Replies to the first Reviewer

We are very grateful for the referee's valuable comments. We address your suggestions point-by-point below and improve our manuscript presentation throughout. Major changes include the evaluation of discretisation error, iteration error, and measurement error.

In this reply, the comments of the referee are marked in black and the replies in blue. We hope that our revised manuscript is now more accessible to the general readers.

The authors have made a serious effort in responding to the reviewer comments and this has improved the manuscript considerably. There are still a number of language problems (see below). The manuscript might benefit from additional language editing.

Reply:

We thank the referee for these valuable comments. In this version, we have improved the structure of the manuscript throughout, particularly in the evaluation of discretisation error, iteration error, and measurement error. Please see our replies to your major comments point-by-point below.

Comments

In section 4 there is a discussion of the errors, which is very much appreciated. The authors talk about truncation error, discretisation error, iteration error, measurement error and random error. Perhaps a brief recollection of the definitions of each of these would be in place here.

Reply:

The definitions of these errors were in fact mentioned in paragraphs 2 – 4 in section 4. To make them clear, we have now spelled out the names of the errors – the discretisation error, iteration error and measurement error or random error – in the first paragraph of section 4. We additionally added in the first paragraph that we briefly introduce them (in the following paragraphs) before analysing them analytically in Appendix A-C.

I do not understand why an assessment of the iteration error would have to await real mission data. I tend to disagree: This is precisely the advantage of applying the technique to a model for which you know the exact solution: It allows to evaluate all the error contributions. By applying the method using synthetic data with zero measurement error and zero random error and a very fine discretization, the limit of the iteration should differ from the exact solution only by the truncation error. For a very fine discretization error, however, this truncation error should be small. One can easily compare any iteration step to the limit and evaluate how the iteration error decreases. It would similarly be not difficult to examine the combined effect of discretization and truncation: use zero measurement and random error and compare to the solution obtained with the exact solution. And one could similarly evaluate the effects of measurement errors and random errors. I admit that it may be difficult to disentangle the truncation and discretization errors, except if one constructs a model problem that has only linear and quadratic variations, so that the truncation error is known to be zero. In short: I find it a pity that not all the error types are studied (numerically) in this manuscript, as the examples are so inviting and offer the perfect occasion to do so.

Reply:

We agree with the referee on these valuable comments. To address your comments, we have performed detailed analyses and added additional discussion as the following, using the dipole field case as an example.

In Appendix A, we have now evaluated the iteration error. To demonstrate this point, we increase the number of iterations to 1000. We find that the relative errors decrease with the number of iterations as shown in Figure A1. The relative errors indeed converge to minimal values (less than 0.01% for the linear gradients and less than 2% for the quadratic gradients) after 100th iteration. To exclude the effect of the truncation error, we hold the configuration of the 7-S/C constellation while scaling down the distances between satellites by a factor of 100. Due to this reduction, the high-order truncation error converges to zero, leaving only the iteration error. Figure A2 shows the relative errors in the absence of the truncation error. We find that the relative errors of the linear gradients decreased to 0 and those of the majority quadratic gradients decreased to less than 0.1%. Therefore, we conclude that the error generated during the iteration process is relatively small given that the number of iterations is above 100.

In Appendix B, we have now evaluated the discretisation error. To introduce the discretisation error, we assume that the magnetic field value at the measurement point is the average along the satellite's trajectory for a duration of 0.25 seconds before and after the point, in the direction of the satellite's motion. We scaled down the distances between satellites by a factor of 100 so that the high-order truncation error converges to zero. Since there is no measurement error, only discretisation error remains. Figure B1 shows the variation in relative errors of the linear and quadratic gradients with respect to the iteration numbers, with a discretisation error introduced. The relative errors converge after 100th iteration. The relative errors of linear gradients are less than 0.012%, while those of majority quadratic gradients are less than 0.1%. Therefore, we conclude that the discretisation error is relatively small.

In Appendix C, we have now evaluated the measurement error by introducing 0.1% and 1% measurement error (by a reduction of the measurement magnitude). Other sources of errors are minimized as mentioned above. Figures C1 and C2 shows the variation in relative errors of the linear and quadratic gradients with respect to the iteration numbers for the dipole field case, for 0.1% and 1% measurement errors, respectively. Again, we find that the relative errors converge to minimal values after 100th iteration. Indeed, we find that the measurement error is of the same order as the accuracy of the instrument.

While the div B = 0 test is a useful overall error measure, it would be interesting if the authors could point out what it tells us about each of the types of errors, e.g, how do these errors vary with the discretization step, with the number of iterations, with the level of measurement and random errors?

Reply:

Thanks for the referee's comments. Please see our response above in complementary to Section 4 and Appendix A-C in the revised manuscript.

Minor comments

- line 81: environments -> environment

Reply:

The correction has been made accordingly. Thanks.

- line 160: equation -> equations

Reply:

The correction has been made accordingly. Thanks.

- line 165: SPractical -> Practical

Reply:

The correction has been made accordingly. Thanks.

- line 178: better: "… from central differences of the magnetic observation time series"

Reply:

The correction has been made accordingly. Thanks.

- line 201: as the following -> as follows:

Reply:

The correction has been made accordingly. Thanks.

- line 202: in order the solution exists -> in order for the solution to exist

Reply:

The correction has been made accordingly. Thanks.

- line 203: constellation -> constellation

Reply:

The spelling of this word has been modified accordingly. Thanks.

- line 270: The characteristic size of the S/C -> The characteristic size of the S/C constellation

Reply:

The correction has been made accordingly. Thanks.

- line 344: which -> whose – or, even better, just drop “which geometry is demonstrated”

Reply:

The sentence has been dropped accordingly. Thanks.

- line 381: drop “with portion of the number of linear and quadratic gradients” as this is made explicit in the next sentence

Reply:

The sentence has been dropped accordingly. Thanks.

- line 386: so symmetrical model magnetic field -> a symmetric model magnetic field

Reply:

The correction has been made accordingly. Thanks.

- line 387: drop “accurate”

Reply:

The word has been dropped accordingly. Thanks.

- line 387: I am not sure I understand the logic and the meaning of the following two sentences. If I understand it correctly, a better formulation would be “The zero components of the magnetic gradients are calculated with the algorithm and checked. Further evaluation of the algorithm with a less symmetric magnetosphere model could be useful.” Please check.

Reply:

The referee understands it correctly. The correction has been made accordingly. Thanks.

- line 392: drop “as the following”

Reply:

The sentence has been dropped accordingly. Thanks.

- line 396: the abbreviation to be used for “seconds” is “s” rather than “sec”

Reply:

The abbreviation has been modified accordingly. Thanks.

- line 398: the discretization errors brought -> the corresponding discretization errors

Reply:

The correction has been made accordingly. Thanks.

- line 453: “fewer”: compared to what?

Reply:

This sentence has been rewritten for clarity. Thanks.

- line 457: “to be able to obtained from” -> “to be obtained by” or “to be available from”

Reply:

The correction has been made accordingly. Thanks.

Replies to the Second Reviewer

We are very grateful for the referee's valuable comments. We address your suggestions point-by-point below and improve our manuscript presentation throughout. Major changes include the evaluation of discretisation error, iteration error, and measurement error.

In this reply, the comments of the referee are marked in black and the replies in blue. We hope that our revised manuscript is now more accessible to the general readers.

Comments on the revised manuscript entitled

Quadratic Magnetic Gradients from 7- and 9-Spacecraft Constellations

submitted by Chao Shen, Gang Zeng, Rungployphan Kieokaew, and Yufei Zhou

General comments

Many concerns have been addressed in the revision. Several specific concerns remain.

Reply:

In this version, we have improved the structure of the manuscript throughout, particularly in the evaluation of discretisation error, iteration error, and measurement error. Please see our replies to your major comments point-by-point below.

Specific comments

Abstract and Key Points

- In the review of the original manuscript, key statements were considered too strong. During the revision, only grammatical errors were corrected, so that the statements now read

"Tests for the situations of magnetic flux ropes and dipole magnetic field have verified the validity and accuracy of this approach."

and

"Magnetic flux ropes and dipole magnetic field testing verifies the validity and accuracy of the approach."

Since the essence has not changed, the statements are still too strong. A complete assessment of the accuracy would require studying all error types and the stability of the model inversion procedure, so the term "verified" in combination with "accuracy" is not applicable in this context. The paper is a proof of concept, i.e., a demonstration of the validity of the approach, and as such the statements may be rephrased as follows:

"The validity of the approach was demonstrated using magnetic flux ropes and dipole magnetic field models."

and

"Magnetic flux ropes and dipole magnetic field testing demonstrated the validity of the approach."

Reply:

The strong statements in the Abstract and Key Points have been rephrased as the referee's advice accordingly. Thanks.

Comparison of new method with analytical modelling:

- Lines 301-305: In the review of the original manuscript (lines 254-258), the reviewer commented "Only total errors after a given number of iterations are discussed. It would be more interesting to get separate assessments of iteration errors and discretisation errors." but this concern was not properly addressed by the authors. Even if "It is not very easy to separate iteration errors and discretisation errors." as stated by the authors in their reply, the limitations of their demonstrations should at least be critically discussed.

Reply:

We agree with the referee on these valuable comments. To address your comments, we have performed detailed analyses and added additional discussion as the following, using the dipole field case as an example.

In Appendix A, we have now evaluated the iteration error. To demonstrate this point, we increase the number of iterations to 1000. We find that the relative errors decrease with the number of iterations as shown in Figure A1. The relative errors indeed converge to minimal values (less than 0.01% for the linear gradients and less than 2% for the quadratic gradients) after 100th iteration. To exclude the effect of the truncation error, we hold the configuration of the 7-S/C constellation while scaling down the distances between satellites by a factor of 100. Due to this reduction, the high-order truncation error converges to zero, leaving only the iteration error. Figure A2 shows the relative errors in the absence of the truncation error. We find that the relative errors of the linear gradients decreased to 0 and those of the majority quadratic gradients decreased to less than 0.1%. Therefore, we conclude that the error generated during the iteration process is relatively small given that the number of iterations is above 100.

In Appendix B, we have now evaluated the discretisation error. To introduce the discretisation error, we assume that the magnetic field value at the measurement point is the average along the satellite's trajectory for a duration of 0.25 seconds before and after the point, in the direction of the satellite's motion. We scaled down the distances between satellites by a factor of 100 so that the high-order truncation error converges to zero. Since there is no measurement error, only discretisation error remains. Figure B1 shows the variation in relative errors of the linear and quadratic gradients with respect to the iteration numbers, with a discretisation error introduced. The relative errors converge after 100th iteration. The relative errors of linear gradients are less than 0.012%, while those of majority quadratic gradients are less than 0.1%. Therefore, we conclude that the discretisation error is relatively small.

In Appendix C, we have now evaluated the measurement error by introducing 0.1% and 1% measurement error (by a reduction of the measurement magnitude). Other sources of errors are minimized as mentioned above. Figures C1 and C2 shows the variation in relative errors of the linear and quadratic gradients with respect to the iteration numbers for the dipole field case, for 0.1% and 1% measurement errors, respectively. Again, we find that the relative errors converge to minimal values after 100th iteration. Indeed, we find that the measurement error is of the same order as the accuracy of the instrument.

- Lines 327/328: In response to the reviewer's comment to the original manuscript (lines 277/278), the author rephrased the statements "The relative error approaches 50%; however, the absolute error is low." to "The relative error approaches 50%; however, the absolute error is just 0.143, which is approaching zero." which creates a new problem because the unit is missing. If measured in "nT.RE^2", the value is 0.143. If measured, e.g., in the SI unit "T.m^2", the absolute error would be about 5800. The quantification of an absolute error in terms of "small" or "large" is meaningful only if compared to a reference value.

Reply:

The unit should be "nT·R_E^2", and it has been added.

- Lines 376/377: In the review of the original manuscript (line 321), the reviewer was concerned about the statement "The relative error approaches 50%; however, the absolute error is low." which was not changed during the revision. The authors are asked to adjust the statement along the lines of the previous comment.

Reply:

The statement has been modified, and the unit has been added.