Supporting information to: Underestimation of Anaerobic Decomposition Rates in *Sphagnum* Litterbag Experiments by the Holocene Peatland Model Depends on Initial Leaching Losses

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S1  Model equations

The litterbag decomposition model combined with each modification of the HPM is obtained from Teickner et al. (2024) (model 1-4, see the supporting information to Teickner et al. (2024) for details). Here, we describe the modules which were added to this model in the different modifications of the HPM. All parameters are listed in Tab. S1.

In models HPMf-LE-peat, HPMe-LE-peat, and HPMe-LE-peat-l0, decomposition rates estimated from litterbag data ($k_{2\text{sample}}$) are modeled with decomposition rates predicted by the HPM decomposition module:

\[
\begin{align*}
  k_{2\text{sample}} & \sim \text{gamma}\left(hpm_k_{2 p1}, \frac{hpm_k_{2 p1}}{hpm_k_{2 \text{sample}}}\right) \\
  hpm_k_{2 p1} & \sim \text{gamma}(hpm_k_{2 p1 p1}, hpm_k_{2 p1 p2}) \\
  hpm_k_{2 \text{sample}} &= f(ldos_{1 \text{sample}}, lwtds_{1 \text{sample}}, m69_{p1}, m69_{p2}, m68_{p1}, m68_{p2}, m68_{p3} \frac{2}{\text{sample}}) \\
  ldos_{1 \text{sample}} &= g(layer\_depth\_midpoint_{1 \text{sample}}, lwtds_{1 \text{sample}}, layer\_total\_porosity_{1 \text{sample}}, lmdosas_{1}) \\
  layer\_total\_porosity_{1 \text{sample}} & \sim \text{beta}(layer\_total\_porosity_{1 p1}, layer\_total\_porosity_{1 p2}) \\
  lmdosas_{1 \text{sample}} & \sim \text{beta}(lmdosas_{1 p1}, lmdosas_{1 p2}) \\
  lwtds_{1 \text{sample}} & \sim \text{normal}(lwtds_{1 p1 \text{sample}}, lwtds_{1 p2}) \\
  hpm_k_{2 p1} & \sim \text{gamma}(hpm_k_{2 p1 p1}, hpm_k_{2 p1 p2})
\end{align*}
\]

(S1)

\textit{ldos}_{1} is short for \textit{layer\_degree\_of\_saturation\_1}. \textit{lmdosas}_{1} is short for \textit{layer\_minimum\_degree\_of\_saturation\_at\_surface\_1}. \textit{lwtds}_{1} is short for \textit{layer\_water\_table\_depth\_at\_surface\_1}. The HPM decomposition module is described in Froliking et al. (2010) (equations (7) to (9)). The Modified Granberg model is
described in Granberg et al. (1999) (equations (1) to (3)) and Kettridge and Baird (2007) (equations (6) to (9)).

In HPMe-LE-peat and HPMe-LE-peat-l0, HPM parameters are estimated from the litterbag data and we assumed the following prior distributions:

\[
\begin{align*}
    m_{68\_p3\_2\_sample} &= \exp(m_{68\_p3\_2\_p2\_species[sample]}) \\
    m_{69\_p1} &\sim \beta(m_{69\_p1\_p1}, m_{69\_p1\_p2}) \\
    m_{69\_p2} &\sim \gamma(m_{69\_p2\_p1}, m_{69\_p2\_p2}) \\
    m_{68\_p1} &\sim \gamma(m_{68\_p1\_p1}, m_{68\_p1\_p2}) \\
    m_{68\_p2} &\sim \gamma(m_{68\_p2\_p1}, m_{68\_p2\_p2}) \\
    m_{68\_p3\_2\_p2} &\sim \text{normal}(m_{68\_p3\_2\_p2\_p1}, 68_{p3\_2\_p2\_p2})
\end{align*}
\]

(S2)

HPMe-LE-peat-l0 in addition modeled the initial leaching loss of *Sphagnum* species as a maximum possible initial leaching loss per species \((hpm\_l\_2\_p1)\) which is modified by the degree of saturation:

\[
\begin{align*}
    l_{\_2\_sample} &\sim \beta(hpm\_l\_2 hpm\_l\_2\_p3, (1 - hpm\_l\_2) hpm\_l\_2\_p3) \\
    hpm\_l\_2\_sample &= \logit^{-1}(hpm\_l\_2\_p1\_species[sample] + hpm\_l\_2\_p2\_ldos\_1\_sample) \\
    hpm\_l\_2\_p1 &\sim \text{normal}(hpm\_l\_2\_p1\_p1, hpm\_l\_2\_p1\_p2) \\
    hpm\_l\_2\_p2 &\sim \text{normal}(hpm\_l\_2\_p2\_p1, hpm\_l\_2\_p2\_p2) \\
    hpm\_l\_2\_p3 &\sim \gamma(hpm\_l\_2\_p3\_p1, hpm\_l\_2\_p3\_p2)
\end{align*}
\]

(S3)
**S2  Prior choices and justification**

Table S1: Prior distributions of all Bayesian models and their justifications. “HPM parameter” is the name of the corresponding parameter in the Holocene Peatland Model (Frolking et al., 2010). When there is no value for “Justification”, the prior was chosen based on prior predictive checks against the data. This prior predictive check tests whether the models can produce distributions of measured variables we expect based on prior knowledge.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HPM parameter</th>
<th>Unit</th>
<th>Prior distribution</th>
<th>Justification</th>
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<tr>
<td>l_2_p1</td>
<td>(g g&lt;sub&gt;initial&lt;/sub&gt;) (logit scale)</td>
<td>normal(-3.5, l_2_p1_p2)</td>
<td>Assumes an average initial leaching loss across all available litterbag data within (95% confidence interval) (0.012, 0.068) g g&lt;sub&gt;−1&lt;/sub&gt;&lt;br&gt;initial</td>
<td></td>
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<tr>
<td>l_2_p2</td>
<td>(g g&lt;sub&gt;initial&lt;/sub&gt;) (logit scale)</td>
<td>normal(0, l_2_p2_p2)</td>
<td>Assumes an average initial decomposition rate across all available litterbag data within (95% confidence interval) (0.024, 0.131) yr&lt;sup&gt;−1&lt;/sup&gt;</td>
<td></td>
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<tr>
<td>k_2_p1</td>
<td>(yr&lt;sup&gt;−1&lt;/sup&gt;) (log scale)</td>
<td>normal(-2.9, k_2_p1_p2)</td>
<td>Assumes an average α across all available litterbag data within (95% confidence interval) (1.451, 2.473)</td>
<td></td>
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<tr>
<td>phi_2_p2_p1</td>
<td>(-) (log scale)</td>
<td>normal(5, phi_2_p2_p1_p2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>phi_2_p2_p2</td>
<td>(-) (log scale)</td>
<td>normal(0, phi_2_p2_p2_p2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>phi_2_p2_p3</td>
<td>(-) (log scale)</td>
<td>normal(0, phi_2_p2_p3_p2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>phi_2_p2_p4</td>
<td>(-) (log scale)</td>
<td>normal(0, phi_2_p2_p4_p2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k_2_p1_p2</td>
<td>(yr&lt;sup&gt;−1&lt;/sup&gt;) (log scale)</td>
<td>half-normal(0, 0.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k_2_p2_p2</td>
<td>(yr&lt;sup&gt;−1&lt;/sup&gt;) (log scale)</td>
<td>half-normal(0, 0.4)</td>
<td></td>
<td></td>
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<tr>
<td>layer_total_porosity_1</td>
<td>L&lt;sub&gt;pores&lt;/sub&gt;L&lt;sup&gt;−1&lt;/sup&gt;sample</td>
<td>beta(12, 3)</td>
<td>The total porosity was not reported in any study and therefore we assumed an average value of 80% with a standard deviation of 10%, roughly based on values reported for low-density Sphagnum peat (Liu and Lennartz, 2019).</td>
<td></td>
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<tr>
<td>layer_minimum_degree_of_saturation_at_surface_1</td>
<td>L&lt;sub&gt;water&lt;/sub&gt;L&lt;sup&gt;−1&lt;/sup&gt;pores</td>
<td>beta(0.9, 17.1)</td>
<td>This parameter comes from the modified Granberg model. The prior distribution assumes a minimum degree of saturation at the surface in different litterbag experiments of (95% confidence interval) (0.001, 0.191)</td>
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<tr>
<td>layer_water_table_depth_to_surface_1</td>
<td>cm</td>
<td>normal(average reported WTD, 3)</td>
<td>The average was set to the average water table depths reported in the litterbag studies.</td>
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<tr>
<td>hpm_k_2_p1</td>
<td>(g g&lt;sub&gt;initial&lt;/sub&gt;) (logit scale)</td>
<td>normal(-2.2, 0.3)</td>
<td>Assumes a maximum possible initial leaching loss across all available litterbag data within (95% confidence interval) (0.058, 0.167) g g&lt;sub&gt;−1&lt;/sub&gt;&lt;br&gt;initial</td>
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Further Information on Bayesian Data Analysis

Monte Carlo Standard Errors Monte Carlo standard errors (MSCE) (Vehtari et al., 2021) for the median were at most 0.012 yr\(^{-1}\) for \(k_0\), 0.363 mass-% for \(l_0\), 0.043 for \(\alpha\), 0.401 mass-% for the remaining mass, 0.001 L\(_{\text{water}}\) L\(_{\text{pores}}\) for \(W_{\text{opt}}\), 0.004 for \(c_1\), 0.001 yr\(^{-1}\) for \(f_{\text{min}}\), 0.002 m for \(c_2\), 0.003 yr\(^{-1}\) for \(k_0\) predicted by the HPM modifications, and 0.342 mass-% for \(l_0\) predicted by HPMe-LE-peat-l0. For the 2.5% and 97.5% quantiles, MCSE were at most 0.088 yr\(^{-1}\) for \(k_0\), 0.646 mass-% for \(l_0\), 0.147 for \(\alpha\), 2.742 mass-% for the remaining mass, 0.004 L\(_{\text{water}}\) L\(_{\text{pores}}\) for \(W_{\text{opt}}\), 0.006 for \(c_1\), 0.005 yr\(^{-1}\) for \(f_{\text{min}}\), 0.007 m for \(c_2\), 0.003 yr\(^{-1}\) for \(k_0\) predicted by the HPM modifications, and 0.293 mass-% for \(l_0\) predicted by HPMe-LE-peat-l0.

Power-scaling Power-scaling exponentiates prior (to analyze prior sensitivity) or likelihood (to analyze likelihood sensitivity) distributions by different constants \(\alpha > 0\), where \(\alpha > 1\) means that the scaled component gets more important relative to the other component, and \(\alpha < 1\) means it gets less important (Kallioinen et al., 2024). We varied \(\alpha\) from 0.99 to 1.01 (default option) and identified sensitivity with the cumulative Jensen-Shannon distance and a threshold of 0.05, as suggested in Kallioinen et al. (2024).

The power-scaling sensitivity analysis indicates a weak likelihood for all peat properties for most litterbag experiments, indicating that, not surprisingly, remaining masses alone do not give much information about peat properties. For \(W_{\text{opt}}\), \(c_1\), and \(c_2\) the analysis suggested a prior-data conflict which supports our finding that parameter values different from the standard values are more compatible with the data. For \(k_{0,i}\), the analysis suggested a prior-data conflict for most species, and similar for the parameters with which we modeled how initial leaching losses depend on the degree of saturation. We did not attempt to resolve these conflicts, either because we know from our previous study that the data provide only uncertain information (Teickner et al., 2024) which makes prior-data conflicts more likely, or because we wanted to use HPM standard parameter values as prior information. A future update of our study with more accurate data may address these challenges.

Software All other computations were done in R (4.2.0) (R Core Team, 2022). We computed prior and posterior predictive checks with the bayesplot package (1.9.0) (Gabry and Mahr, 2022) (supporting section S4). Data were handled with tidyverse packages (Wickham et al., 2019), MCMC samples with the posterior (1.5.0) (Bürkner et al., 2023) and tidybayes (3.0.2) (Kay, 2022b) packages. Graphics were created with ggplot2 (3.4.4) (Wickham, 2016), ggdist (3.1.1) (Kay, 2022a) and patchwork (1.1.1) (Pedersen, 2020).
S4  Prior and posterior predictive checks

Figure S1: Density estimate of 100 sets of remaining masses sampled from the prior distribution of each model (light blue lines) versus density estimate of the measured remaining masses from the litterbag studies.

Figure S2: Density estimate of 100 sets of remaining masses sampled from the posterior distribution of each model (light blue lines) versus density estimate of the measured remaining masses from the litterbag studies.
Figure S3: Density estimate of 100 sets of remaining mass errors (converted to precision) sampled from the prior distribution of each model (light blue lines) versus density estimate of the measured remaining mass errors from the litterbag studies. The x axis is log scaled.

Figure S4: Density estimate of 100 sets of remaining mass errors (converted to precision) sampled from the posterior distribution of each model (light blue lines) versus density estimate of the measured remaining mass errors from the litterbag studies. The x axis is log scaled.
Figure S5: Density estimate of 100 sets of decomposition rates ($k_0$) predicted by the HPM modifications sampled from the prior distribution of each model (light blue lines) versus density estimate of the decomposition rates estimated from the litterbag studies.

Figure S6: Density estimate of 100 sets of decomposition rates ($k_0$) predicted by the HPM modifications sampled from the posterior distribution of each model (light blue lines) versus density estimate of the decomposition rates estimated from the litterbag studies.

Figure S7: Density estimate of 100 sets of initial leaching losses ($l_0$) predicted by HPMe-LE-peat-l0 sampled from the prior distribution (light blue lines) versus density estimate of the initial leaching loss estimated from the litterbag studies.
Figure S8: Density estimate of 100 sets of initial leaching losses ($l_0$) predicted by HPMe-LE-peat-l0 sampled from the posterior distribution (light blue lines) versus density estimate of the initial leaching loss estimated from the litterbag studies.
S5 $k_{0,i}$ estimates in HPMe-LE-peat and in HPMe-LE-peat-l0

Figure S9: $k_{0,i}$ estimates in HPMe-LE-peat-l0 and in HPMe-LE-peat-l0 for each Sphagnum species. Points are average values and error bars are 95% confidence intervals. Sphagnum spec. are samples which have been identified only to the genus level and there are two values here because we defined two separate species in the HPM to estimate maximum possible decomposition rates separately for initial peat samples collected from 10 or 20 cm depth in Prevost et al. (1997).
S6 Marginal posterior distributions of HPM parameters in HPMe-LE-peat and HPMe-LE-peat-l0

Figure S10: Marginal posterior distributions of HPM decomposition model parameters as estimated by HPMe-LE-peat. (a) $k_0$ estimated for each species. Species were assigned to HPM microhabitats as described in the Methods section in the main text. (b) other HPM parameters. Vertical black lines are the standard parameter values from Frolking et al. (2010). *Sphagnum* spec. are samples which have been identified only to the genus level.

Figure S11: Marginal posterior distributions of HPM decomposition model parameters as estimated by HPMe-LE-peat-l0. (a) $k_0$ estimated for each species. Species were assigned to HPM microhabitats as described in the Methods section in the main text. (b) other HPM parameters. Vertical black lines are the standard parameter values from Frolking et al. (2010). *Sphagnum* spec. are samples which have been identified only to the genus level.
Figure S12: Marginal posterior distributions of HPM decomposition model parameters as estimated by HPMe-LE-peat-l0 during the cross-validation. During the cross-validation, one of the cross-validation folds was left out each time and the model was refitted, producing a marginal posterior distribution for each parameter and cross-validation block. (a) $k_{0,i}$ estimated for each species for which data were removed during the cross-validation. (b) Other HPM parameters.
Figure S13: Estimated for $l_0$ from the litterbag data (Predicted with HPM = No) and predicted by different versions of the HPM decomposition module (Predicted with HPM = Yes) (HPMf, HPMf-LE-peat, and HPMe-LE-peat) versus reported (HPMf) or estimated (HPMf-LE-peat and HPMe-LE-peat) average water table depths below the litterbags. Points represent average estimates and error bars 95% posterior intervals. Lines are predictions of linear models fitted to the average estimates. *Sphagnum* spec. are samples which have been identified only to the genus level. Only data for species with at least three replicates are shown.
S8  \( k_0 \) predicted by the HPM versus water table depth below the litter for different studies and species

Figure S14: \( k_0 \) estimated with the litterbag decomposition model from Teickner et al. (2024) (Predicted with HPM = No) and predicted by the HPM decomposition module with standard parameter values (HPMf, Predicted with HPM = Yes) versus reported average water table depths below the litterbags for different species and studies. Points represent average estimates and error bars 95% posterior intervals. Lines are predictions of linear models fitted to the average estimates. *Sphagnum* spec. are samples which have been identified only to the genus level. Only data for species with at least three replicates are shown. Error bars exceeding 0.5 yr\(^{-1}\) are clipped.
Figure S15: $k_0$ estimated with the litterbag decomposition model in HPMe-LE-peat-l0 from the litterbag data (Predicted with HPM = No) and predicted by the HPM decomposition module with parameter values estimated from the litterbag data (HPMe-LE-peat-l0, Predicted with HPM = Yes) versus estimated average water table depths below the litterbags for different species and studies. Points represent average estimates and error bars 95% posterior intervals. Lines are predictions of linear models fitted to the average estimates. *Sphagnum* spec. are samples which have been identified only to the genus level. Only data for species with at least three replicates are shown. Error bars exceeding 0.5 yr$^{-1}$ are clipped.
S9 Depth profiles of predicted decomposition rates with $W_{opt}$ estimated by HPMe-LE-peat-l0 or set to its standard value for *S. fallax*

![Graph showing decomposition rates predicted with HPMe-LE-peat-l0 for *S. fallax* (hollows), using either the standard value for $W_{opt}$ or the $W_{opt}$ value estimated by HPMe-LE-peat-l0 versus depth of the litter below the peat surface. The horizontal line is the average water table depth.](image)

Figure S16: Decomposition rates predicted with HPMe-LE-peat-l0 ($k_{0,\text{modified}(\text{HPMe-LE-peat-l0})$) for *S. fallax* (hollows), using either the standard value for $W_{opt}$ or the $W_{opt}$ value estimated by HPMe-LE-peat-l0 versus depth of the litter below the peat surface. The horizontal line is the average water table depth.
S10  Results for HPMe-LE-peat-l0-outlier

Figure S17: Marginal posterior distributions of HPM decomposition model parameters as estimated by HPMe-LE-peat-l0-outlier. (a) $k_0$ estimated for each species. Species were assigned to HPM microhabitats as described in the Methods section in the main text. (b) other HPM parameters. Vertical black lines are the standard parameter values from Frohling et al. (2010). *Sphagnum* spec. are samples which have been identified only to the genus level.

S11  Prediction uncertainties of HPMe-LE-peat-l0

To illustrate that the HPM decomposition module implies large uncertainties if its parameters are estimated from available litterbag data, we simulate decomposition of *S. fallax* and *S. fuscum* litter during 50 years, either incubated under a degree of saturation of 0.6 $L_{\text{water}} L^{-1}_{\text{pores}}$, or 20 cm below the water table. The results are shown in Fig. S18.
Figure S18: Fraction of initial mass remaining of *S. fuscum* and *S. fallax* versus incubation duration as predicted by HPMe-LE-peat-l0, assuming average species α and uncertainty of remaining masses averaged across litterbag experiments. (a) Shows predicted fractions of initial mass remaining and (b) predicted fractions of initial mass remaining for one individual sample. Samples are either incubated in the saturated zone 20 cm below the water table, or in the unsaturated zone 10 cm above the water table. Shaded areas are 50, 80, and 95% confidence and prediction intervals, respectively.

### S12 R code to predict $k_0$, $l_0$, and remaining masses with HPMe-LE-peat-l0

HPMe-LE-peat-l0 and functions to predict $k_0$ and $l_0$ for different species and water table levels are available via the R package hpmdpredict (Teickner and Knorr, 2024). To make predictions, one first has to define some variables like the incubation duration. Here, we predict remaining masses and initial leaching losses for *S. fuscum* incubated at a degree of saturation of 0.6 $L_{water} L_{poles}^{-1}$ during the first five years.

```r
d <-
tibble::tibble(  
  incubation_duration = seq(from = 0, to = 5, length.out = 30),
  m0 = 1,
  layer_degree_of_saturation_1 = 0.6,
  layer_water_table_depth_to_surface_1 = 20,
  sample_depth_lower = 10,
  hpm_taxon_rank_value = "Sphagnum fuscum"
)
```

Next, one can pass this data frame to `hpmd_predict_fit_4()` which makes the predictions.
library(hpmdpredict)
d <- hpmdpredict::hpmd_predict_fit_4(newdata = d)

To illustrate the result, we plot predicted remaining masses versus incubation time:
library(ggplot2)
library(ggdist)

d |> ggplot(aes(ydist = mass_relative_mass * 100, x = incubation_duration)) +
stat_lineribbon() +
scale_fill_brewer() +
labs(
  y = "Fraction of initial mass (%)",
  x = "Incubation duration (yr)"
)

Further information are available from the package documentation.

References
Kallioinen, N., Paananen, T., Bürkner, P.-C., and Vehtari, A.: Detecting and diagnosing prior and likelihood sensitivity with power-scaling, Statistics and Computing, 34, 57, https: