Reply to the RC2

In this file, the comments from RC2 are in **black font**; our replies below are in **blue font**.

Overall reply: After several days of careful evaluation of our model, we are thrilled to inform you that we have derived a new SAS function analytically for water in outflow rivers. The transport and releasing of rain water and river water have been integrated into this new SAS function successfully. The new SAS function can be calculated analytically, without check whether the beta distribution is appropriate at each time step. In this way, the whole calculation process following the same procedures in previous TTD literature. Therefore, this new SAS function makes our model simpler and easier to understand. Below is a brief introduction to our improved model, followed by a point-by-point response to your comments.

In our new SAS function for outflow rivers, we introduce a new variable called the event rain water threshold age (Figure 1). As shown in Figure 1, for water age below this threshold age, only rain water is released from the lake; for water age above this threshold, both rain water and river water can be released. The threshold age can equal 0, which means there is no event rain water in the outflow. The physical interpretation of the event rain water threshold age is that some rainfall enters the lake very close to the outflow rivers, however there is always a distance between the inflow rivers and the outflow rivers.

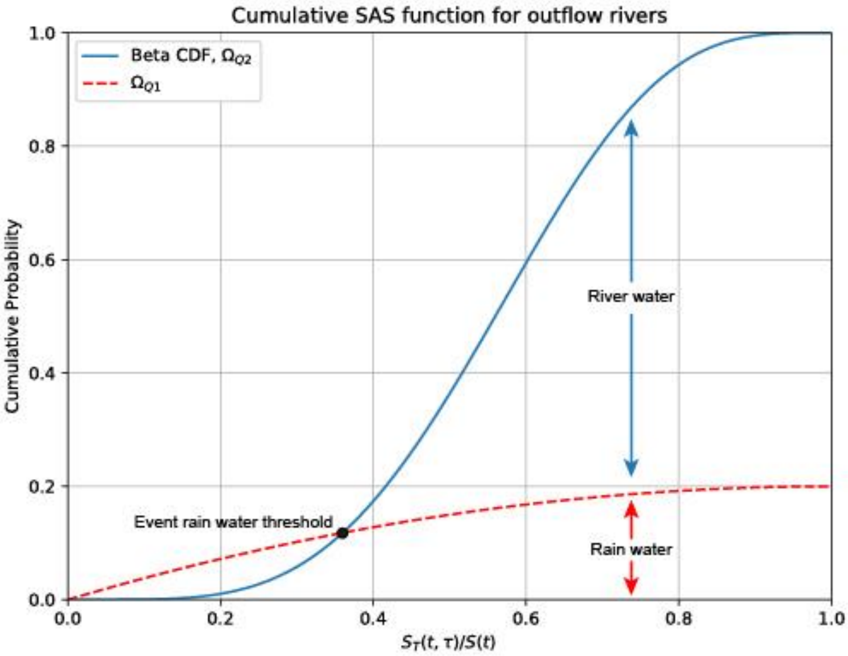

Figure 1. Illustration of the new SAS function for outflow rivers and the event rain water threshold.

Let τ_e represent the event rain water threshold age, then the new cumulative SAS function $\Omega_Q\big(t, S_T(t,\tau)\big)$ is defined as follow:

$$\Omega_Q\big(t, S_T(t,\tau)\big) = \begin{cases} \Omega_{Q1} & \text{if } \tau < \tau_e \\ \Omega_{Q2} & \text{if } \tau > \tau_e \end{cases} \qquad (1)$$

where Ω_{Q2} is the traditional cumulative SAS function. In this study, we use the beta distribution to characterize Ω_{Q2}; and Ω_{Q1} remains to be determined and is the cumulative proportion of the volume of rain water younger than age τ in outflow rivers. Moreover, the value of the event rain water threshold age also remains to be determined. Note that the event rain water threshold age is the intersection point of Ω_{Q1} and Ω_{Q2} (Figure 1), so we will derive Ω_{Q1} first. Then, the event rain water threshold age can be calculated.

(1) Derivation of Ω_{Q1}

In the rainfall mixing model (line 282 and line 303), the volume of rain water aged τ in outflow rivers, i.e., $F_{out\_rain}(t,\tau)$ is tracked as:

$$F_{out\_rain}(t,\tau) = \overline{F_{out_T}}(t,\tau) \cdot f(t,\tau) = F_{out}(t)\,(1-\Omega_Q) \cdot f(t,\tau) \tag{2}$$

where $\overline{F_{out_T}}(t,\tau)$ is the volume of water older than age τ in outflow rivers, $f(t,\tau)$ is the rainfall mixing factor. Let's divide Ω_{Q1} into two parts based on the event rain water threshold point: $\Omega_{Q1\_\mathrm{left}}$ and $\Omega_{Q1\_\mathrm{right}}$. For $\Omega_{Q1\_\mathrm{left}}$, i.e., $\tau < \tau_e$, the water in outflow is all event rain water, so $\Omega_Q = \Omega_{Q1\_\mathrm{left}}$. Substituting $\Omega_Q = \Omega_{Q1\_\mathrm{left}}$ into equation (2), we have:

$$F_{out\_rain}(t,\tau) = F_{out}(t)\,(1-\Omega_{Q1\_\mathrm{left}}) \cdot f(t,\tau) \tag{3}$$

Then, $\Omega_{Q1\_\mathrm{left}}$ can be calculated based on its definition and equation (3):

$$\Omega_{Q1\_\mathrm{left}} = \int_0^{\tau} \frac{F_{out\_rain}(t,\tau)}{F_{out}(t)}\,\mathrm{d}\tau = \int_0^{\tau} (1-\Omega_{Q1\_\mathrm{left}}) \cdot f(t,\tau)\,\mathrm{d}\tau \tag{4}$$

By differentiating both sides of Equation (4) with respect to τ, we obtain:

$$\frac{\mathrm{d}\Omega_{Q1\_\mathrm{left}}}{\mathrm{d}\tau} = (1-\Omega_{Q1\_\mathrm{left}}) \cdot f(t,\tau) \tag{5}$$

Then, $\Omega_{Q1\_\mathrm{left}}$ is solved from equation (5):

$$\Omega_{Q1\_\mathrm{left}} = 1 - e^{-\int_0^{\tau} f(t,\tau)\mathrm{d}\tau} \tag{6}$$

(2) Determination of event rain water threshold age

Since the event rain water threshold age τ_e is the intersection between $\Omega_{Q1\_\mathrm{left}}$ and Ω_{Q2}, this threshold age can be solved by setting $\Omega_{Q1\_\mathrm{left}} = \Omega_{Q2}$, i.e.,

$$\Omega_{Q2} = 1 - e^{-\int_0^{\tau_e} f(t,\tau)\mathrm{d}\tau} \tag{7}$$

(3) Analytical form of the SAS function for outflow rivers:

Finally, the analytical form of Ω_Q is:

$$\Omega_Q\big(t, S_T(t,\tau)\big) = \begin{cases} 1 - e^{-\int_0^{\tau} f(t,\tau)\mathrm{d}\tau} & \text{if } \tau < \tau_e \\ \Omega_{Q2} & \text{if } \tau > \tau_e \end{cases} \tag{8}$$

where Ω_{Q2} is the cumulative beta distribution. ($\Omega_{Q1\_\mathrm{left}}$ in equation (8) is actually the cumulative backward TTD for $\tau < \tau_e$. The transformation between the cumulative age-ranked SAS function and the cumulative backward TTD is ignored here, see Harman 2015, http://dx.doi.org/10.1002/2014WR015707.)

(4) Compositions of water older than τ_e

For water older than τ_e, Ω_{Q2}, the beta distribution, is equal to the cumulative SAS function for all water (i.e., Ω_Q). According to the rainfall mixing model, the cumulative proportion of rain water

$\Omega_{Q1\_\mathrm{right}}$ is:

$$\Omega_{Q1\_\mathrm{right}} = \int_{\tau_e}^{\tau} (1 - \Omega_{Q2}) \cdot f(t,\tau)\, \mathrm{d}\tau + 1 - e^{-\int_0^{\tau_e} f(t,\tau)\mathrm{d}\tau} \tag{9}$$

and the cumulative proportion of river water in outflows is $\Omega_{Q2} - \Omega_{Q1\_\mathrm{right}}$, which is shown in Figure 1.

(5) Summary

The reason we introduce a new SAS function is that the beta distribution may failed to quantify the proportion of event young rain water. In the system where the event young rain water exists, as shown in Figure 1, the proportion of event young rain water is always underestimated, if the SAS function for $\tau > \tau_e$ is used for the case of $\tau < \tau_e$. This new SAS function is in analytical form and will replace our previous numerical method for the SAS calculation. The modification of the SAS function will not change the results of TTD, RTD in Lake Taihu. The new model and its results, especially the value of event rain water threshold age, will be updated in our revised manuscript.

Below are our point-by-point replies:

This manuscript introduces a travel time model that deals with the problem of tracking water age in hydrologic systems characterised by multiple input fluxes. This is relevant for lake studies, which typically have 2 inputs (river inflow and rainfall). The authors apply the model to the case study of Lake Taihu, China, and calibrate the model parameters against 24 monthly values of mean isotope composition of the lake water. The authors use the calibrated model to discuss water age dynamics and rain/river water partitioning within the lake and the output fluxes.

I anticipate that, while I appreciate this work, I am afraid there are major technical issues in the solution proposed by the authors that prevents publication of the current version of the paper, but I think that similar results produced under alternative – and possibly simpler solutions may be worth of publication.

Reply: Huge thanks for your valuable suggestion, which will make our revised manuscript much better than the current version. After carefully consideration of your suggestion, we have put forward a simpler and new solution, which encapsulates the transport and mixing of rain water and river into a single SAS function. This new SAS function is introduced in our overall reply and will be added to our revised manuscript.

The authors touch on a very interesting and challenging problem that is multiple-input tracking in lumped models. Most of the water transit time literature (see the review work that myself and many colleagues have written in 2022, https://doi.org/10.1029/2022WR033096) dealt with systems with one single input, i.e. rainfall. Therefore, the effort put in place by the authors fills an important gap and is to be credited. The text, while needing some English proofread, is understandable and the approach undertaken by the authors appears rigorous and is described in full detail.

Reply: We appreciate your recognition of the contribution of our manuscript. Your acknowledgement has provided us with great encouragement to improve the TTD model in our manuscript.

A key point of the paper is a new solution to the tracking of multiple sources within a lumped model. The authors call their solution "Rainfall mixing" or "rainfall tracking". I think this solution is problematic on some fronts:

1. The manuscript seems to often confound space with age. Lumped age models do not explicitly account for space. The effects of physical processes (like advection and dispersion) can be effectively reproduced by using a lagged transit time distribution or SAS function, but those processes are not directly modelled. Similarly, water age models do not account for any physical mixing within the storage and they only prescribe how the output fluxes remove waters of different ages from the storage (which is why the community tends to speak about "random sampling" rather than "well mixing"). The idea that rainfall water is well mixed with pre-existing lake water while river water is not is unsuitable to this lumped framework and should rather be translated into SAS language and equations. Translating the different transport processes of two very different inputs into a single SAS function may be challenging and highlights the limitations of lumped water age models.

Reply: We accept this comment. We will check through the manuscript to ensure the SAS language is consistently used when discussing our rainfall mixing model and the water age model. In the model development sections, where mixing in space is most often mentioned to discussing the rainfall mixing model, we will immediately translate these discussions into SAS language after these sentences. For example, in lines 239-240, after introducing the concept that rainwater is well mixed vertically with pre-existing lake water parcels, we will add the following sentence: "This indicates that the ratio between the volume of aged rainwater and the volume of lake water older than that rainwater remains fixed. Thus, in the outflow rivers, rainwater aged τ is randomly sampled from the mixed lake water older than age τ, and the random sampling probability or ratio in the outflow rivers is equal to the mixing ratio in the lake." In this way, the physical meaning of our model remains clear when we introduce our model in SAS language. This description approach is commonly used in SAS literature (Pangle et al. https://doi.org/10.1002/2016WR019901; Kim et al 2022, https://doi.org/10.1029/2020WR028959; Wilusz et al, https://doi.org/10.1029/2019WR025140; etc.).

Regarding the issue you mentioned in the last sentence, the new SAS function we proposed in our previous response has successfully integrated the transport of rain water and river water in the lake into a single SAS function. Therefore, this will not be an issue in our revised manuscript.

2. The rainfall mixing approach is difficult to understand and after reading it multiple times I still am not sure I could follow all the steps. My understanding is that ultimately the "candidate" beta-shaped SAS functions are checked at any time step and if some constrain is not respected, they are modified. The effect of this transformation is difficult to follow, but I think it generally increases the amount of young water released to the outflow, to effectively simulate the "preferential" release of rain water. The manuscript should clarify the effect of this modification more explicitly, for example by showing some examples of candidate and modified SAS

function or by showing a simulation with and without those constraints. I think it is also important that this more complex approach is justified in terms of model performance, i.e. that the model with the modification performs significantly better than a “traditional” model.

Reply: In our new SAS function, the steps for check the “candidate” beta-shaped SAS functions are canceled. The reason we need to check the beta-shaped SAS function is that it may fail to capture the proportion of event young rain water in outflow rivers, as shown in Figure 1. For detail, please see our overall reply at the beginning. In our new improved model, the SAS function is clearly defined in its analytical form, with no need to check the constrains. Therefore, this major issue will no longer be a problem in our revised manuscript.

3. Monthly time steps for the solution of the water age balance using the Euler Forward scheme is potentially coarse. I invite the authors to verify that the numerical accuracy is not compromised by the use of large time steps.

Reply: We accept this comment. A large Euler Forward scheme for partial differential equations may cause two problems: stability and accuracy. For the stability problem, this is not an issue, as our calculated TTD, RTD, and deuterium concentration do not blow up and remain within a reasonable range. However, we have not yet examined the accuracy issue as you suggested. We will add a discussion of model accuracy to the manuscript.

Below are the preliminary results of the verification of model accuracy as influenced by time step:

To verify the model accuracy, we interpolated our monthly lake volume data into datasets with several different time steps. For other input data, such as rainfall, ET, and river inflow and outflow rates, we continued using the monthly averaged data. We then reran our model with different time steps: dt=1, 0.5, 025, 0.125 months. The calculated deuterium concentrations are shown in Figure 2.

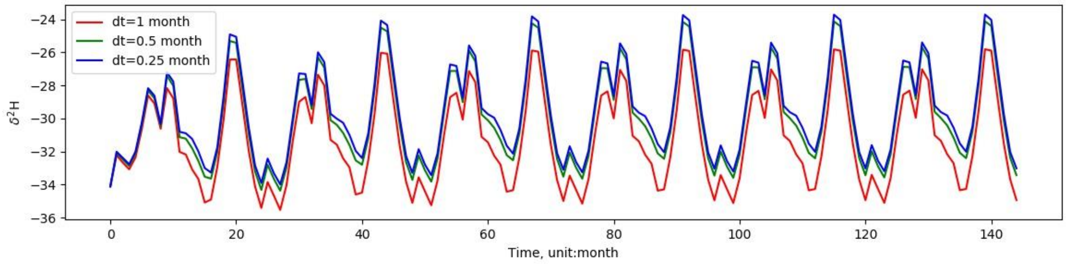

Figure 2. Influence of time steps on deuterium concentration calculation. dt=0.125 month is not displayed, as it is very close to the curve for dt=0.25.

In Figure 2, the deuterium concentration increases slightly with smaller time steps. Referring to Figure 13 in our manuscript, a larger time step may have the most significant influence on the estimation of evaporation fractionation factor for deuterium, but it has little effect on the parameters for the SAS function.

Therefore, we will revise our manuscript using the model with a smaller time step of dt=0.25 month and add a paragraph to discuss the error caused by different time steps.

I believe these major issues need to be addressed before discussing additional minor comments. However, I reiterate that an improved or simplified multiple-input tracking system would likely make the paper worthy of publication.

Reply: With our newly developed SAS function, we believe the multi-input-tracking system will be much simpler and easier to understand and implement than our previous model. We also appreciate your comments on the time step, as they have significantly increased our model's accuracy.

Thank you again for your valuable comments; they will greatly improve the quality of our manuscript!