

This work applies a Physics-Informed Neural Network (PINN) as a data-driven tool to estimate ice thickness across glaciers located on Spitsbergen in Svalbard. A physics-based loss function used in training the PINN is designed to penalize solutions diverging from a modified form of mass conservation. The neural network is also provided with a number of other data inputs, including surface velocity, surface slope, elevation, positional parameters, and values providing assumed relationships between surface and depth-averaged velocities. The authors explore their results using a cross-validation scheme designed to avoid problems with spatial correlation.

PINNs have seen an increasing number of uses within glaciology in the past few years. Their intrinsic ability to mix together known physics with poorly calibrated constants and sparse and/or noisy measurements make them an appealing modeling tool for underdetermined problems. This work is novel in training a single PINN over a heterogeneous domain consisting of multiple glaciers (effectively all of Spitsbergen) and in mixing physics-informed methods with purely data-driven methods.

The authors position their work as a proof of concept, largely demonstrating the ability to scale PINN-based methods to larger domains by mixing in non-physics-informed methods to produce continuous estimates in regions where no suitable boundary conditions exist.

In my view, the manuscript is much improved and offers a constructive contribution to the ongoing challenges of building useful models from sparse in-situ data. The availability of the authors' code and quality documentation adds to the value of the work.

### **Framing of the work**

- In your response to the last round of reviewer comments, you made clear that you view your work as primarily a proof of concept. In that case, I'd suggest making clear in both the abstract and the introduction that your purpose with this article is to demonstrate a new technical method.
- Viewed as a technical proof of concept, I see what sets your work apart as primarily (a) the application of physics-informed techniques to a heterogeneous spatial area spanning multiple catchments enabled by (b) a fusion of physics-informed and purely data-driven loss functions.

If this is consistent with your view, I would suggest a few changes in your literature review:

- Since your work fuses data-driven and physics-driven approaches, consider briefly reviewing non-physics-informed machine learning applications. Some that you might consider including:
  - Ice sheet scale thickness estimates: Leong and Horgan, 2020.
  - Glacier-scale SMB: Bolibar et al., 2019
  - Glacier-scale thickness: Haq et al., 2021

- In addition to reviewing the Teisberg et al., 2021 application of PINNs to ice thickness mapping, also consider incorporating variational inference-based approaches such as Brinkerhoff et al., 2016.
- I'm not sure I follow the comment about "without further consideration of bed properties" - perhaps expand on this if it's a key difference.
- In framing your work as covering "an entire region," I think it's important to note that the significance of this is the heterogeneity of the region in question due to it being composed of multiple glaciers separate catchments. Notably, your domain is of roughly comparable size to the Rutford Ice Stream, Byrd Glacier, and the Amery Ice Shelf, the subjects of three of the works you cite. This is not to take away from what you are doing. It's just important to note that the significance is about the heterogeneity more than the pure spatial scale.

### Physical model

- Section 2.2 is much clearer now.
- Line 79: I don't think that SIA necessarily implies neglecting the temperature dependence of the rate factor. See, for example, Larour et al., 2012. I suggest separating the constant A part out as a separate assumption you are adding.
- Equation 3 feels a bit abstract without knowing the selected values of  $l_{\text{lower}}$  and  $\beta$ . Since these are specified later in the text, I suggest specifying their value/ranges briefly here (and referring readers to the appropriate sections below for details).

### PINN Evaluation

- This is an extremely tricky issue, as there are not yet well established metrics for evaluating PINNs and this application is ever more challenging due to the combination of physics-informed and non-physics-informed loss functions. In general, the authors are doing a good job of discussing these nuances.
- I am not yet convinced by the analysis of the leave-one-out results and, in particular, in the overfitting conclusion. I would suggest evaluating the physics-informed loss function alone on the results of each of the 7 test glaciers when its ice thickness data was and wasn't included in the training (when it was or wasn't the left-out glacier, that is). Depending on the results of this...

- Case 1: The physics-informed loss is about the same for each glacier with the thickness data left out or not.

In this case, it seems that the model has found two different thickness maps which are roughly equally physically-plausible. This would be a very interesting result. I would argue this does not suggest "overfitting". The obvious follow-on question is: if you add in one point (or some very small amount of ice thickness data), does your model now predict the right thickness map?

- Case 2: The physics-informed loss is much higher with the thickness data left out.

In this case, I would agree with your "overfitting" assessment, but it's interesting

that your model is not finding a physically-plausible result without being guided by a few thickness measurements.

### Additional minor comments

- Line 12: I find the opening claim that surface velocity is proportional to the fourth power of ice thickness to be potentially confusing. I assume you are referring to the paragraph following Eq. 8.36 in Cuffey and Paterson, which says that surface velocity is proportional to  $H^4 * \alpha^3$ . Notably, however, ice thickness is inversely proportional to surface slope (Eq. 8.9). I would recommend softening the claim that ice thickness is the single most important input and citing a specific chapter of Cuffey and Paterson (8, I assume) to make it easier for readers to find the derivation you reference.
- Line 13: The importance of bed topography also needs a citation.
- Line 72: “encounter” doesn’t make sense to me here. Perhaps you meant “counter” or “balance”?
- Line 78: “lamellar” -> “laminar”
- Line 111: Missing period after “Appendix B”
- Line 126 / Eq. 7: Equation 7 is not, in my opinion, a physics-aware constraint. I would argue that it is a heuristic smoothing regularization.
- Line 178: The units of the ratio here are a little confusing. I think the units here are  $(m/yr)/(m/m)$ , which simplifies to  $m/yr$ . I recognize that is the same as  $yr^{-1}*m$ , however I would suggest either writing out the full un-simplified form or sticking to the more conventional  $m*yr^{-1}$ .