In the following we address the comments by Referee 1 point-by-point. The referee comments are provided in italic; our responses in bold font.

Response to Referee 1:

General comments:

This Opinion paper reviews the challenges and limitations of constraining future climate response using emergent constraints and discusses an alternative approach, which combines climate-invariant controlling factor analyses (CFA) and machine learning. The authors demonstrate the advantages of CFA, along with the remaining challenges and potential applications on model tuning. Overall, the paper is well-structured, and I have no major concerns with the paper. The following comments are meant to improve the clarity of the article.

We thank the referee for the overall positive comments and constructive suggestions, which have helped to improve our manuscript.

Specific comments:

- *Emergent constraint is a fundamental concept for this topic, and I believe a clearer definition is needed in the Introduction section before introducing the associated limitations. The authors have provided more details when discussing the difference between CFA and emergent constraints, but I recommend adding one to two sentences in section 1.2.2.*

Good idea, we have added the definition from the abstract in the paper by Williamson et al. (2021) as a first sentence in section 1.2.2:

"The emergent constraint approach uses the climate model ensemble to identify a relationship between an uncertain aspect of the future climate and an observable or variation or trend in the contemporary climate (Williamson et al., 2021)."

- *Figure 1: "In (b), internal variability uncertainty for individual ensemble members …" Since only one ensemble member for each model is shown, the figure technically didn't provide any information regarding internal variability uncertainty. Alternatively, the authors may consider adding an inset figure in Fig 1b to show the internal variability for one model, or at least remove the phrase "for individual ensemble members" in the caption. In addition, I suggest adding a dashed line at year 2050 to highlight the difficulty of distinguishing projected warming by that year.*

We agree that we do not illustrate internal variability uncertainty in isolation here, because we do not show multiple ensemble members for the same climate model and climate forcing scenario. To clarify this issue, we have changed the sentence in question to:

"Internal variability uncertainty across the 34 simulations makes it difficult to, e.g., answer the question of how much the region is projected to have warmed by the year 2050, even in the absence of model uncertainty."

Additionally, we have added the suggested dashed dark line to Figure 1 (b).

- *Figure 2: The final observational constraint (delta_y_constrained combined with prediction error) is shown as light blue line in the bottom right figure, but it is different from "delta_y_constrained" (light blue color) in the equation. Please consider revising the figure to make them consistent. For instance, the black dashed distribution could be changed to a light blue dashed line, and the light blue distribution could become black.*

Well spotted - thank you for pointing out this inconsistency. We have adjusted the figure and figure caption accordingly.

- *Figure 2: What is the temporal resolution of the observations in the top right figure? The text mentioned they are monthly-mean data but it doesn't seem correct. Please clarify this.*

Yes, indeed, again well-spotted. This was meant to be a sketch without going into detail. However, we have changed the corresponding sentence in the figure caption, also to reflect that an advantage of CFAs is that one might work with flexible time resolutions to derive the observational constraint relationships:

"Out-of-sample predictive skill is evaluated in each case on held-out test data; illustrated here for a hypothetical test year 2012 on daily data."

In addition, we have added the following clarification concerning the extrapolation to future scenarios to avoid confusion:

"Next, it is tested if the relationships learned also hold under climate change scenarios (annually averaged for visualization purposes). This step is only possible for climate models; demonstrated here for two example SSP projections."

Technical corrections:

- *Figure 2 caption: "the violet lines the predictions of the functions are fed with the model-consistent changes in the controlling factors." It seems like a verb is missing in the sentence. Same for Figure 4's caption: "the solid red lines the linear regressions".*

We have changed the first sentence to improve clarity:

"The black lines mark the actual climate model responses; the violet lines mark the predictions if the functions are fed with the model-consistent changes in the controlling factors (which, if approximately climate-invariant relationships were indeed established, should replicate the actual responses)."

The second sentence we have revised to:

"The dashed red lines mark the prediction intervals, whereas the solid red lines show linear regressions fitted to the data."

- *L256: The uncertainty arises not just from changes in cloud cover but from changes in cloud properties, including cloud height, cloud optical depth, etc. Please consider rephrasing it.*

We have changed the sentence to:

"Changes in cloud properties (amount, optical depth, altitude) are the leading uncertainty factor in global warming projections under increasing atmospheric CO2."

- *L280: duplicated "be"*

We have removed the duplication.

- *L331: add a comma (,) after "…to be non‑linear (Carslaw et al., 2013a)"*

Done.

- *L337: duplicated "either"*

We have removed the duplication.