- The title is misleading: the approach is not semi-analytical. It is a numerical model. I am also uneasy with the notion of vertical propagation on PV when the latter is materially conserved in absence of diffusion and that there is no vertical advection in QG.
- I am uneasy with the justification of the equations using the depth z instead of ρ, as the justification (based on crude finite difference) differs significantly from the rigorous derivation (based on vertical averaging, see e.g. [1]) of layered models. There must be other ways to justify the change (such as arguing that at the relevant order in Ro, isopycnals are flat - corrections only dynamically matter at higher order in Ro).
- line 145: the statement is technically erroneous. $\partial\psi/\partial z$ in equation (2) is a rescaled buoyancy anomaly. The vertical velocity is proportional to the material derivative of this buoyancy anomoly. So, although $\partial\psi/\partial z = 0$ implies no vertical velocity, the reciprocal is not true: no vertical velocity does not imply $\partial\psi/\partial z = 0$.
- Please clarify where equation (5) comes from.
- Equation (12) is unecessarily general since only $U_1 \neq 0$. It would be simpler to just define $\bar{\psi}_1$ in equation (12) and state in the text that $\bar{\psi}_j$, $j = 2, 3, 4$ is set to 0 at $t = 0$. There is no point in plotting curves $\bar{\psi}_j = 0$ in Fig. 3(a).
- The parameter β appears in the equations but its value is not given (unless I am mistaken).

[1] V. Zeitlin, Geophysical Fluid Dynamics: understanding (almost) everything with rotating shallow water models, OUP, 2018