We define the auxiliary field functions as follows:

$$G_1(x, y, t, h) = -\frac{1}{S(x, y)} \frac{\partial T_x(x, y)}{\partial x}, \tag{1}$$

$$G_2(x, y, t, h) = -\frac{1}{S(x, y)} \frac{\partial T_y(x, y)}{\partial y}, \tag{2}$$

$$G_3(x, y, t, h) = \frac{1}{S(x, y)} \left[T_x(x, y) \frac{\partial^2 h}{\partial x^2} + T_y(x, y) \frac{\partial^2 h}{\partial y^2} + Q(x, y, t) \right]. \tag{3}$$

The characteristic curve is then parameterized by:

$$\Gamma(\lambda) = \big(x(\lambda), y(\lambda), t(\lambda), h(\lambda)\big)$$

and determined by solving the following characteristic system:

$$\frac{dt}{d\lambda} = 1, \tag{4}$$

$$\frac{dx}{d\lambda} = G_1(x, y, t, h), \tag{5}$$

$$\frac{dy}{d\lambda} = G_2(x, y, t, h), \tag{6}$$

$$\frac{dh}{d\lambda} = G_3(x, y, t, h). \tag{7}$$

From the derivation, it follows that $t = \lambda$, aligning the characteristic system with the representation used in the manuscript. This approach ensures that each variable clearly corresponds to its role in the characteristic framework, separating them distinctly from their usage in the general field equations.