  
462 than with HRET. This result is logical since the intensity of the coherent internal tide depends on the  
463 length of the time series analyzed: a longer time series allows a better estimate of the coherent signal  
464 which is therefore smoother (Ansong et al.,2015; Zhou et al.,2015; Nash et al.,2012). The separation  
465 of M2 from O1 is not ensured with 104 cycles of SWOT 1-day data, however, in this region the  
466 amplitude of the internal tide is negligible at O1 frequency compared to M2 (see Figure 1 in Tchilibou  
467 et al.,2022), so M2 ITkars\_pca is thus quite reliable.

468 The maps of N2 and S2 highlight the contamination of ITkars by signals other than the coherent  
469 internal tide, and particularly by very small scales. We hypothesize that the contamination is due to  
470 the leakage of nonlinear waves, incoherent internal tides, and ocean variability in the harmonic  
471 analyses. Regarding ocean variability, part of it is not captured by DUACS and was therefore not  
472 subtracted from the SLA. Moreover, subtracting the mesoscale, as we have done, is itself a possible  
473 source of error in estimating the internal tide (Zaron and Ray, 2018). One way to reduce the effects of  
474 contamination by ocean circulation would be to apply a simultaneous internal tide and mesoscale  
475 inversion method as proposed by Ubelmann et al. (2022). The combination of PCA and harmonic  
476 analysis gives semi-diurnal ITkars\_pca maps (M2, S2 and N2) with similar patterns. The amplitude of  
477 N2 ITkars\_pca deduced from SWOT is of the same order as that in the new product HRET14 (Zaron and  
478 Elipot 2024). The result is encouraging for S2, especially as the length of the 1-day observations is not  
479 sufficient to correctly separate it from waves such as Sa and Ssa, whose periods are identical to those  
480 of the annual and semi-annual variation of the ocean. A longer time series is needed to better separate  
481 the internal tide components from SWOT observation, and we will consider analyses of the 21-day  
482 SWOT science orbit data when the time series will be long enough.

483 PCA has improved our estimate of the internal tide model from the SWOT KaRin data. From the  
484 PCA we kept the first two main modes (PCA1 and PCA2) and considered them as the coherent internal  
485 tide given their fairly stationary character. Thus, the coherent internal tide accounts for 24.51% (14.05  
486 of PCA1 and 10.46 of PCA2) of SLA variance in 1-day SWOT observations, a proportion in line with the

487 studies of Zaron (2017) and Egbert and Erofeeva (2021) in this region. The coherent internal tide  
488 isolated through the PCA consists of mode1, mode 2 noticeable in PCA1 and PCA2. The fact that the  
489 coherent internal tide signal is projected onto two main modes of the PCA is an open question. The  
490 principal components of PCA1 and PCA2 are shifted by 3 to 4 days, about a quarter of the aliased  
491 frequency of M2, which could correspond to a phase quadrature, as there is between the imaginary  
492 and real parts needed to reconstruct a sinusoidal signal. Instead of being the real and imaginary parts  
493 of a signal, PCA1 and PCA2 could also represent the same phenomenon and highlight its evolution in  
494 area 2 in the middle of the swaths: the moderation of internal tides in area 2 with PCA1 and their  
495 intensification with PCA2. This type of PCA/EOF behavior is observed in the case of ENSO studies in the  
496 Pacific (Takahashi et al., 2011). The peaks on the wave number spectrum of PCA1 and PCA2 are shifted  
497 by few kilometers at the mode 1 and mode 2 scales, suggesting a change in wavelengths relating to  
498 changes in stratification conditions as suggested by Barbot et al. (2021). A longer series of Cal/Val  
499 observations could have helped to better distinguish PCA1 from PCA2.

500 The principal components (time series) of PCA1 and PCA2 provide an overview and an opportunity  
501 to study the daily variability of internal tide amplitude, a perspective difficult to access with  
502 conventional altimetry missions. This opportunity to learn more about the temporal variability of the  
503 internal tide using a single high-resolution mission is lost, or at least postponed, with SWOT's switch  
504 to its 21-days scientific orbit. One of the limitations of using PCA to analyze SWOT data is probably its  
505 sensitivity to track length. The total variance is distributed differently in the principal components  
506 depending on whether the track is long or short, or whether ocean dynamics change significantly along  
507 the track. It would be interesting to look at this point in the perspective of a global model, for example.  
508 We are curious to know how the PCA will behave in the case of multi-track use, and at their crossing  
509 points.

510 In the context of 1-day SWOT observations, the use of PCA can be useful in determining wave  
511 frequencies of interest for the development of the coherent internal tide model. The combination of  
512 PCA and harmonic analysis further reveals the observational potential of SWOT. We are currently  
513 working on other SWOT tracks in various ocean regions to test the robustness of our method  
514 combining PCA and harmonic analysis. We also plan to explore in situ observations of the SWOT Cal/Val  
515 and other databases to understand better our results. Work remains to be done to confirm the  
516 presence of mode 3 in the coherent internal tide signal in this region. The incoherence of the internal  
517 tide and its interaction with the circulation are other issues to be addressed with these SWOT data.

518 The final issue addressed in this study was the correction of the internal tide signal in the SWOT 1-  
519 day observations. On average, the HRET model corrects only 6% of the SLA variance over the Cal/Val  
520 period, while ITkars based on SWOT observations corrects 12% (Table 2). These percentages are low  
521 due to the high degree of internal tides incoherency in this part of the ocean. However, they indicate  
522 that the HRET correction is not efficient enough. It would be more relevant to directly evaluate the  
523 internal tide signals on 1-day SWOT observations and then use it as a correction model. The harmonic  
524 analysis may be sufficient if the aim is to apply the tidal model to the same data, but if not, the previous  
525 step of the PCA is recommended to obtain a more realistic model. The question of correcting internal  
526 tides and improving internal tide models will also remain a challenge for the exploitation of SWOT's  
527 21-day cycles.

528 **Data availability:** The SWOT Level (L3) V0.3 products are available on the CNES (Centre National  
529 d'Etudes Spatiales) cluster for expert users and for other users on the Aviso website  
530 (<https://www.aviso.altimetry.fr/en/missions/current-missions/swot/access-to-data.html>).

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536

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542

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