Reply to the reviewer's comments

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Title: "Exploring the influence of spatio-temporal scale differences in Coupled Data Assimilation"

Thank you for your constructive and helpful comments. We provide our responses in blue, while for the proposed modifications to the manuscript we use **bold** text. We also provide the line numbers from the new-corrected manuscript.

Reviewer #1:

This manuscript explores the optimal strategy for initializing coupled climate prediction systems by comparing between strongly and weekly coupled data assimilation. Through a series of experiments, the authors have reach to the two conclusions described in the last section. While the conclusion is reasonable to me, I am not fully convinced of the significance of the manuscript. The conclusions presented in Chapter 5 were largely consistent with previous studies. This also indicates that the findings obtained in this study are quite limited. Consequently, I cannot be very positive to the paper because this study seems to be a simple extension of previous studies. I have some suggestions for improving the manuscript, as described below.

R: We thank the Reviewer for their comments and criticism. We recognize now that we did not convey adequately the motivations and the novelties in the original version of our manuscript. We hope to be able to rectify this in our revised version. Specific answers to your queries are provided below.

In our opinion, existing literature on the added value of weakly over strongly coupled DA (WCDA and SCDA, respectively) has often been discordant, with some concluding about improvements and others showing a degradation depending on the configuration. Several factors can influence the conclusions: e.g. spatio-temporal scale separation, model error, sampling error, ad-hoc fixes (e.g. localization).

Our main research objective is to unveil the possible connections between the underlying dynamical properties of the forecasting model and the performance of coupled DA. For this reason, our study intentionally leverages a simple, yet nonlinear and chaotic, system to thoroughly assess the performance of WCDA and SCDA in a wide range of spatio-temporal scale separation that are observed in the real climate system. The identical twin experiment setup allows us to eliminate model bias, the low computational cost made possible the use of a large ensemble size to avoid or mitigate sampling error. Furthermore, having a full control on the model's dynamical features we were able to compare a wide range of parameter configurations while keeping the average rate of error growths within the same level. The latter fact has been pivotal to accomplish a comparison among various model configurations where the interest was in understanding the impact of varying the spatio-temporal scale separation across model compartments rather than on the effect of changing observation interval (which was always set to a given, fixed, multiple of the error doubling time).

Another key consequence of our idealized experimental setup is that we did not need localization, otherwise necessary in higher dimensions. By modifying the error covariances, localization breaks dynamical consistency (in return it provides statistical appropriateness) thus rendering extremely intricate to disentangle the role of spatio-temporal scale separation in shaping the error covariances.

Previous literature on the topic has addressed CDA in specific settings. To the best of our knowledge, our work is the only study that analyses the issue with both spatial and temporal scale separation and observation networks. For instance:
Tondeur et al. (2020) analyses the impact of the temporal scale separation on each CDA approach. This paper explores the observation network and assimilation frequency using a fixed temporal scale separation. On the other hand, Evensen et al. (2024) analyses spatial scale separation. Furthermore, the study by Miwa and Sawada (2024) explores the CDA approaches using several model configurations, such as coupling strength and three different temporal scales, using a fixed spatial scale. In this study, they managed to link the relationship between coupling strength and the chaoticity of the system. **In this context, our study's originality roots in the explicit combination of several spatio-temporal scale separations, observational networks, and the dynamics of the underlying system to better understand CDA performance**.

(1) Usage of more complex model(s): One of the main discussions in coupled data assimilation is how to differentiate real and erroneous error covariance. Therefore, exploring a better localization strategy is essential for coupled data assimilations. However, the present manuscript uses a very simple top model, which is unsuitable for investigations on localization. It is also important to investigate optimal observation frequency (=data assimilation) of the fast and slow-mode models for the coupled data assimilation.

R: We agree that localization is essential to the success of CDA in a realistic, high-dimensional, framework. Nevertheless, the main objective of our manuscript is to link the spatio-temporal scale separations to the performance of CDA. With that scope in mind, we see it as a strength that the system does not require localization. We could thus analyse the effect on CDA of the different degree of instability and spatio-temporal scale separation in the dynamical model. Localization would hide the effects we aim at studying.

We also agree that the assimilation frequency has a large influence on the results. Precisely in the light of this, we have designed our comparison of CDA at different spatiotemporal scales by changing the observation frequency such that the interval between successive observations was fixed to 1/5 of the error doubling time (proportional to the first Lyapunov exponent). As such we ensure that all experiments are compared at similar error level and that the differences only relate to the scale separation.

(2) To investigate various coupled data assimilation strategies: Kurosawa et al. (2023; NPG) investigated various options of coupled data assimilation, as indicated in Figure 2. I would suggest investigating such options together with the sensitivity investigations on observation frequency, ensemble size and localization.

R: We thank the Reviewer for pointing out a paper that we did not consider in our original manuscript, which we are now including as one of our references. In the study by Kurosawa et al. (2023) the SCDA and WCDA experiments are evaluated in a system using atmosphere and land observations. Since in our study we use an atmosphere-ocean system, we consider atmosphere and ocean (instead of land) observations. Most of the assimilation strategies proposed by Kurosawa et al. (2023) were already considered in our study; however, there were two (2) configurations that we did not consider. Using Kurosawa's notation for the experiments, the 'missing' configurations are $A_{A\times}L_{AL}$, and $A_{AL}L_{\times L}$ (both SCDA); where A and L indicate Atmosphere and Land, respectively. These configurations correspond to cases in which both components are observed but only one cross-update is performed. For example, the configuration $A_{A\times}L_{AL}$ considers a well observed system (observations for atmosphere and land) thus, performing atmosphere DA and land DA with the additional cross-update from the atmosphere to land. The same applies for the other experiment.

In the context of our experiment, these cases correspond to our SCDA-FULL experiments, with only one cross-update, from the atmosphere or from the ocean. We considered the same experimental settings of the cases shown in the manuscript. The new cases are:

- FULL-A: Atmosphere DA, ocean DA and atmosphere → ocean cross-update.
- FULL-O: Atmosphere DA, ocean DA and ocean → atmosphere cross-update.

We decided to analyse these cases, comparing them with our WCDA-FULL experiment. This comparison will reveal the impact of only the cross-update. We show our results in Figures 1 and 2, with the metric $\Delta\overline{\mathrm{RMSE}}_n^{\mathrm{SCDA}}$ indicating the averaged difference in RMSE between WCDA and SCDA. $\Delta\overline{\mathrm{RMSE}}_n^{\mathrm{SCDA}} > 0$ indicates that SCDA has larger error than WCDA.

These results (Figures 1 and 2) show that in a fully observed system, the additional cross-update from SCDA causes a suboptimal update, therefore the use of WCDA is better than SCDA. These results reinforce the idea that the estimated cross covariance, used for SCDA, suffers from the sampling error, and that the linear analysis update of the EnKF is suboptimal for the cross-update (see our answer to Reviewer 2 on this point). The sensitivity is higher as the temporal scale separation increases, especially on the observed component.

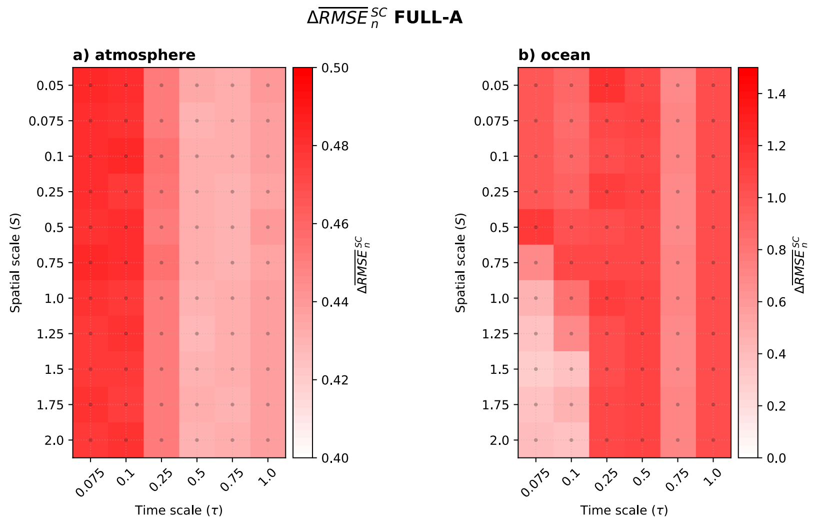


Figure 1: *FULL-A experiment metric* $\Delta\overline{\mathrm{RMSE}}_n^{\mathrm{SCDA}}$ *for a) atmosphere and b) ocean. Red indicates that the SCDA error is larger than the WCDA. Note that the colourmap has different limits, all positive.*

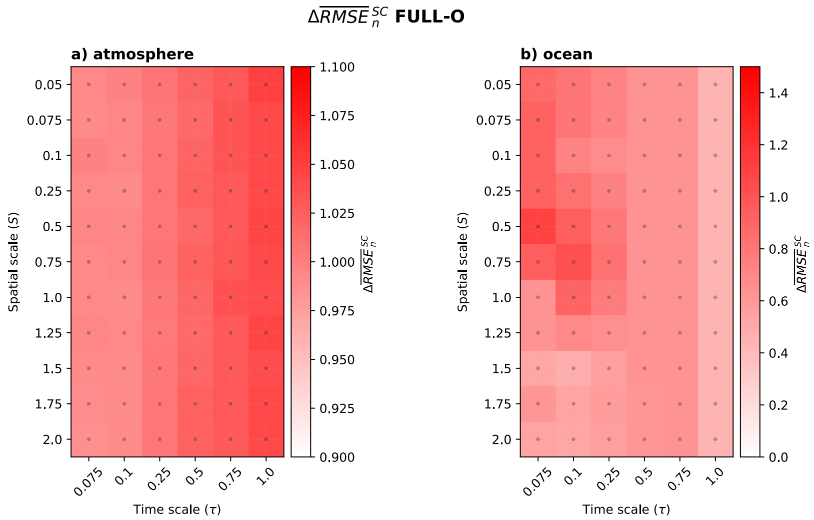


Figure 2: *Figure 1FULL-O experiment metric* $\Delta\overline{\mathrm{RMSE}}_n^{\mathrm{SCDA}}$ *for a) atmosphere and b) ocean. Red indicates that the SCDA error is larger than the WCDA. Note that the colourmap has different limits, all positive.*

Reviewer #2:

The authors tried to clarify when the strongly coupled data assimilation (SCDA) is preferrable to weakly coupled data assimilation (WCDA) by a two-components coupled Lorenz-63 system (one component representing the atmosphere, fast component; the other one representing the ocean, slow component), with changing the parameters of observation networks (FULL, ATM, OCN), spatial scales (S, 0.05 - 2.0), and temporal scales (\tau, 0.075 – 1.0). The data assimilation implemented in this study is EnKF with perturbed observations and adaptive inflation. In WCDA, the assimilation is applied to the individual components separately by using the observations available for that component. In SCDA, the observations from one component impact the other components directly during the assimilation.

The manuscript described the stability analysis of the coupled Lorenz-63 model, which is a simpler version of Tondeur et al. (2020). The results were discussed with respect to the observation networks: (1) in a well-observed system, SCDA degrades the system's performance slightly compared to WCDA; (2) SCDA improves over WCDA when only one component is observed. Similar conclusions have been reported by previous studies. I think the interpretation from the instability analysis on why the SCDA shows different responses from WCDA would be interesting to the readers, whereas the results and conclusions of the paper are not consistent. My concerns are listed as followings.

R: Thank you for your constructive comments and the careful reading of our manuscript. The Reviewer has also well understood our main goal of connecting the CDA performance to the underlying instability properties of the dynamic model. Prompted by her/his criticism, in our revised version of the manuscript we have attempted to improve our discussion and strengthen the interpretation of such connections. We have also tried to clarify the potential inconsistencies as highlighted by the Reviewer.

Major comments:
1. Lines 5 – 7. (a) In full observations, "SCDA and WCDA yield similar performances". Does this mean that the spatial scale (S) and temporal scale (\tau) have very few influences on the coupled data assimilation? (b) If the SCDA performs marginally worse than WCDA could be explained by the approximation in the EnKF – linear analysis update and sampling error, I will encourage the authors to explicitly describe how the linear analysis update and sampling error in the SCDA differs from those in the WCDA.

R: We are grateful to the Reviewer for pointing to these potential inconsistencies. Accordingly, we have revised the text to make these statements more precise and discuss them clearly in our results and discussion section. We respond to your comments:

a) In our original abstract (Lines 5-7) we wrote that in the FULL experiment "SCDA and WCDA yield similar performances" because the difference between each method is around 0.1% and 1% of the climatological (FREE) RMSE for atmosphere and ocean, respectively. Therefore, we consider that both methods perform similarly. Nevertheless, the performance, albeit generally similar, clearly depends on the spatio-temporal scales (S and \tau values); we discussed this in greater detail in Sect. 4.1.

b) SCDA provides a slightly worse update than WCDA in the FULL coverage observation network. We speculate that this difference is due to the approximations done with the EnKF; the linear analysis update and sampling error. In SCDA, the presence of sampling error is larger relative to the size of the state vector -- in SCDA, the state vector is 6 whereas it is 3 in WCDA for the update of the individual components. Also, the cross-component covariances are small (so more prone to sampling error) and becoming more non-linear as the scale separation increase. A linear analysis can lead to a degradation and non-linear iterative approach should be more suitable.

In the revised version of the manuscript, we modify our abstract and main conclusion to account for the influence of a) the spatio-temporal scales and b) the impact of the approximation inherent to the EnKF; the revised text now reads:

The abstract:
Lines 5 – 10: "**In the fully observed scenario, SCDA and WCDA yield similar performances. However, some little differences are present, and we conjecture these are due to the SCDA being more sensitive to the approximations at the basis of the EnKF present in the cross-update – linear analysis update and sampling error. This sensitivity increases as the temporal scale separation increases, especially on the slow-large scale component.**"

We also updated our first conclusion from our Summary and main findings (Sect. 5.1) as:
Lines 363 – 369: "In a well-observed system, the potential for improvements over WCDA is very limited as observations from both components constrain the system nearly optimally already. We even find that sometimes SCDA degrades the system's performance. This is possibly due to the approximation in the DA method – linear analysis update and sampling error. The state vector during the assimilation with **SCDA is 6 whereas it is 3 with WCDA for the update of the individual components. Consequently, the sampling error is larger relative to the dimension of the state vector with SCDA than with WCDA. Furthermore, the cross-components covariances are often weaker and their non-linearity growth as the scale separation increases. The linear approximation during the analysis with the EnKF can yield a degradation. When the time scale separation (and to a lesser extent the spatial scale separation) is large, a nonlinear update (e.g. Evensen 2024) may be better suited.**"

2. Lines 7 – 8. When observations are only in one of the components, "SCDA systematically outperforms WCDA" is contradictory with the discussions in Chapter 4.3 and Figure 12 (a), where the dotted area means SCDA degrades over WCDA (Figure 10).

R: We thank again the Reviewer for pointing to a possible overstatement. From the figures that you mention it is obvious that, for the atmosphere in our OCN experiment (Fig. 12a), SCDA's improvement over WCDA has a clear dependence on the spatio-temporal scale separation. Thus, we modified our conclusion as:

Lines 10 – 12: "**When observations are only in one of the components, the spatio-temporal scale separation determines SCDA's performance.** In this scenario, the largest improvements are found when the observed component has a smaller spatial scale…"

3. Lines 8 – 10. "The spatio-temporal scale separation determines SCDA's performance in this scenario, and the largest improvements are found when the observed component has a smaller spatial scale." (a) The first part of this sentence says that the spatio (S) -temporal (\tau) scale affects the SCDA's performance, whereas the second part says that only the spatial scale (S) affects the performance, which is not consistent. (b) In Figure 12, when only the ocean was

observed, the large improvements were found when the spatial scale is larger, which is contradictory with the sentence in the abstract.

R:

a) We clarified this apparent contradiction. Now it reads:
Lines 11 – 13: "When observations are only in one of the components, the spatio-temporal scale separation influences SCDA's performance. In this scenario, the largest improvements are found when the observed component has a smaller spatial scale. **The fast-to-slow update has a larger benefit with a larger temporal scale separation. Meanwhile, with the slow-to-fast update, the improvement is limited to instances when the temporal scale separation is less than one-half.**"

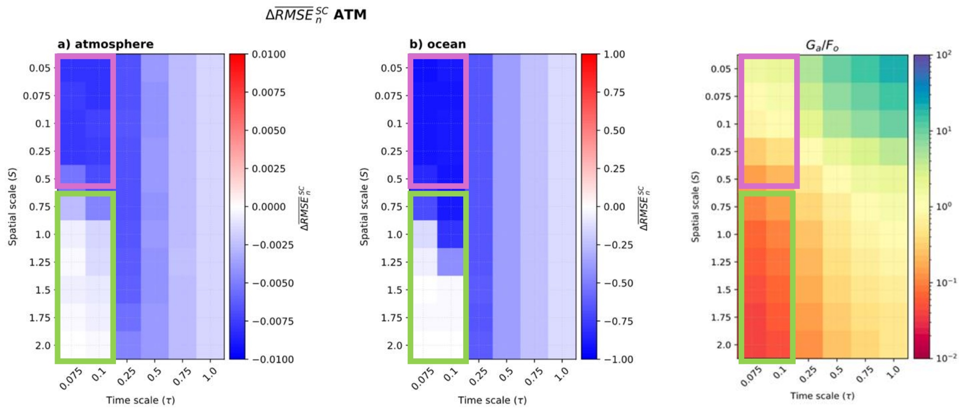
b) We understand the confusion of the reviewer. The actual spatial scale of the ocean is inversely proportional to the parameter S (Sect. 2.1), which we estimated using the 'Energy ratio' between the two components (Fig. 3a). In Fig. 12a the largest improvements in the unobserved atmosphere occur when $S \geq 1$ and it is maximum at $S = 2$, \tau = 1. This corresponds to the configuration where **the observed ocean has a smaller spatial scale** (relative to the atmosphere) and similar time scale. It is thus, in agreement with the abstract. To clarify this, we modified our manuscript in Sect. 2.1, to explicitly indicate that the ocean's spatial scale is inversely proportional to S. The manuscript now reads:

Line 102 - 114: " **We use energy *E* to estimate each component's spatial scale.** The energy E of the two components … The relative energy content of each component (Fig. 3a) shows that the energy of the ocean (E_o), **and hence the spatial scale separation**, is mostly inversely proportional to S and that the temporal scale has only a little influence on it. **Therefore, the ocean's spatial scale increases as $S > 1$.**"

4. Lines 10. "This suggests that SCDA of fast atmospheric observations can potentially improve the large-slow ocean component." This sentence is contradictory with the discussion in the Chapter 4.2, Lines 326 – 327: "This result can be explained as the atmosphere → ocean error propagation is smaller (Fig. 8); thus, the atmospheric data has no impact over the ocean."

R: The discussion in Sect. 4.2 on the lines 326-327 refers to the regions where the spatio-temporal scale parameters are $S \geq 0.75$ and $\tau < 0.25$ (green boxes in the figure below), indicating a relatively **smaller-slower ocean**. Over this region, Fig. 11 shows that SCDA has no significant impact compared to WCDA (note that the improvement is close to zero). This is supported by what we found in Fig. 8, which shows the competing direction of error propagation. Over the region $S \geq 0.75$ and $\tau < 0.25$ Fig. 8 shows that the dominant direction of error propagation is from the ocean to the atmosphere (i.e, red shading). Thus, this indicates that the assimilation of atmospheric observations towards the **smaller-slower ocean** has little impact, as we show in Fig. 11.

On the other hand, the conclusions for Line 10 "This suggests that SCDA of fast atmospheric observations can potentially improve the large-slow ocean component" refers to the region where the spatio-temporal parameters are $S < 0.75$ and $\tau < 0.25$ (magenta boxes in the figure below). These parameters indicate a relatively **larger-slower ocean**. Over this region, we conclude that the well constrained atmosphere and the high frequency cross-update towards the ocean help to reduce the error when using SCDA. Therefore, our main conclusion is not contradictory with our findings, but it refers to a different region of the spatio-temporal scales combinations that we analysed.



Red : ocn -> atm
Blue: atm -> ocn

References:

Tondeur, M., Carrassi, A., Vannitsem, S., and Bocquet, M.: On Temporal Scale Separation in Coupled Data Assimilation with the Ensemble Kalman Filter, Journal of Statistical Physics, 179, 1161–1185, https://doi.org/10.1007/s10955-020-02525-z, 2020.

Kurosawa, K., Kotsuki, S., & Miyoshi, T. (2023). Comparative study of strongly and weakly coupled data assimilation with a global land-atmosphere coupled model. *Nonlinear Processes in Geophysics*, *30*(4), 457–479. https://doi.org/10.5194/NPG-30-457-2023

Evensen, G., Vossepoel, F. C., & Leeuwen, P. J. van. (2024). Iterative ensemble smoothers for data assimilation in coupled nonlinear multiscale models. *Monthly Weather Review*, *1*(aop). https://doi.org/10.1175/MWR-D-23-0239.1

Miwa, N., & Sawada, Y. (2024). Strongly Versus Weakly Coupled Data Assimilation in Coupled Systems With Various Inter-Compartment Interactions. *Journal of Advances in Modeling Earth Systems*, *16*(3), e2022MS003113. https://doi.org/10.1029/2022MS003113