

Dear Editor,

We are very grateful for the referee's valuable comments. In this revised version of the manuscript, we mainly modified the discussion regarding the various types of errors that arise when applying our method in Section 4 and Appendix. We address the referee's suggestions point-by-point below as well as improve the manuscript's presentation throughout.

In this reply, the comments of the referee are marked in black and the replies in blue. We hope that our revised manuscript is now more accessible to the general readers. Thank you so much for your editorial work.

Kind regards,

Chao Shen and Gang Zeng, on behalf of the co-authors

### Replies to the first reviewer

The authors have responded to the reviewer comments. It is appreciated that also the language has been revised thoroughly.

There are still a few minor points that are unclear, possibly incorrect.

Reply:

We thank the referee for these valuable comments. In this version, we have improved the manuscript throughout, particularly in the evaluation of discretisation error, iteration error, and measurement error as discussed in Section 4 and detailed in Appendix. Please see our replies to your major comments point-by-point below.

Line 20: ropes -> rope

Reply:

The correction has been made accordingly. Thanks.

Line 408: during measurement period -> during the measurement period

Reply:

The correction has been made accordingly. Thanks.

Line 411: accessed -> assessed

Reply:

The correction has been made accordingly. Thanks.

Line 411: spatial resolution can indeed be due to S/C motion during the measurement, but it also can be the S/C motion in between two successive measurements – that

depends on how the instrument works. It would be good to mention that possibility.

Reply:

Another definition of the spatial resolution has been added accordingly in Lines 397 – 398. Thank you.

Line 419: solution -> solutions

Reply:

The correction has been made accordingly. Thanks.

Line 426: “More advanced evaluation ...” : One can study iteration error in the best possible way using artificial examples. I do not see how satellite data would improve your evaluation of this property of the algorithm. I propose to simply drop that phrase.

Reply:

Thanks for the referee’s comments. The sentence has been dropped accordingly. Thanks.

Line 431: “the relative errors of the gradients associated to the measurement errors are expected to be the same magnitude as the measurement errors”: It is a generally known property of numerical differentiation that the relative errors on the gradients are often much larger than those of the measurements, especially as you are typically differencing values that are very similar in magnitude.

Reply:

This sentence has been dropped accordingly. Thank you.

Additionally, to better evaluate the measurement errors as suggested by your comment below, we impose random Gaussian error to evaluate our algorithm instead of 0.1% decrease for each measurement in Appendix D. Please see our detailed response below.

Line 484-486: “We also note that ... not perfectly synchronized”: I appreciate that you mention these two aspects; it would be even better if you could also say why these are important. For instance, if there are systematic measurement errors, that may lead to an error that is not compensated for by this algorithm.

Reply:

Thanks for the referee’s comments. Nowadays, the sampling time resolution of the detectors are already very high, and the temporal variations of the magnetic field or magnetic spatial gradients can be obtained from the time-series data. So, this algorithm only focuses on the magnetic spatial gradients, and the magnetic measurements from different spacecraft should be simultaneous. Furthermore, a homogeneous set of instruments onboard the constellation may not be achieved so that systematic error may arise. The total systematic error can be analysed by the well-established error theory. These clarifications have now been incorporated to the manuscript in Section 5 where we provide conclusions and perspectives.

Line 490: drop “expected to be”: Can be dropped: you have computed them ...

Reply:

The “expected to be” has been dropped accordingly. Thank you.

Line 495: I think the word “verified” better reflects the results of your work than “demonstrated”; alternatively, you could say “validated”

Reply:

The word “demonstrated” has been replaced by the word “verified” accordingly. Thanks.

Caption figure A1: Make clear that these are relative total errors, just to indicate that this is not the iteration error alone.

Reply:

Thanks for the referee’s comments. The phrase “relative errors” has been replaced by “total relative errors” throughout Appendix.

Appendix A: “It can be suggested that the error generated during the iteration process is relatively small.” Can you replace this by “The iteration error clearly goes to zero asymptotically.” ? If not, I think there is a problem.

Reply:

Thanks for the referee’s comments. The sentence has been replaced accordingly.

Figure A2: I would expect to see here plots of the asymptotic values as a function of separation; the variation with iteration number is not relevant here. It is nice to see how the truncation error changes as you reduce the separation. If you decide to stick to just comparing the two separation scales, it would be better to present that comparison in a small table; there is no need for a figure then.

Reply:

Thanks for the referee’s comments. The variation of truncation error with the distance between satellites has been investigated in appendix A (see Figure A1).

Idem for Figure B1.

Reply:

Thanks for the referee’s comments. The variation of truncation error with the discretization step has been investigated in appendix C.

Appendix C: You state that “each measurement is decreased by 0.1%” That is not a correct way to evaluate the effects of random measurement errors. You should impose gaussian errors with a specified distribution width on the measurements.

Reply:

Thanks for the referee's comments. The random Gaussian error has been imposed to evaluate this algorithm in Appendix D.

Figure C1 and C2: The variation with iteration number is not relevant. You could present the results in the table or make a figure with the error as a function of the magnitude of the measurement error.

Reply:

Thanks for the referee's comments. The random gaussian errors with mean 0 and standard deviation 0.01 are imposed. The following Figure shows the relative total errors of the linear and quadratic gradients at various barycentres for the dipole field case. The measurement error cause large errors for the quadratic magnetic gradients. In Appendix D, it is indicated that the observation data should be filtered to remove noise.

