

This paper studies the coupling between the troposphere, stratosphere, and mesosphere by gravity waves during a sudden stratospheric warming (SSW). By using an analytical reflection coefficient R to characterise the spectrum of waves that is transmitted/reflected at the tropopause and stratopause during SSW events, the study offers a novel perspective on the role of gravity waves in atmospheric coupling and the observed effect of a cooling mesosphere during an SSW. Given the knowledge gaps in this area and the need for better representation of gravity wave effects in models, I believe this paper adds value and recommend it for publication provided that the following comments are addressed.

- Whilst the data in the figures supports the conclusion drawn regarding the changes in R during this idealised SSW, I am confused by the key facts used to illustrate the takeaway points. Firstly, please could you provide a reason for the $R < 0.4$ threshold used to delineate transmitted/reflected waves in figures 2 – 5, it seems like an arbitrary choice. Please then could you provide a reason for the $\Omega = 0.2$ and $\Omega = 0.1$ thresholds in Figs 2/4/5 and Fig 3, respectively? $\Omega < 0.2$ and $\Omega < 0.1$ mark most likely transmitted waves given $R = 0.4$, however the Ω thresholds don't appear to reflect what the figure tells you. For example, in Fig. 2, almost all Ω have $R < 0.4$ for $V_h < 0.5$, the text in line 149 however implies that only $\Omega < 0.2$ have $R < 0.4$ for $V_h < 0.9$. The same critique applies to Fig 3 and L 155, Fig 4 and L 170, Fig 5 and L 177. These numbers are used in the discussion in the paper, so are a significant aspect which needs addressing before the paper is published.

These thresholds really have no physical meaning. When I drew these Figs. I thought it might be easier for comparison between Figs. to mark some frequencies and reflection coefficient values. Therefore, I chose $R=0.4$ and $\Omega < 0.2$. In this version of the article, I have deleted these thresholds.

- The assumptions used in deriving the reflection coefficient, and the set-up of the two scenarios: no-SSW, and SSW, are highly idealised. I.e., The atmosphere from the tropopause to the mesosphere is not isothermal and is highly variable compared to the two idealised scenarios given. Please could you address the sensitivity of your results to deviations from the assumptions and idealisations, and the validity of using this form of the reflection coefficient to the real atmosphere? Perhaps you could support its validity by calculating R from data of observed or modelled SSW events, which should hopefully show that R increases at the tropopause, decreases at the stratopause, and in such a way that suggest similar changes in gravity wave fluxes to those reported in this paper.

In the article the atmosphere from the tropopause to the mesosphere is not isothermal because there is a temperature stratification from the upper troposphere/lower stratosphere with $T=220$ K to the middle stratosphere with $T=240$ K. The similar temperature stratification is between the middle stratosphere ($T=240$ K) and upper stratosphere/lower mesosphere with $T=270$ K, this is no-SSW case. During the SSW, there are temperature changes in the stratosphere and mesosphere and temperature stratification is different: the upper troposphere/lower stratosphere has the same temperature $T=220$ K, while the temperature in the middle atmosphere increases for 25 K, $T=265$ K. The temperature of the upper stratosphere/lower mesosphere also changed. This part of the atmosphere cools and the temperature decreases for 25 K, to $T=245$ K. This is the same approach as in the article -Phases and amplitudes of acoustic-gravity waves II: The effects of reflection, Astronomy and Astrophysics. 278, 617-626 (1993) by Marmolino et al. in abstract: “ **We study wave reflection caused by the temperature stratification of the solar atmosphere, assumed to be a succession of two layers of different temperatures...**” Page 620, Fig. 2: “Amplitudes and phases of the reflection and transmission coefficients R and T vs. frequency at $k_x=1.31$ Mm^{-1} . Solid lines refer to **a succession of two layers suitable to represent the photospheric stratification ($T_l=5000$ K (this means T in the lower layer); $T_u=4500$ K (this means T in the upper layer)...**” The same is in my article

where the succession of the two layers with temperatures $T_1=220$ K and $T_2=240$ K represents the atmospheric stratification between the upper troposphere/lower stratosphere and middle stratosphere; also the temperatures $T_2=240$ K and $T_3=270$ K represent the stratification between the middle stratosphere and the upper stratosphere/lower mesosphere.

I can support my results because in these articles the results are the same as mine: Namely, my Fig. 3 shows that SSW events prevent GWs propagation from the troposphere towards the stratosphere, which is consistent with known scientific results:

Wang, L. and Alexander, M. J.: Gravity wave activity during stratospheric sudden warmings in the 2007–2008 Northern Hemisphere winter, *J. Geophys. Res.-Atmos.*, 114, D18108, <https://doi.org/10.1029/2009JD011867>, 2009;

Hindley, N., Wright, C., Hoffmann, L., Moffat-Griffin, T., and Mitchell, N.: An 18 year climatology of directional stratospheric gravity wave momentum flux from 3-D satellite observations, *Geophys. Res. Lett.*, 47, <https://doi.org/10.1029/2020GL089557>, e2020GL089557, 2020;

Wicker, W., Polichtchouk, I., and Domeisen, D. I. V.: Increased vertical resolution in the stratosphere reveals role of gravity waves after sudden stratospheric warmings, *Weather Clim. Dynam.*, 4, 81–93, <https://doi.org/10.5194/wcd-4-81-2023>, 2023.

Also, the absence of gravity wave breaking in the mesosphere explains the mesospheric cooling during an SSW as in:

Holton, J. R.: The influence of gravity wave breaking on the general circulation of the middle atmosphere, *Journal of the Atmospheric Sciences*, 40, 10, doi:10.1175/1520-0469(1983)040<2497:TIOGWB>2.0.CO;2, 1983;

Liu, H. L. and Roble, R. G.: A study of a self-generated stratospheric sudden warming and its mesospheric-lower thermospheric impacts using the coupled TIME-GCM/CCM3, *J. Geophys. Res.* 107, 18, doi:10.1029/2001JD001533, 2002.

Stephan, C. C., Schmidt, H., Zuelicke, C., and Matthias, V.: Oblique gravity wave propagation during sudden stratospheric warmings, *Journal of Geophysical Research: Atmospheres*, 125, e2019JD031528, <https://doi.org/10.1029/2019JD031528>, 2020; citation: “During the course of the SSW the mesospheric GW momentum flux (GWMF) turns from mainly westward to mainly eastward. **Waves of large phase speed (40–80 m s⁻¹) dominate the eastward GWMF** during the peak phase of the warming. In addition, the contribution of slow phase speed waves to the total GWMF decreases dramatically and waves of phase speeds > 40 m s⁻¹ dominate the eastward GWMF during the peak phase of the warming. My result for the phase velocity of gravity waves that propagate from the stratosphere to the mesosphere during SSW is 65 ms⁻¹<Vh<280 ms⁻¹. These waves have the best chance to pass the upper stratosphere/lower mesosphere temperature discontinuity.

In article- Dörnbrack, A., Gisinger, S., Kaifler, N., Portele, T. C., Bramberger, M., Rapp, M., Gerding, M., Faber, J., Žagar, N., and Jelić, D.: Gravity waves excited during a minor sudden stratospheric warming, *Atmos. Chem. Phys.*, 18, 12915–12931, <https://doi.org/10.5194/acp-18-12915-2018>, 2018, page 12921: “Applying this relationship results in an estimate of the scaled intrinsic frequency being $\omega/f \approx 1.4$ – 1.7 for the stratospheric layers on 30 January. This means that the observed waves are dominated by intrinsic frequencies much smaller than the **buoyancy frequency $N \approx 0.02$ s⁻¹**.” My result is that gravity waves with frequency of $\omega < 0.005$ s⁻¹ have the best chance of propagating from the troposphere. This frequency is 4 times smaller than $N \approx 0.02$ s⁻¹.

In article- High Resolution Dynamics Limb Sounder observations of the gravity wave-driven elevated stratopause in 2006, by France, J. A. Et al., JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 117, D20108, doi:10.1029/2012JD017958, 2012, page 7: “ There is **an increase in GW KMF** beginning on 5 January that maximizes on 8 January, **which extends from the lower stratosphere to the lower mesosphere**. The largest amplitudes of GW KMF (gravity wave kinetic momentum flux) on this date occur in the lower mesosphere. The temperature contours indicate an increase in temperature at the stratopause following the increase in GW KMF between 5 and 8 January.” My Fig. 5 shows that during SSW gravity waves have lower reflection coefficient values and a higher chance of propagating from the stratosphere to the mesosphere.

The same is in article- Satellite observations of middle atmosphere gravity wave absolute momentum flux and of its vertical gradient during recent stratospheric warmings, Atmos. Chem. Phys., 16, 9983–10019, <https://doi.org/10.5194/acp-16-9983-2016>, 2016, by Ern, M., et al, on page 10000: “ Finally, it should be mentioned that, in all winters considered, enhanced values of gravity wave potential drag are preferentially found in the upper stratosphere and in the mesosphere (i.e., at altitudes above about 40 km).”

The waves with frequencies in my article are found in the atmosphere at the altitudes about 55-60 km, as in article Nina, A. and Čadež, V.: Detection of acoustic-gravity waves in lower ionosphere by VLF radio waves, Geophysical Research Letters, 40, 18, 4803-4807, <https://doi.org/10.1002/grl.50931>, 2013. These waves are presented in Fig. 3, bottom, on page 4805. These waves are gravity waves with frequencies lower than Brunt-Väisälä frequency of 0.02 s^{-1} . In my article gravity waves that propagate from the stratosphere to the mesosphere have a frequencies lower than 0.019 s^{-1} .

About the deviations from the assumptions and idealizations- It is known that -...” at the bottom side of the thermal layers (where the temperature profile is increasing in altitude) the positive temperature gradient with increasing altitude signifies an increase in atmospheric stability and reduction of vertical mixing. In contrast, at the topside of the thermal layer the negative temperature gradient with decreasing altitude implies a reduction in atmospheric stability to the point that the atmosphere may become convectively unstable, thus possibly supporting the development of turbulence,” page 4 in MESOSPHERE INVERSION LAYERS AND STRATOSPHERE TEMPERATURE ENHANCEMENTS by John W. Meriwether and Andrew J. Gerrard, Reviews of Geophysics, 42, RG3003 / 2004. Namely, the turbulence can not be described by the linear theory applied in my article. In addition, the potential for the nonlinear interaction also exists. This can induce gravity wave breaking at lower altitudes in the atmosphere and also the generation of the secondary gravity waves (Gavrilov, N. M. and Kshevetskii, S. P.: Identification of spectrum of secondary acoustic-gravity waves in the middle and upper atmosphere in a high-resolution numerical model, Solar-Terrestrial Physics, 9, 3, 86-92, doi: 10.12737/stp-93202310, 2023.). Linear approximation can be understood as a limitation of this work.

- It is not clear how equations 5 come from equations 1 – 3, this appears to be a novel step in deriving the acoustic gravity wave dispersion relation, so would be insightful to have the steps elaborated on with their physical meaning. The reader is pointed to more detail in Jovanovic (2016), however no extra details to what is shown in this paper are given.

This is not a novel step. This procedure is used in Pinter, B. Čadež, V. M., and Roberts, B: Waves and instabilities in a stratified isothermal atmosphere with constant Alfvén speed – revisited, Astron. Astrophys. 346, 190–198, 1999. I used the same approach in the Sect. 2. Also in the article – Diagnostics of plasma in the ionospheric D-region: detection and study of different ionospheric

disturbance types, A. Nina, V. M. Čadež, L. Č. Popović, and V. A. Srećković, in *The European Physical Journal*, Vol. 71, Issue 7, 2017.

According to your comments I include some new details in this section.

- Please ensure that all symbols are correctly defined. E.g., no variables in equations (1 – 3) are defined except the constants.

I did it. I thought it wasn't necessary because p , T , and ρ have usual meanings.

- For brevity, it would probably be more appropriate to refer to the troposphere-stratosphere boundary as the tropopause and the stratosphere-mesosphere boundary as the stratopause. All terms appear in the paper, so I suggest picking a single convention for consistency in the paper.

The term boundary is important because of the model in Sect. 3. On the other hand, in the real atmosphere there is a tropopause and a stratopause between the troposphere/stratosphere and between the stratosphere/mesosphere, respectively. I have changed the boundary term in Section 4 (Results) to include atmospheric temperature stratification. Therefore, there is no boundary between the troposphere and stratosphere (nor tropopause); there is only a temperature discontinuity between the upper troposphere/lower stratosphere with a temperature of 220 K and middle stratosphere with a temperature of 240 K. The same is for the stratosphere/mesosphere- there is no stratopause, just a temperature discontinuity between the middle stratosphere with a temperature of 240 K and the upper stratosphere/lower mesosphere with a temperature of 270 K in case without SSW. As suggested by another reviewer, I need to include the term stratopause in the discussion because during SSW events the stratopause drops from about 55 km to about 40 km and heats the middle and lower stratosphere.

- Please could the labels on the figures be made in a larger font, at least to match the font size in the paper.

The Figs. 2, 3, 4, 5 are made in mini page format with a width of 0.48 cm. This allows the Figs. to be placed side by side, which is convenient for comparing them. I changed their width to 0.494 cm. Its a maximum width for the minipage Figs. Of course, it is always possible to make larger Figs., but they won't be side by side.

- Please could you also review the spelling and grammar, as many instances of grammatical errors were spotted in the paper.

I tried to do that.