



1 Consistency-Checking 3D Geological Models

2 Marion N. Parquer¹, Eric A. de Kemp¹, Boyan Brodaric¹ and Michael, J. Hillier¹

3
4 Correspondance: Eric A. de Kemp
5 eric.dekemp@canada.ca
6 orcid.org/0000-0003-0347-5792

7
8 ¹Geological Survey of Canada,
9 Three-dimensional Earth Imaging and Modelling Lab
10 601 Booth Street, Ottawa, Canada, K0E 1E9

12 Abstract

13
14 3D geological modelling algorithms can generate multiple models that fit various mathematical and geometrical
15 constraints. The results, however, are often meaningless to geological experts if the models do not respect accepted
16 geological principles. This is problematic given the expected use of the models for various downstream purposes,
17 such as hazard risk assessment, flow characterization, reservoir estimation, ~~natural~~ storage, or mineral and energy
18 exploration. Verification of the geological reasonableness of such models is therefore important: if implausible
19 models can be identified and eliminated, it will save countless hours, ^{of} computational and human resources, ~~as well as~~
20 ~~minimize user problems.~~

21 *rather vague?*

Consistency with what?

22 To begin assessing geological reasonableness, we develop a framework for consistency-checking and test it with a
23 proof-of-concept tool. This framework consists of a space of consistent and inconsistent geological situations that
24 can be held between a pair of geological objects, and the tool assesses a model against the space to identify
25 (in)consistent situations. Both the framework and tool are successfully applied to several case studies as a promising
26 first step toward automated assessment of geological reasonableness.

27 ** space, or set of rules? If space, what are the dimensions?*

28 **Keywords** – geological knowledge, geological consistency, 3D geological modelling, temporal relation, spatial
29 relation, polarity

31 1 Introduction

32 Geomodelling techniques are often deployed to bridge the spatial gaps between explored areas, including gaps in
33 stratigraphic structure, property distribution, and target extent. Increased data availability, and rising societal need for
34 natural resources, have recently stimulated development of ^{Computational} ~~advanced~~ geomodelling modelling techniques such as
35 stochastic simulation (Lajevardi and Deutsch, 2015), time-varying modelling (Hinojosa, 1993), Bayesian techniques
36 (de la Varga and Wellmann, 2016), and direct perturbation of models or data (Lindsay et al., 2012). Wrapped into
37 ^{increasingly} ~~growing~~ complex workflows (de Kemp et al., 2016), these new techniques can operate with scarce and
38 heterogeneous data, are frequently deployed to model less accessible and more complex terrains, and often produce
39 a wide range of ~~equiprobable~~ models and associated uncertainties (Wellmann and Caumon, 2018).

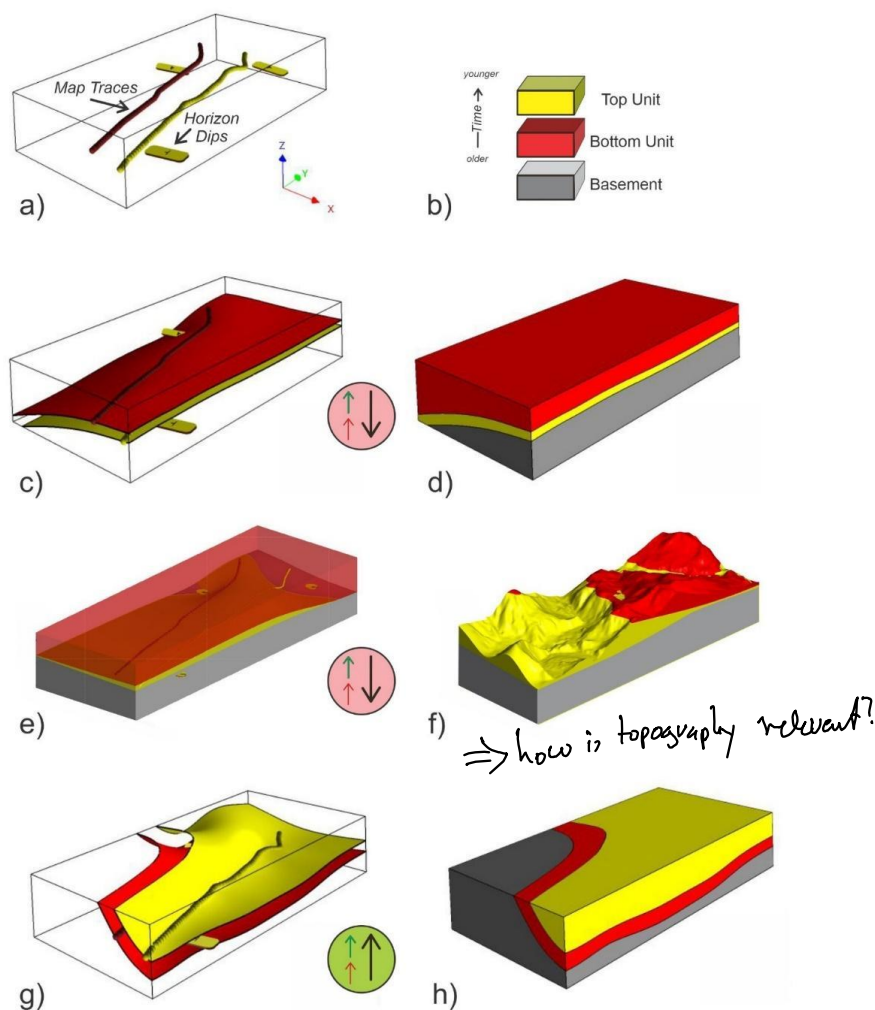


1
2 However, several problems can arise from these ~~advanced~~ techniques. For example, accuracy issues associated with
3 scarce data can become magnified and lead to geologically questionable spatial interpolations, such as older
4 geological units deposited on younger units (Figure 1). These issues might be further compounded by biases at each
5 modelling step or decreases in the reliability of the data as the number of participants increases (Bond, 2015). Data
6 may also become irrelevant due to scale discrepancies (Snyder et al., 2014), and degraded due to re-sampling to
7 meet coarser scale requirements or to suit algorithms that imprecisely fit data (Hillier et al., 2021). As data scarcity
8 and loss necessarily impact the accuracy and credibility of any model, multiple equiprobable realizations are often
9 generated in the hope that some model, or the mean of models, comes closer to representing reality and minimizing
10 uncertainty. Equiprobable models, though, are not necessarily geologically possible (Deutsch, 2018). Indeed, some
11 of the more data-driven 3D modelling methods can generate results that respect the data, but do not necessarily
12 respect established geological principles (Lyell, (1833) 2022). Conversely, purely knowledge-driven 3D modelling
13 methods might respect geological principles, ^{Lyell didn't like that loss!} or norms, but ~~might~~ ^{often do} not fit the underlying data (Bai et al., 2017). Thus,
14 amongst a multitude of equiprobable models, it is unavoidable that a non-negligible number of them might produce
15 geologically unreasonable results. This is especially a challenge for hypothesis testing in natural systems, or
16 developing AI training sets, which might involve billions of such models.
17 The number of models is not an issue? (Their quality is).
18 The number of possible models therefore must be reduced, and this reduction can be accomplished in two ways: (1)
19 by building fewer and better models, or (2) by excluding inappropriate models. The first solution involves having
20 more and better data and knowledge, or improved algorithms. Increasing the amount of data in study areas, possibly
21 from diverse sources such as geophysical or structural data (Giraud et al., 2020, 2024; Wellman and Caumon, 2019;
22 Hillier et al., 2014; Grose et al., 2019; de la Varga et al., 2019), or improving data quality, increases overall accuracy
23 and reduces the ^{differences between model realizations} number of possible models. Such results ^{also} might be achieved from increased constraints,
24 including refinements of knowledge such as input stratigraphy or augmentation of algorithms with implicit and rule-
25 based modelling (Bertoncello et al., 2013; Bai et al., 2017). Problematically, however, these solutions likely require
26 the acquisition of new data or knowledge, which is often impossible. It also might require the development of more
27 robust algorithms to improve model quality (Jessell et al., 2010; Cherpeau et al., 2010; Ranalli, 1980), such as
28 physics-based modelling approaches (Shokouhi et al, 2021; Hobbs et al. 2021), ^{however these} which are not yet mature.

Interpolation
quirks
(e.g. boldface)
Should also
be mentioned
here

Why equiprobable?

hypothesis testing?
Expand/explain or
remove.



1

2 **Figure 1.** Example of *unreasonable* 3D geological models (c) to (f). Sparse input data (a) includes two separate
 3 depositional horizon traces and 3 (gently dipping) bedding constraints (yellow tablets) indicating depositional
 4 tops upward. The event history (b) has an older unit (red) deposited below the younger unit (yellow).
 5 However, use of the Lajaunie (1997) implicit method in SURFE (Hillier et al. 2014, 2021) results in the older
 6 unit deposited over the younger unit (c)-(d), which is unreasonable in the absence of other events. Similarly,
 7 using commercial software from LeapFrog Geo (Seequent), in (e) without topography and in (f) with
 8 topography, also indicates an unreasonable geological sequence with the older unit on top. In contrast, (g) and
 9 (h) show reasonable models generated with SURFE software tuned to respect a minimal horizon thickness and
 10 depositional history. Circled arrows show a deposition polarity vector for the older unit (red arrow), younger
 11 unit (green arrow), and the temporal direction (black arrow, from older to younger unit), as well as the
 12 geological plausibility of the situation: a green background indicates consistency with geological principles
 13 (aligned vectors), and the red background denotes an inconsistent scenario (unaligned vectors).
 14



1 The second solution, which involves model exclusion, can be accomplished ^{by} ~~manually or automatically~~: (i) manually ~~by~~
2 ~~by~~ having a geologist inspect and reject models using accumulated expertise; or (ii) automatically ~~by~~ performing a
3 ~~rapid~~ computer-driven check to eliminate poor instances, during or after model construction. A significant
4 disadvantage of the manual approach is lack of reproducibility: as implicit knowledge can vary between and within
5 geologists (Brodaric *et al.*, 2004; Brodaric 2012, Bond 2015), it is unlikely manual corrections would be
6 reproducible for more than a few models, and the selection of a certain model would likely be unexplainable. In
7 contrast, if knowledge is made explicit (Brodaric and Gahegan, 2006), automatic approaches could be reproducible
8 and explainable, as per the consistency-checking approach in this paper. A critical aspect of this approach therefore
9 is the explicit digital encoding of ~~knowledge, as well as its~~ ^{geological rules and their} integration into geo-modelling workflows. Although
10 integration techniques like rule-based geomodelling (Pyrzcz *et al.*, 2015) and implicit modelling (Jessell *et al.*, 2014)
11 are quite common, they typically incorporate a limited range of ^{geological rules} ~~knowledge~~. Extending this range also is not new,
12 e.g. early work focuses on capturing knowledge from a geological map, cross-section, or other field record (Harrap,
13 2001; Burns, 1975; Burns and Remfry, 1976; Burns *et al.*, 1978; 1969), but only recently have we begun such
14 extension for 3D geo-modelling (e.g., Jessell *et al.*, 2021; Rauch, *et al.*, 2019). In addition to limitations in
15 knowledge range, there exist accompanying limitations in its use, as the knowledge is utilized primarily *a priori* for
16 model-building rather than *a posteriori* for model evaluation. Key goals for a consistency checker then include an
17 expansion of the range of knowledge plus its effective application for consistency evaluation.
18
19 A first step to such expansion and evaluation might be the utilization of all information from a geologist's
20 observation sheet. However, it is very unusual to incorporate all such knowledge in a 3D model: much of it remains
21 reported on a map, e.g. as colours, abbreviations or symbols, and the rest in the map legend, in related articles and
22 reports, or in the mind of the geologist. In particular, the geological legend as we know can be incomplete (Harrap,
23 2001) and does not always contain the entire stratigraphic and structural history, prompting the development of a
24 'legend language' as a first attempt to formalize geological map knowledge for the purpose of checking the
25 consistency of traditional 2D geological maps (Harrap, 2001).
26
27 More recently, the topological aspects of geological maps and models have received significant attention as a
28 pertinent knowledge representation (Thiele *et al.*, 2016a, b; Le *et al.*, 2013). This includes the spatial relations
29 between discrete elements of a 3D model, ~~particularly~~ those unchanging under continuous deformation (Crossley,

more important
on the time of
visualisation
challenges

Specifically



important why? please expand.

1 2005), such as adjacency, inclusion or intersection. An important aspect is the dimensionality of the spatial objects,
2 which might be 0D (a point), 1D (a line), 2D (a surface), or 3D (a volume). Spatial relations between such objects
3 have been widely examined, with distinct relationships identified between 2D regions (Egenhofer and Franzosa,
4 1991) as well as 0D, 1D, 2D, and 3D regions (Zlatanova et al., 2004). They also have been applied to certain types
5 of geological objects (Schetselaar and de Kemp, 2006), providing a basis for the spatial component of geological
6 knowledge, and underpin efforts in knowledge-driven 3D geological model construction (Zhan et al., 2022; 2019).
7 However, they are not yet applied to the evaluation of geological models, especially in combination with temporal
8 relations, despite being applied to the evaluation of models in other domains (e.g. Van Oosterom, 1997; Gong and
9 Mu, 2000; Arora et al., 2021; Nikoohemat et al., 2021; Bezhanishvili et al. 2022).

*Arguably
not true:
many software
=> Check if es
a generated
mesh is
"watertight".*

10
11 In this paper we develop a knowledge framework and proof-of-concept tool for automated consistency-checking of
12 3D geological models. The knowledge framework consists of a space of (in)consistent geological situations holding
13 between nine kinds of geological objects, with each situation being a unique combination of a spatial, temporal and
14 polarity relations. ~~The proof-of-concept tool then assesses the relations in a geo-model against the space to identify~~
15 (in)consistencies and is successfully applied to several case studies. ~~The framework is presented in Section 2, the~~
16 ~~tool is described in Section 3, several case studies are outlined in Section 4, some additional thoughts on the~~
17 ~~consistency checker and geological reasonableness are presented in Section 5, and the paper concludes with a brief~~
18 ~~recap in section 6.~~

*Delete for
brevity*

19 20 **2 Geological ~~Consistency-Checking~~ Framework**

21 Geological knowledge is the accumulation of results from thousands of years of human inquiry into our natural
22 environment, with modern formal geological knowledge emerging in the mid 1800's (Lyell, (1833) 2022; Rothery,
23 2016). Such knowledge is found in digitally archived articles and books (e.g. Kardel and Maquet, 2012) as well as
24 online products and courses (e.g. Fattah, 2018). It is particularly useful to help understand the often hidden and
25 unobserved subsurface of the Earth. However, the various possible sources of data (e.g. surface mapping, boreholes,
26 geophysical surveys) generally cannot provide sufficiently uniform and continuous information for a volume of
27 interest. Supplementary geological knowledge is required for improved interpretation between sometimes extremely
28 sparse observations (Groshong, 2006; Frodeman, 1995), especially when coupled with new data integration
29 techniques and approaches (Giraud et al. 2020; Wellmann and Caumon, 2018).

*is this
relevant?*



1

2 For consistency-checking purposes herein, we distinguish between data and knowledge, with data being
3 observational and geological knowledge being either local or universal. Data then includes any form of observation
4 used to understand a specific geological area, e.g. bedding top indicators, structural orientations, fault and horizon
5 contacts or seismic picks, or geophysical readings. Local knowledge applies to a specific area but is not
6 observational: it is interpretational and includes things such as the local stratigraphy and process history. In contrast,
7 universal geological knowledge, or ‘norms’, apply to ^{all} ~~different~~ geographical areas, and includes things such as
8 general laws, principles, process types, and classification systems, e.g., Walther’s Law, ~~uniformitarianism~~, the
9 notion of deposition, rock type classification. Significantly, data and knowledge are interconnected insofar as
10 knowledge is inferred from data, and the data is contextualized by knowledge during observation and interpretation
11 (Brodaric et al, 2004). Indeed, both data and knowledge are required to arrive at any interpretation, including a 3D
12 geo-model. Consistency then can be seen as the degree of agreement between a model, ^{its underlying} and the relevant data and
13 knowledge. However, current modelling techniques are primarily focused on ensuring and assessing data
14 consistency, with knowledge consistency less developed. ^{For example,} ~~one~~ implicit modeling techniques typically optimize fit to
15 data, and assume stratigraphic consistency, but such consistency might not be achieved by all techniques (see Figure
16 1), and further might not be reflected in all geometric realizations due to idiosyncrasies of ~~spatialization~~ ^{interpolation?} algorithms
17 (Hillier et al., 2021).

Is topology
(e.g. A crosses B)
knowledge
or data
under this
definition?

18

19 Therefore, some geological models can still fail to respect basic geological norms. To determine knowledge
20 consistency for a 3D geo-model, we expect local knowledge to be typically derived from a 2D map legend, cross
21 section, or associated report, whereas universal knowledge is preexistent, in our case as core types of geological
22 objects, foundational spatial, temporal and polarity relations. Truth ~~T~~ables then denote all possible combinations of
23 these relations for pairs of object types, with each combination being a situation identified as consistent or
24 inconsistent. Knowledge consistency is finally assessed by traversing the spatial relations between pairs of objects in
25 a geo-model, using the local knowledge to determine object types, temporal relations, and polarities of the objects,
26 which together form an index into the Truth ~~T~~ables to determine the (in)consistency of a specific situation.

27

28

29



1 2.1 Geological Objects and Polarity

2 The objects in a 3D geo-model (geo-objects) are, for the purposes of this paper, representations of instances of nine
3 distinct ~~geological object~~ types: depositional unit, intrusion unit, extrusion unit, metamorphic unit, fault, erosion
4 surface, fold volume, and linear and planar fabric. Note that horizons, understood as the top or bottom surfaces of a
5 volume, are excluded from the list of geological objects primarily because, in effect, they imply a volume and are
6 thus already included. This does not exclude the top or bottom surfaces of volumetric entities from being
7 represented in 3D geo-models, but they are not distinct geological object types in this paper and are converted to 3D
8 volumes for consistency-checking in our proof-of-concept tool. Additionally, we utilize two types of polarity
9 associated with geological objects. The first type of polarity is an internal vector pointing in the direction of creation
10 or destruction of an object's material: e.g. for depositional units, from the base or oldest part of the geological body
11 to the top, in the direction of material creation; and for erosional surfaces from the top to bottom of the eroded rock
12 body, in the direction of material destruction. Volumetric geo-objects are assigned a gross internal polarity valid for
13 the object as a whole, but some immaterial objects, of lower-dimensionality, lack polarity as they are not associated
14 with material growth or destruction, e.g. fault surfaces. Those with a gross internal polarity might also have many
15 local polarities distributed throughout the object, constituting in sum a polarity field for the object and forming a
16 basis for determining its gross polarity. The second type of gross polarity is an age direction vector shared by two
17 geo-objects, pointing from the older to the younger object and set parallel to one of the internal polarities of the
18 objects. Thus, collectively, there possibly exist three gross polarity vectors associated with a pair of geo-objects: the
19 growth direction vector within each object, and the age direction vector across the objects. The alignment of these
20 nicer way to define it! vectors then helps determine the geological plausibility of the situation (see Figure 1). The nine types of geo-objects,
21 and associated polarities, include:

- 22 • Depositional unit: a rock volume primarily generated by gravitational, water or air mediated, material
23 accumulation processes, and associated with a time interval. The gross internal polarity is unidirectional
24 and perpendicular to the upper depositional surface (Figure 2a): for a given sedimentary unit, it points up in
25 the opposite direction of gravity at the moment of deposition.
26 • Extrusion unit: a rock volume primarily generated by igneous extrusive processes and associated with a
27 time interval. The local internal polarities typically point radially upwards to a proximal vent or feeder
28 facies. This includes internal polarities associated with deposition of eruptive material, which is affected by
29 gravity and tends to flow downhill, but with airfall material accumulating upward. A gross internal polarity

what about
"alteration
Unit"
(e.g. the
Potassic zone
in a porphyry
system).

surely the polarity of a fault is its slip vector? Possibly worth mentioning even if this isn't used.
this is a topological relation, not a vector!



- 1 vector thus points upwards at the time of formation, similar to sedimentary units. However, extrusive units
2 with variable growth directions in subglacial situations are an exception, having chaotic eruptive
3 depositional internal polarity vectors that cannot be characterized by a single gross vector.
- 4 • Intrusion unit: a rock volume primarily generated by igneous subterranean processes and associated with a
5 time interval. Its internal polarities radiate from a core region towards the cooling host rock contact
6 surfaces (Figure 2c), with a gross internal polarity set to a representative direction. Many configurations for
7 the growth gradients in these bodies exist, but in general the emplacement contacts with host rocks are
8 similar to unconformities, in that they tend to be truncating earlier material through magmatic erosion,
9 assimilation or expansion (Annen, 2011).
 - 10 • Metamorphic unit: a rock volume primarily generated by deep thermal-kinetic processes and associated
11 with a time interval. The internal polarities are perpendicular to the metamorphic isograd and pointing to
12 the lower metamorphic grade or into the host protolith (Figure 2d). However, in many cases a gross internal
13 polarity vector can be set pointing upwards from a core heat source. This holds for a regional perspective,
14 in which we can envision the earth's regional geothermal gradient as pointing from hotter-deeper to cooler-
15 shallower lithospheric material. It also holds for a local perspective, in which the location of the source of
16 metamorphism and hence the local gradient may be easier to establish from metamorphic aureoles around
17 intrusions.
 - 18 • Fault surface: a spatial (immaterial) surface between displaced rock volumes that were once continuous,
19 and associated with ^{one or more instantaneous displacements} ~~a time instant or interval for the displacement activity~~ (Figure 2e). The surface lacks
20 internal polarity, as it is never constituted by any material. We are not, at this stage in our approach,
21 considering polarities from kinematic properties distributed on faults such as slip displacement directions.
 - 22 • Erosion surface: a spatial surface where a rock volume has completely eroded via a mechanical process. It
23 is associated with a time interval or instant indicating the end of the erosion process. Its gross internal
24 polarity points towards the eroded unit, in the direction of material destruction (Figure 2f); typically, this
25 direction will be opposite to that of an overlying sedimentary unit.
 - 26 • Fold volume: a rock volume affected by ductile tectonic deformation processes with an associated time
27 interval. The fold volume can be composed of depositional, intrusive, extrusive, or metamorphic material,
28 since these units can all become folded, as can an earlier fault or erosional surface. The gross internal
29 polarity points in the direction of structural vergence or tectonic transport direction (Figure 2g). For our

} what about hangingwall vs footwall concepts and their link to age & offset??



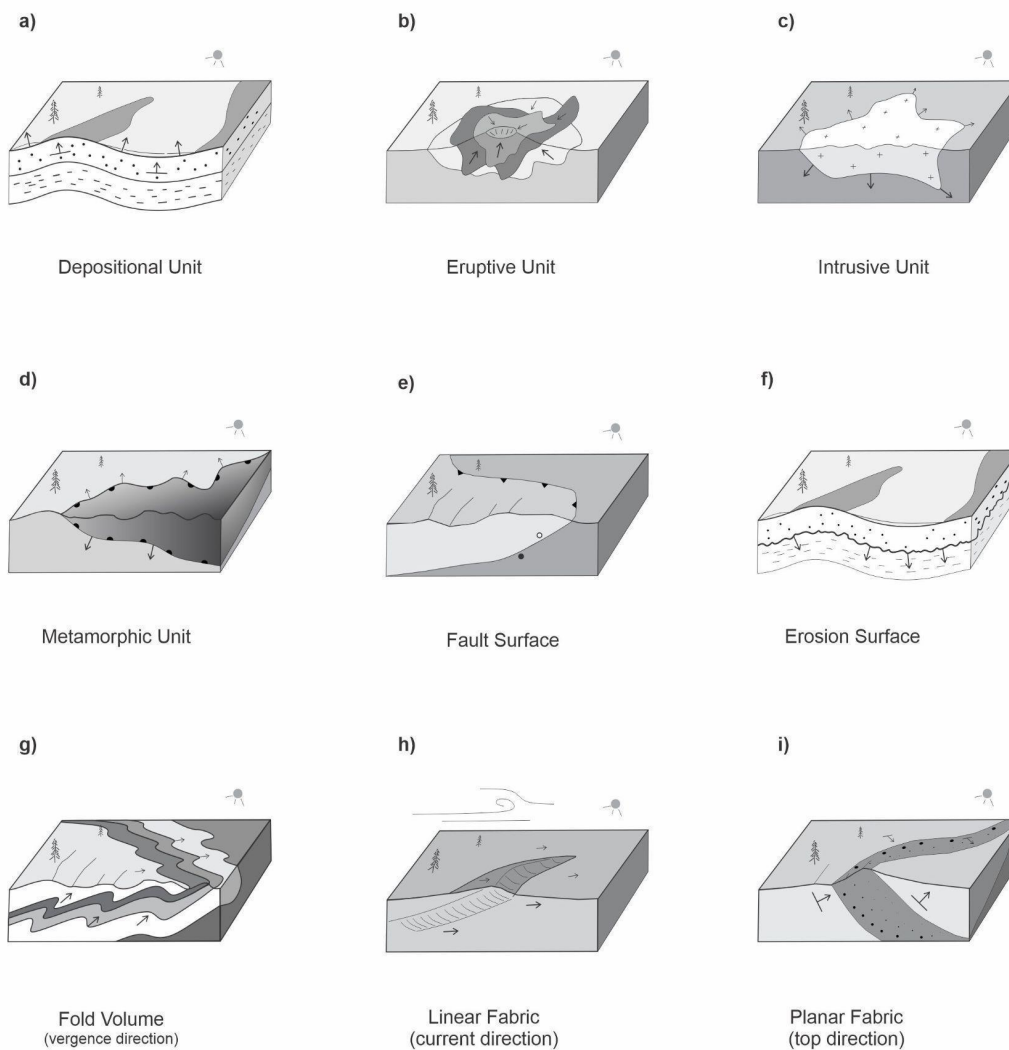
→ Consider relating to
"deformation volume" or
"strain volume" or "tectonic volume"??

1 purposes, nappes, duplex structures and orogenic scale tectonic units could be classified as fold volumes.
2 They may contain multiple folding events and have fold-fold relations. We do not treat these in this study,
3 but consider they could be dealt with using a similar consistency approach. For example, an older fold set
4 cannot re-fold a younger generation of folds.

- 5 • Linear fabric: A penetrative preferential linear orientation within a pre-existing rock material, with an
6 associated time interval. Some objects such as paleocurrents could have a unidirectional (Figure 2h) or
7 bidirectional gross internal polarity such as from tidal currents, but a fold hinge line has no preferred
8 direction (polarity).
- 9 • Planar fabric: A penetrative preferential planar orientation within a pre-existing rock material, with an
10 associated time interval. A primary planar deposition fabric has a normal with a positive upward polarity at
11 the time of formation (Figure 2i), (i.e., bedding top observations). A metamorphic planar fabric has a
12 normal with no polarity unless it can be derived from its host metamorphic unit. Igneous fabrics may have
13 derived polarity direction based on crystal accumulation, igneous flow layering or emplacement contact
14 directions. All fabrics (linear and planar) will have spatial, temporal topological relations and possible
15 polarity properties, that could be considered in determining feature-to-feature relation validity.

16

17 Field geologists typically infer these geo-objects and associated geological histories by interpreting common spatial
18 and temporal patterns across field sites. These patterns involve geological relations held between geological objects,
19 thus providing a simple topological framework for possibly complex geological situations. Major geological
20 relations, or situations, further can be decomposed into combinations of spatial, temporal, or polarity relations. For
21 example, if Sandstone-A is a depositional unit and Granite-B is an intrusion unit, and Sandstone-A is *intruded-by*
22 Granite-B, then we also expect a spatial relation such as Sandstone-A *spatially meets* Granite-B, a temporal relation
23 such as Sandstone-A is *temporally met by* ^{older than?} Granite-B, and the gross internal polarities are either *aligned* or *opposed*.
24 In this way, the *intruded-by* relation might be decomposed into a specific combination of a spatial, temporal, and
25 polarity relation. The task of a consistency checker then can be seen as verifying the validity of such a relation
26 combination/situation.



→ these do not seem to fit here?

1

2

3

4

5

6

Figure 2. Examples of geological objects with polarities, symbolized with black arrows. Note there is no polarity for fault features (e).



1

2 **2.2 Spatial Relations**

3 Prominent formalisms for binary spatial relations are derived from two main approaches (Galton 2009): Region
4 Connection Calculus (RCC) and the 9-intersection model (9I; Egenhofer, 1989; Egenhofer et al., 1993, Egenhofer
5 and Franzosa, 1991). In this paper we informally adapt the 9I approach, implemented for 0, 1, 2 or 3D objects and
6 512 possible spatial relations (Zlatanova et al., 2004). However, these 512 possibilities are drastically reduced for
7 typical geological situations in 2D and 3D (Schetselaar and de Kemp, 2006), resulting in 40 spatial relations for the
8 nine geological object types, as shown in Figure 2; then for any pair of spatial objects only one spatial relation can
9 hold. These relations can be represented as a three-part tuple, as shown in Tuple Equation 1. The tuple is also
10 directed or not, depending on the symmetry of the relation, given that asymmetric relations are directional and
11 symmetric relations are not directional; e.g. *meets* is symmetric, so if A *meets* B then B meets A, thus *meets* is not
12 directional; but if A *contains* B then it cannot be the case that B *contains* A (or A *is contained by* B), so *contains* is
13 asymmetric and directional. The symmetric spatial relations from Table 1 are *is disjoint with*, *meets*, *overlaps*,
14 *equals*, *intersects*, and the remaining relations are asymmetric. Symmetric relations also are their own converse,
15 whereas asymmetric relations have distinct converses, such as A *contains* B and B *is contained by* A.

16

17

$$\text{Entity}_A \left\{ \begin{array}{l} \textit{isdisjointwith} \\ \textit{meets} \\ \textit{overlaps} \\ \textit{contains} \\ \textit{iscontainedby} \\ \textit{covers} \\ \textit{iscoveredby} \\ \textit>equals} \\ \textit{intersects} \end{array} \right\} \text{Entity}_B \quad (1)$$

18

19



Does A contain B in both cases?
Surely these situations are topologically different.



Alteration or Metamorphism:



⇒ Cannot both be "contains".

[A & B occupy the same space]

Intuition: A and B cannot occupy the same space.

Give a geological twist?

"overlies"

"intrudes"

"Abuts"/"Touches"

"Overprints"

Consider referring to avoid confusion from the geological meaning of these terms

Spatial Relations	3D/3D	3D/2D	3D/1D	2D/2D	2D/1D	1D/1D
A is disjoint with B						
A meets B						
A overlaps B						
A contains B						
A is contained by B						
A covers B						
A is covered by B						
A equals B						
A intersects B						

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9

Table 1. The 9 spatial relations between two geological objects of 1/2/3 dimensions. Blank gray cells denote impossible spatial relations after Egenhofer (1989); Egenhofer et al. (1993); Egenhofer and Franzosa (1991); and Zlatanova et al. (2004).



1 **2.3 Temporal Relations**

2

3 Temporal relations are required to establish a temporal ordering between geological objects (Perrin et al., 2011).

4 Though the temporal position of a geological object is not always known (Michalak, 2005), the temporal ordering

5 between objects can be derived from the timeline of associated generative events (Galton, 2009; Claramunt and

6 Jiang, 2001). As with spatial relations, dimensionality plays a role: temporal relations can be categorized according

7 to the nature of the time duration (of the event) with 3 potential combinations: period/period, period/instant, or

8 instant/instant. Building on Allen's definitions (Allen, 1983), this leads to 14 distinct temporal relations, including

9 converses (e.g. *A precedes B* and *B is preceded by A*), as shown in Table 2, for the nine geological object types;

10 moreover, for any pair of objects only one temporal relation can hold. Of note is the *is incomparable to* relation,

11 which indicates the temporal ordering is unknown due to unavailable temporal knowledge about one or both objects.

12 Tuple Equation 2 illustrates the three-part tuple for expressing these relations. The symmetric relations are *equals*

13 and *is incomparable to*, with the remainder being asymmetric.

14

15

$$\text{Entity}_A \left\{ \begin{array}{l} \textit{precedes} \\ \textit{meets} \\ \textit{overlaps} \\ \textit{is finished by} \\ \textit{contains} \\ \textit{starts} \\ \textit{equals} \\ \textit{is incomparable to} \\ \textit{is started by} \\ \textit{is during} \\ \textit{finishes} \\ \textit{is overlapped by} \\ \textit{is met by} \\ \textit{is preceded by} \end{array} \right\} \text{Entity}_B \quad (2)$$

16

17

18

19



"follows" and "followed by" may be better, to avoid confusion with spatial terms?

temporal relations	period / period	period / instant	instant / instant
A precedes B B is preceded by A			
A meets B B is met by A			
A overlaps B B is overlapped by A			
A starts B B is started by A			
A finishes B B is finished by A			
A during B B contains A			
A equals B			
A is incomparable to B			

1

2 **Table 2.** The 14 temporal relations between two geological objects, after Allen (1983). The temporal timeline
3 advances from left to right in each cell. Blank gray cells denote impossible temporal relations, and blank white
4 cells denote unknown temporal relations.

5
6
7
8
9

10 **2.4 Polarity Relations**

11 A polarity relation can be determined from up to three independent component polarities (discussed earlier in
12 Section 2.1); the two internal polarities, dependent on the type of geo-object and processes governing its formation,
13 and the age polarity. The internal and age polarity vectors, can be compared to determine if they are 'aligned' or
14 'opposed', such that:

- 16 • Aligned polarity relation: the vectors are roughly parallel, such that each vector is within 90° of every
17 other vector.
- 18 • Opposed polarity relation: a vector is oriented in an opposite direction to the others, such that one
19 vector is at least greater than 90° from one of the others.

20 If an object does not have internal polarity, such as a fault, or the internal polarity is missing due to lack of
21 knowledge, then the polarity relation defaults to aligned. Importantly, polarity alignment or opposition does not

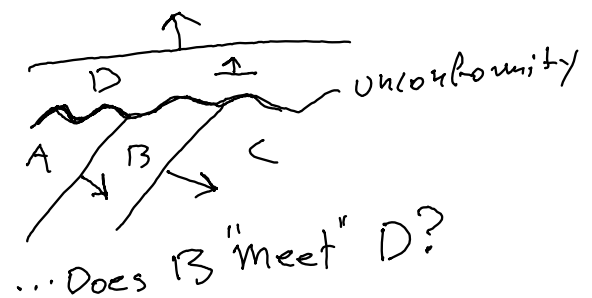
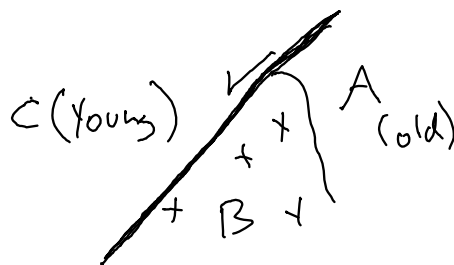
Clarity is essential for the best part of the work. This is a calculation of the...

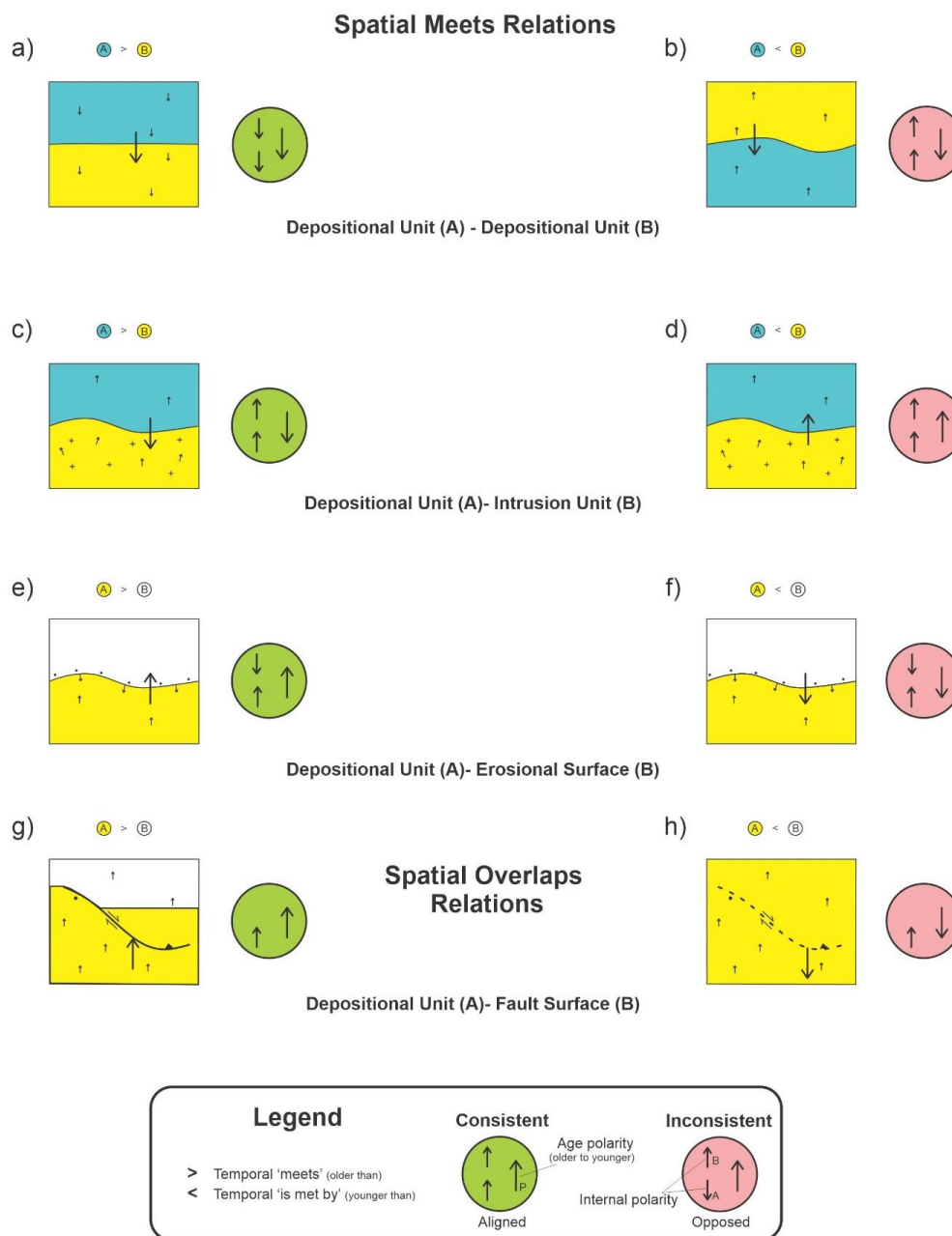


1 necessarily determine (in)consistency alone, as such determination requires consideration of the spatial and temporal
 2 relations. For example, opposed internal polarity can indicate either inconsistency or consistency: e.g. touching
 3 depositional units with opposed internal or temporal polarities are inconsistent (Figure 3b), because such units must
 4 create material in the same spatial and temporal direction; but a touching depositional unit and erosional surface
 5 with opposed internal polarity are consistent, because the surface must erode material towards the older unit (Figure
 6 3e)*. The internal polarity relation also might not play a determining role in assessing (in)consistency, as the spatial
 7 and temporal relations may individually or together be determining factors: e.g. the (in)consistency of an intrusion
 8 into a host depositional unit can be determined without regard to internal polarities (Figure 3c-d), as the intrusion
 9 must be younger and touching the unit, otherwise some interceding object such as a fault or erosional surface is
 10 missing from the model; similarly for a depositional unit and a fault surface (Figure 3g-h), as the unit must pre-exist
 11 the fault. Note ^{that} we do not consider growth faults to be strictly synchronous within a full unit, since at least some of
 12 the material needs to pre-exist ^{date} the faulting. On the other hand, there may not be any age polarity information
 13 available other than what can be derived from the internal process vector directions. In the case of intrusions this can
 14 be observed from cooling margins, inclusions along contacts and other textures indicating internal growth and
 15 assimilation direction. Also note cases needing all three polarity components might be unaligned if the age polarity
 16 is unaligned with either of the internal polarities (see Figure 1.c-f). A general framework for (in)consistency
 17 therefore must consider the spatial, temporal, and polarity relations. This is accomplished by using these relations as
 18 an index into 'Truth Tables' representing geological norms and specifying the (in)consistency of the situation (see
 19 Section 2.5).
 20

use appropriate term from Table 1 (meeting?)

** therefore units deposited immediately above a depositional unit do not touch/meet the units below it (e.s. overturned basement strata?)*





1

2 **Figure 3.** Examples of consistent (green circle) and inconsistent (red circle) polarity configurations for the spatial
 3 meets and overlaps relations; included are two vectors for internal polarity (small arrows) and a third age
 4 polarity vector pointing to the younger object (large arrow). A consistent scenario between two depositional
 5 units is depicted in (a), allowing overturning since overturning happened AFTER the binary relation has been
 6 established. An inconsistent scenario between such units is depicted in (b), as the age polarity is opposed to



1 both internal polarities, implying a third event between the unreasonable deposition of older on younger
2 material. In (c) the deposition - intrusion unit scenario is consistent, as the age vector is aligned with the
3 intrusive process, whereas in (d) it is not allowed because the host depositional unit is younger than the
4 intrusion unit. In both (c) and (d) the internal polarities do not impact consistency evaluation, as age is the
5 determining factor. In (e) and (f) the erosional surface needs a pre-existing material to erode, thus the direction
6 of material reduction of the eroding surface moves into the older underlying depositional material. For a
7 consistent scenario the age vector points from older unit to younger erosional surface (e), and for an
8 inconsistent scenario, the age vector points from the younger surface to older unit (f). In (g) the fault has no
9 internal polarity but needs a material object to displace. When the depositional unit pre-exists, as in (g), the
10 relation is valid; however, in (h) the depositional unit is younger and did not exist when the fault evolved, so
11 the relation is inconsistent.
12

13 2.5 Truth Tables

14 Many geological principles, known implicitly to geologists, must be considered in assessing, even grossly, the
15 consistency of the spatial, temporal, and polarity relations between two geological objects. Amongst the foremost
16 are stratigraphic considerations (Ziggelaar, 2009; Aubry et al., 1999):

- 17
- 18 ● principle of lateral continuity: in general, a given object has the same age over its full extent.
 - 19 ● principle of actualism: past objects are formed by processes (tectonism, magmatism, deposition ...)
20 acting in the same way as today.
 - 21 ● principle of paleontological identity: two objects with the same fossils are considered contemporary.
 - 22 ● principle of superposition: without structural disruption events, a given object is younger than the
23 object it covers and older than the one covering it. \Rightarrow distinguish them from "cover" in table
 - 24 ● principle of horizontality: sedimentary objects deposit in nearly horizontal orientation; in general, a
25 non-horizontal sedimentary sequence is generally deformed after its deposition.
 - 26 ● principle of cross-cutting: a given material layer is older than objects cross-cutting it.
 - 27 ● principle of inclusion: an object included into another object is older than the including object, except
28 when a younger object internally displaces the enclosing object (i.e. dyke, sill, migmatite melt phase).
- 29

30 For the nine types of geological objects considered herein, 45 valid pairwise combinations of objects are identified
31 that respect these principles: $45 = (2 + 9 - 1)! / 2! (9 - 1)!$. For each object combination, a ternary "Truth Table"
32 establishes all possible consistent and inconsistent spatial-temporal-polarity relations: spatial relations along one
33 side, temporal relations along another side, and internal polarities along a third side; each table cell then can be
34 marked as consistent or inconsistent for the pair of objects. Although the 45 tables exist in draft form, for the



- 1 purposes of this paper we concentrate on the 7 tables relevant to the case studies (see the supplementary data
- 2 provided at de Kemp, 2024).

		TEMPORAL RELATIONS								POLARITY	
		A precedes B	A meets B	A overlaps B	A is finished by B	A contains B	A starts B	A equals B	A is incomparable to B		
SPATIAL RELATIONS	A is disjoint with B									↑↑ aligned ↓↓	↓↑ opposed ↑↓
	A meets B									↑↑ aligned ↓↓	↓↑ opposed ↑↓
	A overlaps B									↑↑ aligned ↓↓	↓↑ opposed ↑↓
	A contains B									↑↑ aligned ↓↓	↓↑ opposed ↑↓
	A is contained by B									↑↑ aligned ↓↓	↓↑ opposed ↑↓
	A covers B									↑↑ aligned ↓↓	↓↑ opposed ↑↓
	A is covered by B									↑↑ aligned ↓↓	↓↑ opposed ↑↓
	A equals B									↑↑ aligned ↓↓	↓↑ opposed ↑↓
										↑↑ aligned ↓↓	↓↑ opposed ↑↓
										↑↑ aligned ↓↓	↓↑ opposed ↑↓
										↑↑ aligned ↓↓	↓↑ opposed ↑↓
										↑↑ aligned ↓↓	↓↑ opposed ↑↓
										↑↑ aligned ↓↓	↓↑ opposed ↑↓
										↑↑ aligned ↓↓	↓↑ opposed ↑↓

3
 4 **Table 3.** Truth Table showing consistent (green) and inconsistent (red) spatial-temporal-polarity relations between
 5 two depositional units. All 14 temporal relations are not included (as columns), for reasons of space and
 6 because the values are duplicated for the inverse temporal relations.

7
 8 The Truth Table in Table 3 illustrates all possible spatial-temporal-polarity relation combinations for two
 9 depositional units. The eight columns represent the temporal relations possible between two intervals of time; the
 10 remaining inverse temporal relations are excluded for reasons of space and redundancy, as the values in each row
 11 are repeated for the temporal inverse, e.g. *A precedes B* and *A is preceded by B* are both red (inconsistent). The
 12 rows represent the possible spatial and internal polarity relations between two depositional rock volumes. Green



*What is material sharing?

1 cells then indicate consistent combinations, red cells inconsistent combinations, with the consistent cells being far
2 less numerous. Indeed, in Table 3, two distinct depositional units can be spatially related only via *is disjoint with* or
3 *meets*, once material sharing is excluded. All combinations are possible for spatially disjoint units, but only aligned
4 polarity is valid for units that spatially meet, because opposed polarities would signal inconsistencies, such as
5 missing events or intermediary objects.

6 ⇒ Again; clarify that units do not "meet" across a fault or unconformity?

7 In addition to the general geological principles, the following criteria are applied to the truth tables:

8 • Order and type of relata: the type of each relatum is fixed across all relations in a Truth Table. For example,
9 for a Truth Table between a depositional unit A and intrusion unit B, A is the first relatum and B is the
10 second relatum for all relations in the table, e.g. A *meets* B, A *precedes* B, B *is preceded by* A, and A *is*
11 *aligned with* B. This ensures all possible relation combinations are considered for the pair of objects.

12 • Material sharing: only some, not all, geological objects can share material: depositional, intrusion,
13 extrusion, or metamorphic units cannot share material *at any time*, because they cannot simultaneously
14 occupy the same space. These units, however, can share material with a fold volume, since the fold acts to
15 geometrically transform other material volumes. Furthermore, although there exist many scenarios in which
16 geological objects share material, e.g. a depositional unit and its parts, such as group and its formations,
17 however for the purposes of this paper we exclude all such cases, mainly because 3D algorithms tend to
18 bulk together geological components into single packages, such as a series, or model them as facies. As a
19 result of this material-sharing limitation, the *overlaps*, *contains*, or *covers* relations, and their converses,
20 can only hold between certain volumetric geological objects.

} Meets this earlier

21 • Spatial relations: the following are assumed: (1) geological objects can be spatially disjoint and possibly
22 very far apart, e.g. on different continents, allowing all temporal relations; (2) only geological objects in
23 their full-dimensional form are valid, e.g. volumes can't be represented as lower dimensional top or bottom
24 surfaces, or a fault surface by a line. Implementation of this approach to consistency-checking thus requires
25 the conversion of all lower-dimensional representations to their full-dimensional counterparts.

26 • Model completeness: 3D geo-models are assumed to be complete. Therefore, any two geological objects
27 that touch or share material cannot have objects missing between them, such as an intermediary erosional
28 surface or fault. Without this assumption, the range of consistent scenarios becomes extremely large, with
29 significantly fewer inconsistent scenarios, and the effectiveness of the approach diminishes. In contrast,



1 with this assumption, inconsistent scenarios can signal (but not identify) the absence of spatial

2 intermediaries, which is useful during model-building.

→ Consider illustrating this important point with e.g. a bedding parallel thrust.

3 Geological Consistency-Checking Tool

5 The consistency checker workflow is presented in Figure 4. This workflow aims to detect the consistency of 3D geo-
6 models given knowledge inputs of:

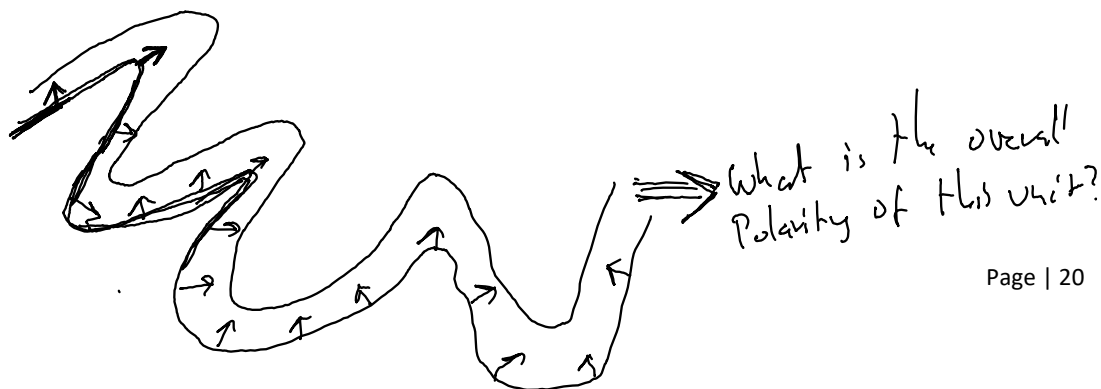
- 7 • a 3D geo-model;
- 8 • local knowledge consisting of relative or absolute ages, internal polarities, types of geological objects, and
- 9 • universal knowledge in the form of geological norms (i.e. Truth Tables). (i.e. Map legends?)

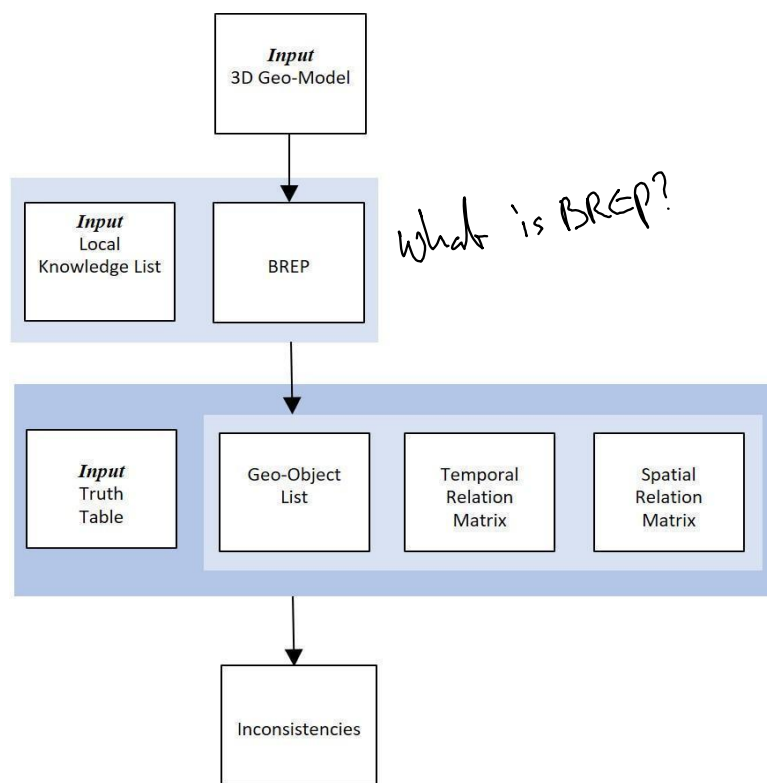
10 After traversal of the 3D geo-model, the consistency checker constructs three intermediary products:

- 11 • a geo-object list, itemizing the geometric objects with internal polarities in the geo-model; → And when polarity varies spatially? (see below)
- 12 • a matrix of temporal relations for each pair of geological objects;
- 13 • a matrix of spatial relations for each pair of geological objects; → Could be adapted such that "meets" is replaced with a directed relation that includes polarity??

14 Then, as per Algorithm 1 (see Appendix): for each pair of geologic objects, the checker obtains their spatial relation
15 from the spatial relation matrix, their temporal relation from the temporal matrix, and their internal polarity direction
16 from the internal polarity property of individual objects. These are all then used as a 3 component (spatial-temporal-
17 polarity) index into a cell within the appropriate geo-object pair Truth Table to determine consistency. Each geo-
18 object pair is navigated to identify any inconsistent regions, which if present are output as a list of inconsistencies in
19 the geo-model. The tool is written using the proprietary Geodes-Solutions spatial toolkit (Botella et al., 2016;
20 Geodes-Solutions; Pellerin 2017), which facilitated spatial navigation and enabled conversion to a full-geometric
21 spatial representation. It was run on a moderately powerful Windows desktop, typically requiring several seconds to
22 minutes to assess a model from the case studies. Note the tool is written strictly to demonstrate proof-of-concept for
23 the framework and its use, and is not meant for widespread deployment being restricted to a specific, older, version
24 of the toolkit and tuned to the specific case studies.

25





1

2 **Figure 4.** Consistency checker workflow.

3

4 **3.1 Local Knowledge List**

5 Local geological knowledge is specific to each study area and, alongside the 3D geo-model, is a primary input to the

6 consistency checker tool. It is found in a variety of sources external to the 3D geo-model, such as databases, map

7 legends, stratigraphic columns, journal articles and other reports. For the synthetic model shown in Figure 5a, a local

8 knowledge list is developed and illustrated in Figure 5b: the list contains the name of the geological object, its type

9 (from the nine possibilities), gross internal polarity and its relative age. For simplicity in the proof-of-concept tool,

10 an object's gross polarity is either up, down, or unknown. Furthermore, if a geological object is an aggregate of

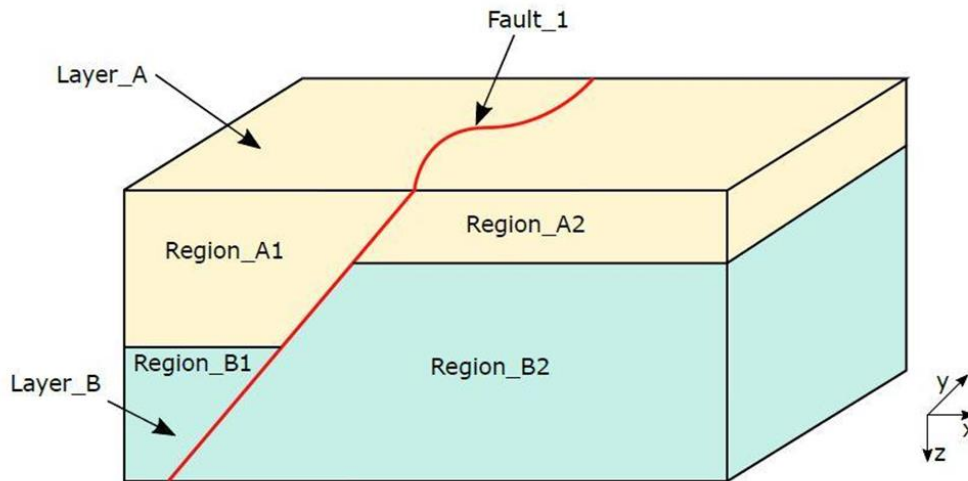
11 parts, then the local knowledge applies to the aggregate and is assumed to be the same for every part. For example,

12 in Figure 5a, the local knowledge for units A and B is assumed to also hold for each of their parts A1, A2, B1, B2;

13 separate local knowledge for these parts, if it existed, would not be used. This enables the spatial, temporal, and

14 polarity to hold between the aggregates, which tends to be the resolution at which the input knowledge is available.

15



1
2
3
4

Figure 5a. example 3D geo-model.

Geological entity name	Geological entity type	Geological entity polarity	Geological entity age
layer_A	depositional unit / intrusion / extrusion / metamorphic unit / fault / erosion surface / fold volume / linear fabric / planar fabric	up / down / unknown	intermediate
layer_B	depositional unit / intrusion / extrusion / metamorphic unit / fault / erosion surface / fold volume / linear fabric / planar fabric	up / down / unknown	oldest
fault_1	depositional unit / intrusion / extrusion / metamorphic unit / fault / erosion surface / fold volume / linear fabric / planar fabric	up / down / unknown	youngest

5
6
7

Figure 5b. Local knowledge list for the model in Figure 5a.

8
9

3.1 Temporal Relation Matrix

10 Temporal relations are also obtained from external sources, including absolute or relative ages for each geological
 11 object, as well as the kind of duration of the geological event (i.e. interval or instant). This knowledge then
 12 determines the appropriate temporal relation between all pairs of geological objects, organized as a temporal matrix
 13 (Figure 6). The matrix is developed manually for our case studies but could be determined automatically from
 14 databases and other digital sources. The *incomparable* relation is chosen if there exists insufficient knowledge to
 15 determine the temporal relation between a pair of objects.

⇒ Note the strong link between spatial and temporal topology (e.g., xcutting relationships)



	layer_A	layer_B	fault_1
layer_A	6	0 / 1 / 2 / 3 / 4 / 5 / 6 / 7 / 8 / 9 / 10 / 11 / 12 / 13	0 1 / 2 / 3 / 4 / 5 / 6 / 7 / 8 / 9 / 10 / 11 / 12 / 13
layer_B	0 1 / 2 / 3 / 4 / 5 / 6 / 7 / 8 / 9 / 10 / 11 / 12 / 13	6	0 1 / 2 / 3 / 4 / 5 / 6 / 7 / 8 / 9 / 10 / 11 / 12 / 13
fault_1	0 / 1 / 2 / 3 / 4 / 5 / 6 / 7 / 8 / 9 / 10 / 11 / 12 / 13	0 / 1 / 2 / 3 / 4 / 5 / 6 / 7 / 8 / 9 / 10 / 11 / 12 / 13	6

Temporal Relations Legend

- 0 = precedes
- 1 = meets
- 2 = overlaps
- 3 = is finished by
- 4 = contains
- 5 = starts
- 6 = equals
- 7 = incomparable to
- 8 = is started by
- 9 = is during
- 10 = finishes
- 11 = is overlapped by
- 12 = is met by
- 13 = is preceded by

1

2 **Figure 6.** Temporal relation matrix for the model in Figure 5a.

3
4 **3.2 Spatial Relation Matrix**

5 This matrix typically requires transformation of the 3D geo-model into a boundary representation (BREP; Banerjee
6 et al., 1981), which ensures all geological objects are represented in their full-dimensional form, e.g. a volume
7 initially represented by its top and bottom surfaces is converted into a mesh of the full exterior limits of the volume,
8 consisting of faces, edges and vertices. A geo-model then can be traversed by following the geometric
9 decomposition of each object and their adjacencies. If objects are named and typed (e.g. as in the Geodes-Solutions
10 BREP solution; Botella et al., 2016; Geodes-Solutions; Pellerin 2017), then such traversal enables building of the
11 spatial relation matrix. Specifically, the consistency checker builds a list containing each geometric object, as well
12 their dimensionality (volume, surface, or line), type (e.g. depositional unit), and name (e.g. "Layer_A"). The list is
13 traversed in order of dimensionality, starting with higher-dimensional objects (volumes) and progressing to lower-
14 dimensional objects (surfaces and lines), with spatial relations determined between pairs of objects by inspecting
15 decompositions and adjacencies. The results are recorded in the spatial relation matrix (Figure 7). For simplicity, a
16 cell in the spatial matrix contains a single value, and the entities being related are the whole objects, e.g. Layer_A
17 (Figure 5a), and not their fragments, e.g. Region_A1 or Region_A2 (Figure 5a). This is obviously problematic as
18 fragments of objects might be spatially related in many ways, e.g. some might touch and others are disjoint, so the
19 wholes can be related in many ways too, requiring multiple values per cell for each pair of wholes: e.g. it is possible
20 Region_A1 has one relation with Region_B1 and a different relation with Region_B2, thus A would have multiple
21 distinct relations with B. In such cases, the most dominant relation is selected, which suffices for our case studies.

Arguably simpler/easier in a discretised (gridded) form?

Consider using the "structural" and "lithological" topology terms from Thiede et. al.



Can the "Equivalent to" temporal relationship resolve this issue? (i.e. by building a graph of the structural topology, but with an expanded temporal topology graph (with many "equivalent" relations) used to distribute temporal information at each lithology).

- 1 To avoid multi-valued cells, a rigorous approach would utilize object fragments for consistency-checking, rather
- 2 than wholes. This issue and the incorporation of local relations is an important topic for future research.

	layer_A	layer_B	fault_1
layer_A	8	0/1/2/3/4 5/6/7/8	0/1/2/3/4 5/6/7/8
layer_B	0/1/2/3/4 5/6/7/8	8	0/1/2/3/4 5/6/7/8
fault_1	0/1/2/3/4 5/6/7/8	0/1/2/3/4 5/6/7/8	8

Spatial Relations Legend:
 0 = disjoint
 1 = meets / is met
 2 = intersects
 3 = overlaps / is overlapped by
 4 = contains
 5 = is contained by
 6 = covers
 7 = is covered by
 8 = equals

3

4 **Figure 7.** Spatial relation matrix for the model in Figure 5a.

5

6 **4 Case studies**

7

8 The consistency checker is tested in four case studies: three synthetic models in which inconsistencies are
 9 introduced, and a real regional geo-model from ongoing project work in Western Canada (Thapa and McMechan,
 10 2019, McMechan et al., 2021). The geo-models are built using a variety of 3D modelling software and underlying
 11 approaches including: Noddy (Jessell, 1981), GOCAD/SKUA (Jayr et al, 2008; Mallet, 2004), GOCAD (Mallet,
 12 1989) and certain extensions, namely SURFE (Hillier, et al. 2014; de Kemp et al. 2017) and SPARSE (de Kemp et
 13 al. 2004).

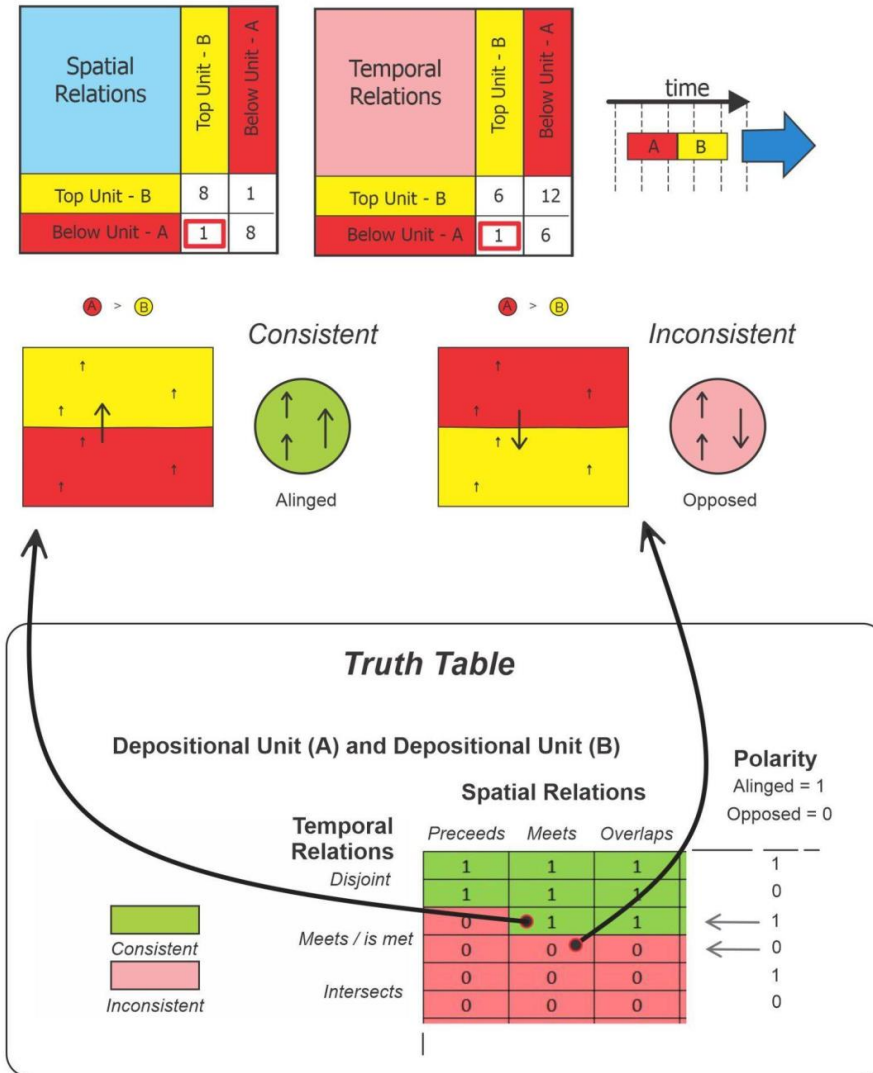
14

15 **4.1 Implicit Case study**

16 A simple but common modelling error occurs when applying certain implicit algorithms to sparse observations of
 17 near parallel, shallow, dipping strata. Then, as depicted in Figure 1, if some older unit data are slightly
 18 topographically higher than younger unit data, algorithm bias can result in older units above younger units. To
 19 assess such a model, the consistency checker requires alignment of the three polarity vectors, two internal and one
 20 temporal: as shown in Figure 8, the Truth Table indicates two depositional units that spatially and temporally meet
 21 are (1) consistent, if the polarity vectors are aligned, and (2) inconsistent, if they are opposed. Therefore, this model
 22 is evaluated as inconsistent, because the temporal polarity vector is opposed to the internal polarity vectors. Note



- 1 that there could be many reasons for the temporal reversal but this is not identified by the checker, e.g. it might be
- 2 algorithmic bias or missing objects, such as absent thrust faults, recumbent folds, or erosional surfaces.



3

4 **Figure 8.** From the simple model in Figure 1, the internal polarity vectors are aligned with each other, but opposed
 5 to the age vector. As the units both spatially and temporally meet, the Truth Table shows this configuration is
 6 inconsistent due to the unaligned age vector.
 7

8

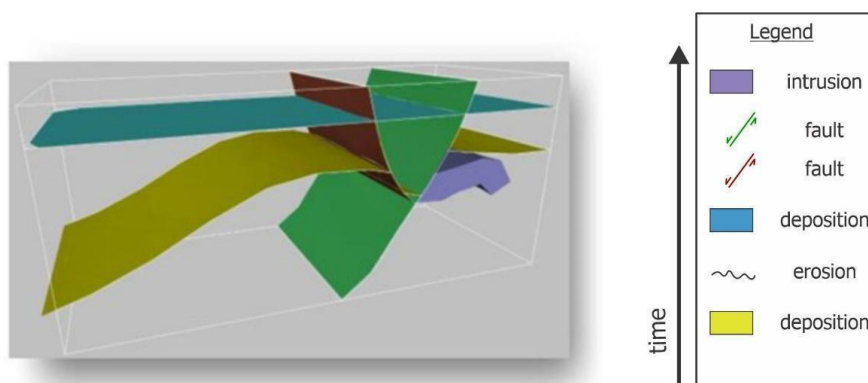
9

10



1 **4.2 GOCAD/SKUA Case Study**

2 This synthetic model contains two depositional units, one intrusion, two faults, one fold, and one erosional feature
3 that is the top surface of the oldest unit (Figure 9). The model is created with GOCAD/SKUA (Mallet, 1989, 2004)
4 using the Structural and Stratigraphic workflow (Jayr et al., 2008); the local knowledge list (Table 4) and temporal
5 matrix (Figure 10) are developed artificially. The spatial matrix (Table 5) includes a variety of spatial relations, such
6 as touching geological units, faults cutting geological units, intrusion units protruding into other geological units, as
7 well as disjoint geological objects. As expected, results from the consistency checker indicate the geo-model is
8 geologically consistent. However, if the event timeline is manipulated to generate inconsistencies without altering
9 spatial relations (Figure 11), then an intersection between the third deposited layer (blue) and the first fault (red) is
10 detected, which is inconsistent with the altered event history (Figure 12).
11



12

13 **Figure 9.** Synthetic geo-model for the GOCAD/SKUA case study: two sedimentary horizons (in yellow and blue),
14 one intrusion (in purple), two faults (in green and red), one fold (in the yellow horizon) and one erosion
15 surface (top of the yellow unit). Horizons define the top of a unit. For simplicity we ignore the folding event
16 that affects pre-erosion sediments.
17



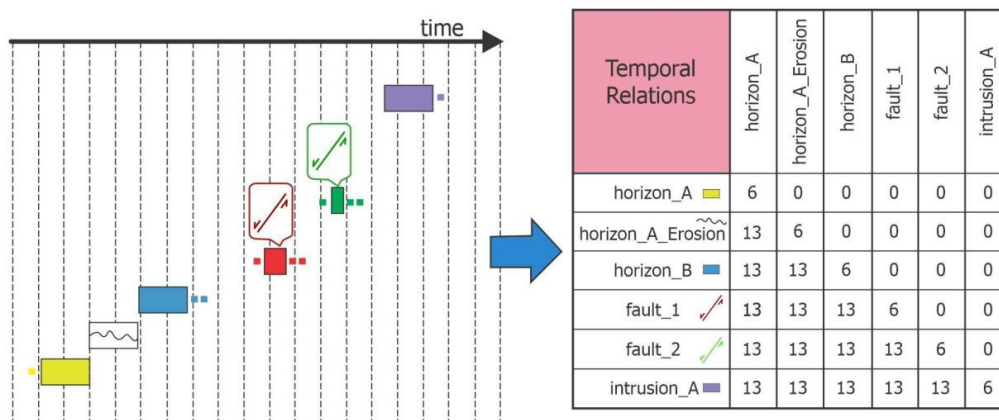
Name	Entity type	Age	Polarity
Intrusion_A	Intrusion Unit	Youngest	Up
Fault_2	Fault Surface	Younger than F1	None
Fault_1	Fault Surface	Older than F2 Younger than HB	None
Horizon_B	Depositional Unit	Younger than HAE	Up
Horizon_A_Erosion	Erosional Surface	Younger than HA	Up
Horizon_A	Depositional Unit	Oldest	Up

1

2 **Table 4.** Local knowledge list for the GOCAD/SKUA case study.

3

4



5

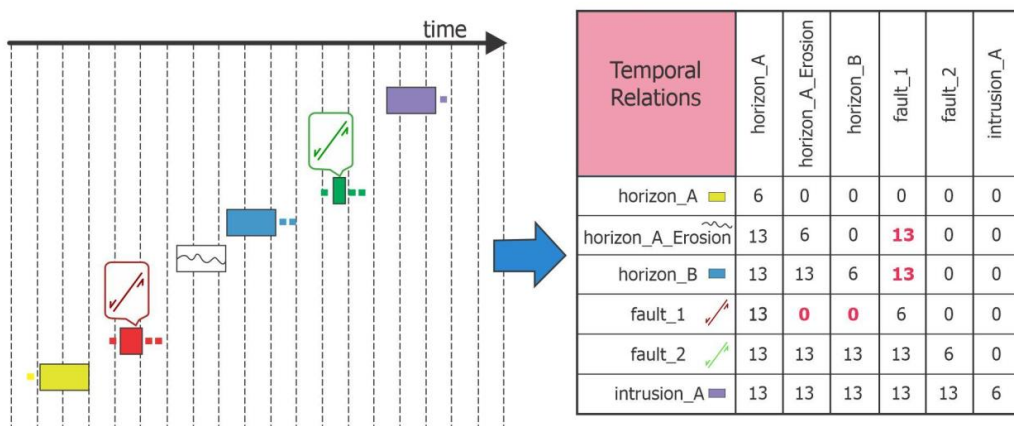
6 **Figure 10.** Event history (left) and temporal matrix (right) for the GOCAD/SKUA case study. For temporal relation
7 codes see Figure 6.



Spatial Relations	horizon_A	horizon_A_Erosion	horizon_B	fault_1	fault_2	intrusion_A
horizon_A	8	1	1	2	2	1
horizon_A_Erosion	1	8	0	2	2	0
horizon_B	1	0	8	2	2	0
fault_1	2	2	2	8	1	0
fault_2	2	2	2	1	8	1
intrusion_A	1	0	0	0	1	8

1

2 **Table 5.** Spatial relation matrix for the GOCAD/SKUA case study. For spatial relation codes see Figure 7.

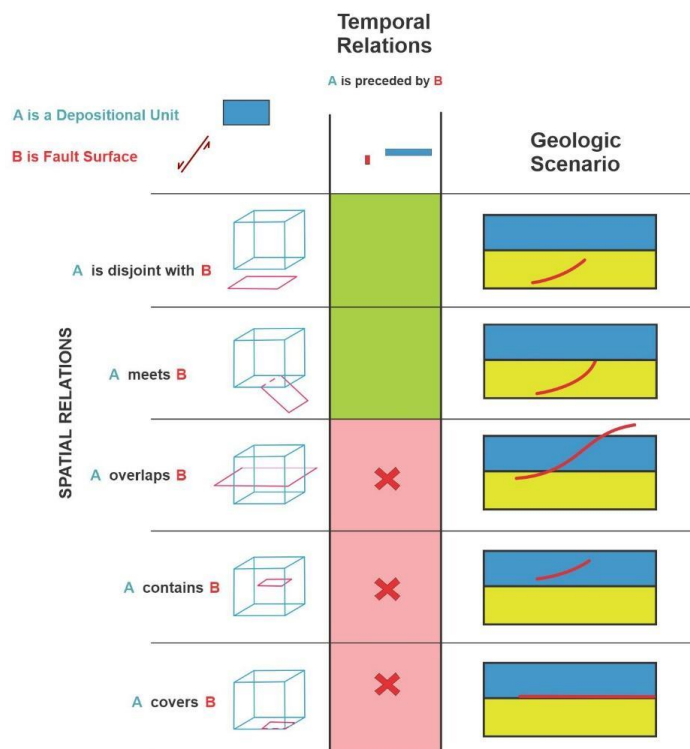


3
4



1 **Figure 11.** Modified event history (left) and temporal matrix (right) for the GOCAD/SKUA case study, with
 2 unfeasible temporal relations (in red). For temporal relations codes see Figure 6.

3



4

5 **Figure 12.** Truth table fragment indicates the relation between fault_1 (earliest fault in red) and Unit B (Blue) is not
 6 consistent with the altered history. The inconsistent conditions (marked 'X') in this case are: A *overlaps* B, A
 7 *contains* B, and A *covers* B, which are all geologically not plausible given the early fault is eroded BEFORE
 8 deposition of UNIT B for the altered history. The acceptable consistent scenarios, in green, that are allowed
 9 are possible because of the temporal A *is preceded by* B relation since there is a time gap to allow the fault to
 10 be preserved in the underlying host rock either with a hiatus in movement of the fault, or an intervening
 11 erosional event introduced with the change in the event history. In all other scenarios Unit B (blue) does not
 12 yet exist to host the fault.
 13

14

15

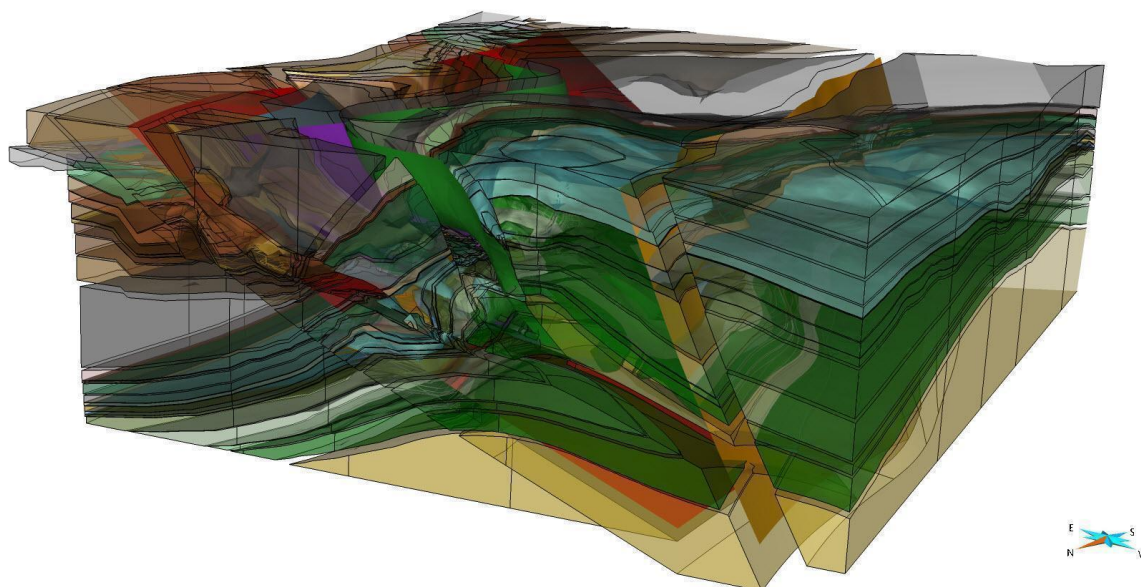
16



1 **4.3 Western Canada Case Study**

2 The geo-model for this real case study (Figure 13) uses data from the Rocky Mountains of the Western Canadian
3 Cordillera, and is built using the GOCAD/SKUA, SURFE, and SPARSE toolkits (Dutranois, et al. 2010, Hillier et
4 al. 2014, de Kemp et al. 2016). It represents a portion of an east verging fold and thrust belt that has telescoped the
5 Paleozoic and basement meta-sediments of the early North American craton margin, with tectonic deformation
6 having produced in-sequence and out-of-sequence thrusts (McMechan et al., 2021, Morely 1988), as well as later
7 normal faults, with fold-fault and horizon relations that complicate original stratigraphy. The event history (Figure
8 14) is simplified with all the sedimentary units depositing sequentially, and incurring some facies changes across
9 major structures, followed by several episodes of faulting with some overlapping in time. The spatial complexity of
10 the model arises from the multitude of entities, from faults crosscutting other faults and impacting the pre-deposited
11 layers. The resulting geometry is composed of 213 objects within the 25 units, and 6 faults, with each object
12 delimited or separated from the rest of the unit by a fault.

13
14
15



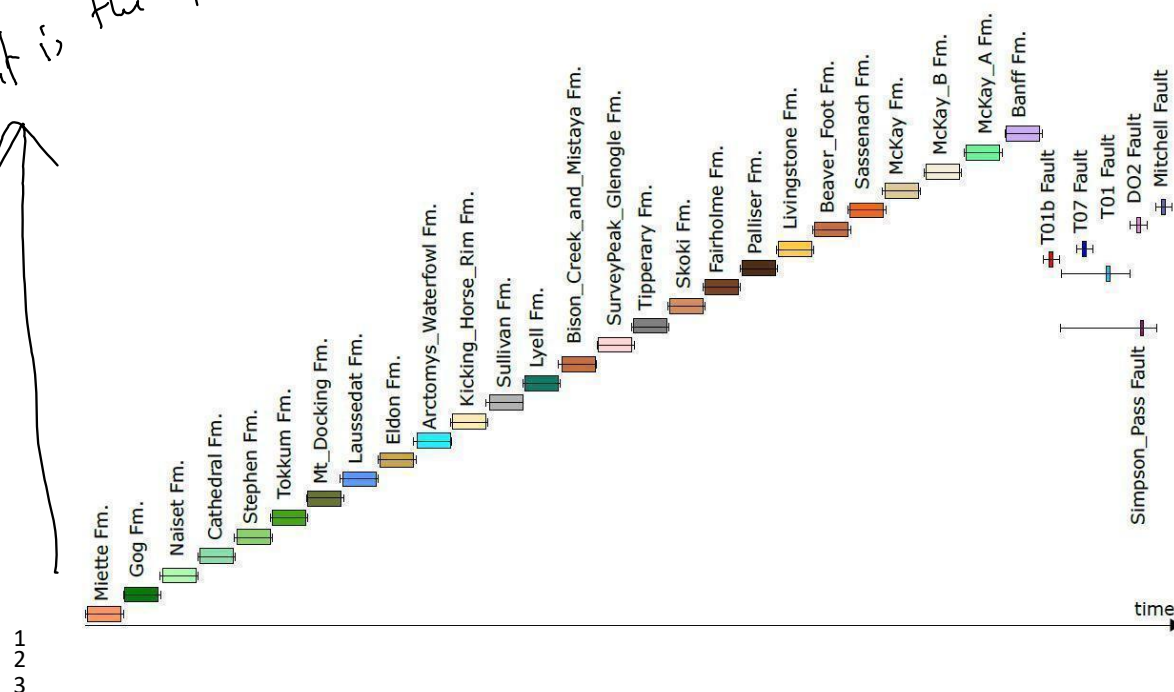
16
17

18 **Figure 13.** Western Canada case study geo-model: 213 component objects of 25 geological depositional units, and 6
19 faults.

20
21



What is the y-axis?



4 **Figure 14.** Event history for the Western Canada case study: horizontal boxes are relative timelines and bars are
5 possible ranges.

7 After compilation of the local knowledge list and temporal relation matrix from external sources, including maps
8 and reports, and development of the spatial relation matrix (Table 6), the consistency checker detects one
9 inconsistency. The inconsistency (Figure 15) involves the spatial containment of one sedimentary unit (Miette) by
10 another (Gog), which is impossible given they cannot occupy the same space, being caused by different depositional
11 processes at different times. The consistency checker not only identifies the kind of inconsistency, through
12 specification of the truth table cell, but it also pinpoints the location of the problem by identifying the inconsistent
13 volumes, after iteration through all the geo-object pairs. Subsequent analysis of the inconsistency suggests it is an
14 artifact of the modelling algorithm and ^{an} inaccurate interpolation of the data.

15
16

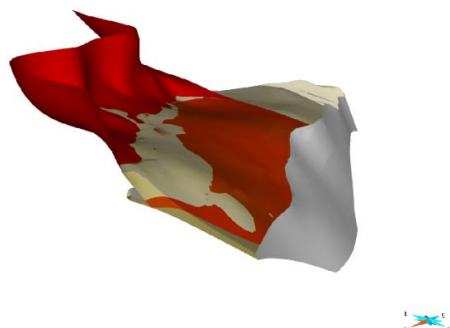


Spatial Relations	Miette	Gog	Naiset	Cathedral	Tokkumm	Mt_Docking	Lausosedat	Eldon	Arctomys_Waterfowl	Kicking_Horse_Rim	Sullivan	Lyll	Bison_Creek_and_Mistaya	SurveyPeak_Glenogle	Tipperary	Skoki	Fairholme	Palliser	Livingstone	Beaver_Foot	Sassenach	McKay	McKay_A	Banff	Kana_Topography	T01b_fault	T07_fault	T01_fault	Simpson Pass_fault	D02_fault	Mitchell_fault	
Miette	8	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	0	2	2	2	
Gog	4	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	0	2	2	2	
Naiset	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	0	2	2	2	
Cathedral	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	0	2	2	2	
Tokkumm	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	0	2	2	2	
Mt_Docking	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	0	2	2	2	
Lausosedat	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	0	2	2	2	
Eldon	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	0	2	2	2	
Arctomys_Waterfowl	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	0	2	2	2	
Kicking_Horse_Rim	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	2	2	2	2	
Sullivan	1	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	2	2	2	2	
Lyll	1	1	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	0	0	0	0	2	2	2	2	2	2	
Bison_Creek_Mistaya	1	1	1	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	0	0	0	0	2	2	2	2	2	2	
Survey_Peak_Glenogle	1	1	1	1	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	0	0	0	0	2	2	2	2	2	2	
Tipperary	1	1	1	1	1	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	0	0	0	0	2	2	2	2	2	2	
Skoki	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	0	0	0	0	2	2	2	2	2	2	
Fairholme	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	8	1	1	1	1	0	0	0	0	2	2	2	2	2	2	
Palliser	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	8	1	1	1	0	0	0	0	2	2	2	2	2	2	0
Livingstone	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	8	1	1	0	0	0	0	2	2	2	2	2	2	0
Beaver_Foot	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	8	1	0	0	0	0	2	2	2	2	2	2	0
Sassenach	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	8	0	0	0	0	2	2	2	2	2	2	0
McKay	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	8	1	1	1	2	2	2	2	2	2	0
McKay_A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	8	1	1	0	2	2	0	2	0	
Banff	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	8	1	2	2	2	2	2	0	
Kana_Topography	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	
T01b_fault	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	8	0	0	1	0	0	
T07_fault	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	8	0	1	1	1	0	
T01_fault	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	1	0	
Simpson Pass_fault	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	1	1	0	8	0	0	
D02_fault	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	1	1	0	8	1	0	
Mitchell_fault	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	0	1	0	0	1	8	

Spatial Relations Legend: 0 = disjoint, 1 = meets, is met by, 2 = intersects, 3 = overlaps, is overlaped by, 4 = contains, 5 = is contained by, 6 = covers, 7 = is covered by, 8 = equals

1
2
3
4
5

Table 6. Spatial relation matrix for the Western Canada case study, with the inconsistent containment relations in red.



6

Figure 15. Inconsistent spatial containment between the Gog (red) and Miette (yellow) units in the Western Canada case study. The Miette is an older unit preceding deposition of Gog material and should not, at this scale, have a ‘Miette contains Gog’ spatial relation.

7
8
9

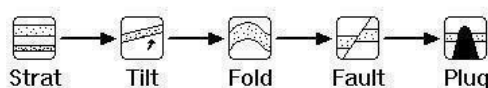


1

2 4.4 Noddy Case Study

3 The synthetic geo-model for this case study is generated using Noddy, which is a 3D rule-based modelling tool
4 (Jessell, 1981) that applies an input list of geological events, or event schema (Perrin et al. 2013), to a volume of
5 interest from which a spatial topology can be generated between objects in the volume. The event history for this
6 case study is quite simple, including 5 major events, deposition, tilting, folding, faulting, and intrusion (Figure 16),
7 from which the local knowledge list and temporal matrix are derived. The resulting geo-model (Figure 17) has an
8 initial depositional sequence involving 6 depositional units (represented by their top horizons), an early tilting event
9 followed by folding and normal faulting, and an intrusive body subsequently injected into all previous geological
10 objects, with the fault cutting all the horizons but not the intrusion body. Navigation of the BREP representation of
11 the geo-model yields a rich temporal matrix (Table 7) and spatial matrix (Table 8a).

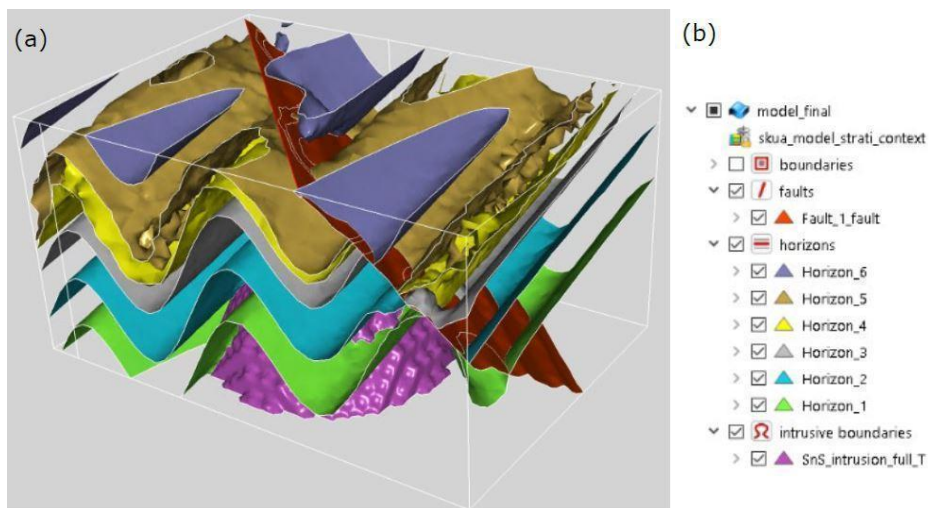
12



13

14 **Figure 16.** Event history for the Noddy case study: the Stratigraphic event is the deposition of 6 geological units in
15 sequence.

15



16

17

18 **Figure 17.** 3D geo-model for the Noddy case study.

19



TIME	Horizon_1	Horizon_2	Horizon_3	Horizon_4	Horizon_5	Horizon_6	Above Horizon 6	Fault	Intrusion
Horizon_1	6	0	0	0	0	0	0	0	0
Horizon_2	13	6	0	0	0	0	0	0	0
Horizon_3	13	13	6	0	0	0	0	0	0
Horizon_4	13	13	13	6	0	0	0	0	0
Horizon_5	13	13	13	13	6	0	0	0	0
Horizon_6	13	13	13	13	13	6	0	0	0
Above Horizon 6	13	13	13	13	13	13	6	0	0
Fault	13	13	13	13	13	13	13	6	0
Intrusion	13	13	13	13	13	13	13	13	6

1

2 **Table 7.** Temporal relations matrix for the Noddy case study. Temporal codes used here; 0 = precedes, 6 = equals,
 3 13 = preceded by.

4
 5

6 As an unconstrained (knowledge based) geo-modelling tool, Noddy will always produce a consistent model.
 7 However, export to GOCAD/SKUA via the DXF file format results in a different spatial relation matrix, as several
 8 geologically consistent but nevertheless suspect spatial relations are introduced. First, every unit spatially *meets* (i.e.
 9 touches) every other unit (Table 8b), which although not impossible is somewhat suspicious, given it contradicts the
 10 original Noddy model. As the resolution of the Noddy model is quite low, it seems likely that unit volume mesh
 11 extents might have been mis-calculated during export to GOCAD/SKUA, to cause the dubious relations. Although
 12 the relations between pairs of objects are individually consistent in this model, it is unlikely that ALL unit pairs
 13 display a *meets* relation.

14



a)

SPACE	Horizon_1	Horizon_2	Horizon_3	Horizon_4	Horizon_5	Horizon_6	Above Horizon 6	Fault	Intrusion
Horizon_1	8	1	0	0	0	0	0	2	2
Horizon_2	1	8	1	0	0	0	0	2	2
Horizon_3	0	1	8	1	0	0	0	2	2
Horizon_4	0	0	1	8	1	0	0	2	2
Horizon_5	0	0	0	1	8	1	0	2	2
Horizon_6	0	0	0	0	1	8	1	2	2
Above Horizon 6	0	0	0	0	0	1	8	2	2
Fault	2	2	2	2	2	2	2	8	0
Intrusion	2	2	2	2	2	2	2	0	8

b)

SPACE	Horizon_1	Horizon_2	Horizon_3	Horizon_4	Horizon_5	Horizon_6	Above Horizon 6	Fault	Intrusion
Horizon_1	8	1	1	1	1	1	1	2	2
Horizon_2	1	8	1	1	1	1	1	2	2
Horizon_3	1	1	8	1	1	1	1	2	2
Horizon_4	1	1	1	8	1	1	1	2	2
Horizon_5	1	1	1	1	8	1	1	2	2
Horizon_6	1	1	1	1	1	8	1	2	2
Above Horizon 6	1	1	1	1	1	1	8	2	2
Fault	2	2	2	2	2	2	2	8	0
Intrusion	2	2	2	2	2	2	2	0	8

1

2 **Table 8.** Spatial relations matrix for Noddy case study, including (a) original model, and (b) after export via DXF to
 3 GOCAD/SKUA. Note the replacement of many *is disjoint with* relations (0) in (a), with *meets / is met by* (1) in
 4 (b).

5



1

2 **5 Discussion**

3 The consistency-checking framework and tool presented in this article are a first step toward the automated
4 assessment of geological consistency in 3D geological models. The approach yields promising results in the four
5 case studies: ^{each of} given minimal knowledge typically accompanying a 3D model, it detects geological inconsistencies
6 that contravene geological principles as captured in the Truth Tables. However, there is much room for
7 improvement in assessing the consistency of complex situations: the checker assesses the validity of a single
8 geological relation in isolation, but as evident from the Noddy case study (Table 8b), a collection of relations can be
9 inconsistent even if each relation is consistent. Assessing such relation combinations remains a future task.

10

11 To help differentiate equiprobable models, another future consideration is the development of consistency metrics
12 for quantitative assessment of the overall quality of a 3D geo-model. These might include a cumulative consistency
13 score to gauge the overall effect of inconsistencies on the model, as well as perhaps targeted consistency scores for
14 specific geological relations. The latter would be particularly useful to differentiate (1) models with few
15 inconsistencies but deep impact on internal model architecture, from (2) models with many inconsistencies but low
16 impact on internal architecture.

17

18 Several aspects of the consistency-checking workflow could be improved:

- 19 ● API: development of a simple API (Application Programming Interface) to the Truth Tables, to enable
20 consistency-checking from a variety of software environments, including possibly those with streamlined
21 spatial navigation mechanisms not necessarily requiring conversion to BREP.
- 22 ● Enhanced Output: from the current application or prospective API, to enhance both formatting and content,
23 such as encoding conflicting objects using knowledge graphs or spatial standards to improve visualization and
24 understanding.
- 25 ● Polarity: more automated tools could be incorporated to determine summary internal polarity for an object, and
26 polarity relations between objects. The internal polarity of an object is rarely available in local knowledge,
27 though potentially can be calculated from the modelling algorithm, e.g. as part of the scalar field gradient
28 direction in implicit modelling. Supplementation from added methods also would be beneficial, e.g. from: point
29 observations at the top horizon of a depositional unit; paleo-traces inside the unit in the case of eroded tops; the



1 cooling surface of an intrusion or extrusion; the regional or contact metamorphic gradient for a metamorphic
2 unit; or fold vergence and principal strain gradients (Fossen, 2016; Alsop, 1999; Finkl, 1984). A further
3 refinement might use local internal polarity vectors to determine polarity relations, rather than summary vectors.

- 4 • Spatial relations: in a robust solution, spatial relations would be evaluated between the fragments of geological
5 objects, not between their wholes, with consistency-checking then occurring over the fragments. It also might
6 be a worthwhile experiment to include lower-dimensional representations as relata in the spatial relations, e.g.
7 allowing surfaces to represent volumetric units.
- 8 • Expanding the range of model inputs to include 2D maps, cross-sections and drill logs. These representations
9 could also be checked with the same approach to see if all representations of a model are collectively consistent.

10 Consistency-checking also could be improved conceptually by: expanding the list of geological objects to include
11 fault types (e.g. normal, reverse, strike slip); expanding the associated knowledge to include fault domains (e.g.
12 upper-crust/thin-skin, deep-crust/ductile); or adding kinematic directions as additional parameters in the Truth
13 Tables, thus expanding the framework space. This would enable, for example, comparison of macro properties such
14 as nature of the deformation system with the observed local kinematic conditions, e.g. thrusting or normal fault
15 displacements.

16

17 As most 3D modeling algorithms and tools typically do not generate solid volumes in which one is fully contained
18 or covered by the other, we have set these relations as invalid for this work, knowing their presence likely indicates
19 a modeling problem and hence an inconsistency. However, as algorithms will no doubt mature, future work should
20 amend the truth tables to reflect the potential validity of such cases. This might include further extended parameters,
21 such as for parthood to indicate if a geological object is validly part of another, e.g. a formation part of a group.

22

23 More generally, broadening the underlying notion of reasonableness, which thus far is roughly equated with
24 consistency, would yield further theoretical gains. An important assumption in the existing approach is the
25 correctness of input geological knowledge. As such knowledge typically reflects the understanding of domain
26 experts, inconsistent models often differ from the expectations of these experts (van Giffen et al., 2022; McKay and
27 Harris, 2016; Burch, 2003). However, the correctness of input knowledge is a dangerous assumption, as it is more
28 likely that input knowledge is incomplete and has gaps, biasing expert expectations. It is necessary then to broaden
29 notions of geological reasonableness beyond the binary categories of consistent and inconsistent. Indeed, if we



1 consider input knowledge might be grossly good (e.g. true) or bad (e.g. false), and models consistent or inconsistent
2 with input knowledge, then four kinds of reasonableness emerge, as per Table 9: reasonable, unreasonable,
3 reasonably bad, and unreasonably bad. Reasonable models, generally preferred, are consistent with good input
4 knowledge and data constraints. Unreasonable models have geological relations inconsistent with good input
5 knowledge. Reasonably bad models have geological relations that fit with the input knowledge, but this knowledge
6 is wrong, or incomplete, so the model is variously questionable. Unreasonably bad models have input knowledge
7 that may be wrong, and the model is also inconsistent, because of algorithm bias, scale/resolution, constraint data
8 configuration or other processing errors. Inconsistent models thus signal a need to adjust the algorithm or investigate
9 the input data and knowledge. Note, however, all models might be useful (Gleeson et al, 2021), as any geo-model
10 from bad knowledge might be preferred to no models, or models with no input knowledge; and an inconsistent
11 model from good knowledge, that is unreasonable, might be preferable to the alternatives, especially in parts where
12 it is actually consistent.
13
14

		Model Consistency	
		Inconsistent	Consistent
Good Knowledge		Unreasonable	Reasonable
Bad Knowledge		Unreasonably Bad	Reasonably Bad

15
16

17 **Table 9.** Types of 3D geo-model consistency.

18

19 It is also noteworthy, and sobering, that an ideal model could arise from any of the four categories, simply because
20 the combination of input knowledge, data, and computational processes just happens to produce the best result.
21 Consistency-checking thus provides only some insight as to whether an ideal model is achieved, as one would hope
22 an ideal model should be consistent more often than not. For example this should be the case when comparing a
23 suite of models and their flow characteristics, with 'reasonable' models matching the real world historical

→ What do you mean by "ideal"?



1 production curves (Melnikova et al., 2012). Mounting evidence suggests ^{that} even a minimum of geological knowledge
2 and improved consistency with this knowledge can improve the utility of models (Giraud et al., 2020).
3
4 Finally, application of the framework to case studies at various scales, using different tools and algorithms, would
5 provide further insight into its utility for: exploring different levels of geological and model complexity (Pellerin et
6 al., 2015); comparing high-resolution to generalized regional models; testing more speculative models; for
7 correlation of jurisdictional bordered models (e.g. comparing number and variety of entities, and consistency with
8 each other) and finally for assessing the range of equiprobable 3D geological models created from probabilistic and
9 future generative AI methods.

10

11 **6 Summary**

12 Due to the increasing complexity of current geo-modelling algorithms, leading to a plethora of models of variable
13 quality, there is a clear need for an approach to check the geological consistency of a model. To this end, the
14 consistency checker framework and proof-of-concept tool successfully verify geo-models in four case studies,
15 confirming consistencies and finding inconsistencies. Inputs include knowledge typically available with any
16 geological model, namely, the spatial-temporal-polarity relations between pairs of geological objects. A specific
17 combination of these inputs serves as an index into a Truth Table documenting a possible geological situation that is
18 either consistent or inconsistent with established geological principles. As the situations in the framework are quite
19 general, there is significant potential to expand the space with additional parameters, such as tectonic or parthood.
20 To better determine effectiveness, further testing also is required on large numbers of models representing a variety
21 of geological situations, at different scales, with models made using diverse tools and algorithms. Altogether, this
22 work represents a first step toward the real-time consistency-checking of geo-models; therefore, it also has the
23 potential to inform consistency-checking during model-building, ^{and} to help increase knowledge constraints in geo-
24 modelling algorithms.

25
26
27
28
29
30
31
32
33
34
35



1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51

Appendix

Algorithm 1: Consistency-checking

Require: *Mspatial* spatial relationship matrix, *Mtemporal* temporal relationship matrix, *LNaturePolarity* nature and polarity matrix of entities, *Truthtable* ‘Truth Tables’ for each pair of geological entities, *LGeologicalEntities* list of all geological entities detected in the 3D model.

Initialize (empty): *Linconsistencies* list of inconsistencies detected inside the given 3D geological model

```
for each GeolEntity in LGeologicalEntities do
  for each GeolEntity in the remaining rows in LGeologicalEntities do
    extract the name, nature and polarity in each geological entity from LNaturePolarity
    given both geological entities, find the corresponding truth table Truthtable
    deduce the polarity relation from both geological entities: aligned, opposed, unknown
    extract the spatial relationship for the pair of geological entities from Mspatial
    transform the spatial relationship into a row in the truth table
    extract the temporal relationship for the pair of geological entities from Mtemporal
    transform the temporal relationship into a column in the truth table
    if the statement found in the corresponding truth table is ‘inconsistent’ then
      for each part in each GeolEntity do
        for each part in each GeolEntity do
          extract the name, nature and polarity of each geological entity inside LNaturePolarity
          given both geological entities, find the corresponding truth table Truthtable
          deduce the polarity relation from both geological entities: aligned, opposed, unknown
          extract the spatial relationship for the pair of geological entities from Mspatial
          transform the spatial relationship into a row in the truth table
          extract the temporal relationship for the pair of geological entities from Mtemporal
          transform the temporal relationship into a column in the truth table
          if the statement found in the corresponding truth table is ‘inconsistent’ then
            for each part in each part of GeolEntity do
              for each part in each part of GeolEntity do
                etc...
              end for
            end for
          end if
        end for
      end if
    end for
    add the statement found in the corresponding truth table to Linconsistencies
  end for
end for
return Linconsistencies
```

52

53 Code and Data Availability

54 Consistency-inconsistency matrices called ‘Truth Tables’ used for determining validity of geological spatial-
55 temporal relations <https://doi.org/10.5281/zenodo.10909160>, (de Kemp, 2024).

56



1 **Video Supplement**

2 There are currently no video files (mp4) related to this article.

3 **Author contributions**

4 Conceptualization by MP, EdK, BB and MH; MP developed the tool; MP, EdK, BB and MH all contributed to the
5 case studies development and the writing of the paper.

6 **Competing interests**

7 The author declares that there is no conflict of interest.

8 **Special Issue Statement:** This contribution is part of the Loop stochastic geological modelling platform –
9 development and applications, edited by Laurent Ailleres

10 (https://gmd.copernicus.org/articles/special_issue1142.html, last access: 6 April 2024).

11 **Acknowledgements**

12 Generous support for this research was provided from the Canada3D project (C3D) of Natural Resources Canada
13 (<https://canada3d.geosciences.ca/>, last access: 6 April 2024). Support from the LOOP project
14 (<https://loop3d.github.io/>, last access: 6 April 2024), Australian Research Council (ARC); (Enabling Stochastic 3D
15 Geological Modelling, LP170100985), in collaboration with the OneGeology initiative is gratefully acknowledged.
16 Thanks to the many collaborators from the LOOP team including Mark Jessell (UWA) for providing the Noddy
17 model used for the first application of the consistency-checking tool. Many thanks to RING ([https://www.ring-
18 team.org/](https://www.ring-team.org/), last access: 6 April 2024) for academic support for use of GOCAD/SKUA software, and Geoid-Solutions
19 libraries (<https://geoid-solutions.com/>, last access: 6 April 2024) for model format conversions from
20 GOCAD/SKUA (Model3D) to VTK. Use of Leapfrog Geo graciously provided by Seequent. This paper is NRCAN
21 contribution number 20230104.
22

23 **References**

- 24 Allen, J. F.: Maintaining knowledge about temporal intervals, *Communications of the ACM*, 26, 832–843, 1983.
- 25 Alsop, G.I., Holdsworth, R.E.: Vergence and facing patterns in large scale sheath folds. *Journal of Structural*
26 *Geology* 21, 1335-1349, 1999.
- 27 Annen, C.: Implications of incremental emplacement of magma bodies for magma differentiation, thermal aureole
28 dimensions and plutonism–volcanism relationships, *Tectonophysics*, 500 (1-4), 3-10,
29 <https://doi.org/10.1016/j.tecto.2009.04.010>, 2011.
- 30 Arora, H., Langenhan, C., Petzold, F., Eisenstadt, V., and Althoff, K.-D.: METIS-GAN: An approach to generate
31 spatial configurations using deep learning and semantic building models, in: *ECPM 2021–eWork and*
32 *eBusiness in Architecture, Engineering and Construction*, pp. 535 268–273, CRC Press, 2021.
- 33 Aubry, M.P, Berggren, W.A., Van Couvering, J.A. and Steininger, F.: Problems in chronostratigraphy: stages, series,
34 unit and boundary stratotypes, global stratotype section and point and tarnished golden spikes, *Earth-Science*
35 *Reviews*, 46, (1–4), 99-148, ISSN 0012-8252, [https://doi.org/10.1016/S0012-8252\(99\)00008-2](https://doi.org/10.1016/S0012-8252(99)00008-2), 1999.
- 36 Bai, H., Montési, L.G.J. and Behn, M.D. :MeltMigrator: A MATLAB-based software for modeling three-
37 dimensional melt migration and crustal thickness variations at mid-ocean ridges following a rules-based
38 approach, *Geochem. Geophys. Geosyst.*, 18, 445–456, <https://doi.org/10.1002/2016GC006686>, 2017.
- 39 Banerjee, P. K., Banerjee, P. K., and Butterfield, R.: *Boundary element methods in engineering science*, McGraw-
40 Hill (UK), 1981.
- 41 Bertonecello, A., Sun, T., Li, H. Mariethoz, G. and Caers, J.: Conditioning Surface-Based Geological Models to Well
42 and Thickness Data. *Math. Geosci.*, 45, 873–893 (2013). <https://doi.org/10.1007/s11004-013-9455-4>, 2013.



- 1 Bezhanishvili, N., Ciancia, V., Gabelaia, D., Grilletti, G., Latella, D. and Massink, M.: Geometric Model Checking
2 of Continuous Space, Logical Methods in Computer Science Volume 18, Issue 4, 2022, pp. 7:1–7:38,
3 [https://doi.org/10.46298/lmcs-18\(4:7\)2022](https://doi.org/10.46298/lmcs-18(4:7)2022).
- 4 Bond, C.E.: Uncertainty in structural interpretation: Lessons to be learnt, *Jour. Struct. Geol.*, 74, 185-200, ISSN
5 0191-8141, <https://doi.org/10.1016/j.jsg.2015.03.003>, 2015.
- 6 Botella, A., Lévy, B. and Caumon, G.: Indirect unstructured hex-dominant mesh generation using tetrahedra
7 recombination, *Comp. Geosci.*, 20, 437–451, <https://doi.org/10.1007/s10596-015-9484-9>, 2016.
- 8 Brodaric, B.: Characterizing and representing inference histories in geologic mapping, *Int. Jour. Geog. Info. Sci.*, 26
9 (2), 265-281, <https://doi.org/10.1080/13658816.2011.585992>, 2012.
- 10 Brodaric, B. and Gahegan, M.: Representing Geoscientific Knowledge in Cyberinfrastructure: challenges,
11 approaches and implementations. In: Sinha, A.K. (Ed.), *Geoinformatics, Data to Knowledge*, Geological
12 Society of America Special Paper 397, pp. 1-20. [https://doi.org/10.1130/2006.2397\(01\)](https://doi.org/10.1130/2006.2397(01)), 2006.
- 13 Brodaric, B., Gahegan, M., Harrap, R.: The art and science of mapping: Computing geological categories from field
14 data, *Comp. Geos.*, 30 (7), 719-740, <https://doi.org/10.1016/j.cageo.2004.05.001>, 2004.
- 15 Burch, T.K.: Data, Models, Theory and Reality: The Structure of Demographic Knowledge. In: Billari, F.C.,
16 Prskawetz, A. (eds) *Agent-Based Computational Demography*. Contributions to Economics. Physica,
17 Heidelberg, https://doi.org/10.1007/978-3-7908-2715-6_2, 2003.
- 18 Burns, K.L.: Analysis of geological events, *Mathematical Geology*, 7, 295–321, 1975.
- 19 Burns, K.L., Shepherd, J. and Marshall, B.: Analysis of Relational Data from Metamorphic Tectonites: Derivation of
20 Deformation Sequences from Overprinting Relations, Proceedings of the International Association for
21 Mathematical Geology (IAMG) 25th International Congress in Sydney, Australia, August 1976, In: *Recent
22 Advances in Geomathematics, An International Symposium*, Edited by D.F. Merriam, Syracuse University,
23 *Computers and Geology* Volume 2, 171-199, 1978.
- 24 Burns, K.L. and Remfry, J.G., 1976, A Computer Method of Constructing Geological Histories from Field Surveys
25 and Maps, *Computers and Geosciences*, Vol. 2, 141-162, 1976.
- 26 Burns, K.L., Marshall, B. and Gee, R.D.: Computer-Assisted Geological Mapping, Proceedings of the Australian
27 Institute of Mining and Metallurgy, 323, 41-47, 1969.
- 28 Cherpeau, N., Caumon, G., and Lévy, B.: Stochastic simulations of fault networks in 3-D structural modelling:
29 *Comptes Rendus Geosciences*, 342, 687–694, 2010.
- 30 Claramunt, C. and Jiang, B.: An integrated representation of spatial and temporal relationships between evolving
31 regions, *Journal of geographical systems*, 3, 411–428, 2001.
- 32 Crossley, M. D.: *Essential topology*, Springer-Verlag, London, 226 pp, ISBN 1-85233-782-6, 226, 2005.
- 33 de la Varga, M., Schaaf, A., and Wellmann, F.: GemPy 1.0: opensource stochastic geological modeling and
34 inversion, *Geosci. Model Dev.*, 12, 1–32, <https://doi.org/10.5194/gmd-12-1-2019>, 2019.
- 35 de la Varga, M., Wellmann, J.F.: Structural geologic modeling as an inference problem: A Bayesian perspective,
36 *Interpretation*, 4(3), pp. SM15-SM30, <http://dx.doi.org/10.1190/INT-2015-0188.1>, 2016.
- 37 de Kemp, E.A.: Truth Tables for consistency-checking 3D geological models, Zenodo,
38 <https://doi.org/10.5281/zenodo.10909160>, 2024.
- 39 de Kemp, E. A., Jessell, M. W., Aillères, L., Schetselaar, E. M., Hillier, M., Lindsay, M. D., and Brodaric, B.: Earth
40 model construction in challenging geologic terrain: Designing workflows and algorithms that makes sense, in:
41 *Proceedings of Exploration'17: Sixth DMEC – Decennial International Conference on Mineral Exploration*,



- 1 edited by: Tschirhart, V. and Thomas, M. D., *Integrating the Geosciences: The Challenge of Discovery*,
2 Toronto, Canada, 21–25 October 2017, 419–439, 2017.
- 3 de Kemp, E.A., Schetselaar, E.M., Hillier, M.J., Lydon, J.W. and Ransom, P.W.: Assessing the Workflow for
4 Regional Scale 3D Geological Modelling: An Example from the Sullivan Time Horizon, Purcell
5 Anticlinorium East Kootenay Region, Southeastern British Columbia, *Interpretation*, Special section:
6 Building complex and realistic geologic models from sparse data, 4(3), p. SM33-SM50, 2016.
- 7 de Kemp, E. A., Sprague, K., and Wong, W.: Interpretive Geology with Structural Constraints: An introduction to
8 the SPARSE © plug-in, Americas GOCAD User Meeting, Houston Texas, 1–16,
9 <https://doi.org/10.5281/zenodo.4646210>, 1 November 2004.
- 10 Deutsch, C.V.: All Realizations All the Time, In: Daya Sagar, B., Cheng, Q., Agterberg, F. (eds) *Handbook of*
11 *Mathematical Geosciences*. Springer, Cham. https://doi.org/10.1007/978-3-319-78999-6_7, 2018.
- 12 Dutranois, A., Wan-Chiu, L., Dulac, J-C, Lecomte, J-F, Callot, J-P and Rudkiewicz, J-L: Breakthrough in basin
13 modeling using time/space frame, *OFFSHORE*, Sept. 1, 2010.
- 14 Egenhofer, M.J.: A formal definition of binary topological relationships, In: Litwin, W., Schek, HJ. (eds)
15 *Foundations of Data Organization and Algorithms*, FODO 1989, Lecture Notes in Computer Science,
16 Springer, Berlin, Heidelberg, 367, https://doi.org/10.1007/3-540-51295-0_148, 1989.
- 17 Egenhofer, M., Sharma J. and Mark D.: A critical comparison of the 4-intersection and 9- intersection models for
18 spatial relations: formal analysis, In: R. McMaster and M. Armstrong (eds), *Autocarto 11*, Minneapolis, MN,
19 1-11, 1993.
- 20 Egenhofer, M. and Franzosa, R., 1991. Point-set topological relations. *International Journal of Geographical*
21 *Information Systems* 5 (2): 161-174.
- 22 Fattah, M.: *Physical Geology*, On line course, UDEMY, (<https://www.udemy.com/course/geology-fundamentalz/>,
23 last access: 6 April 2024), 2022.
- 24 Finkl C.W.:Field Geology. In: Finkl C. (eds) *Applied Geology*. *Encyclopedia of Earth Sciences Series*, vol 3.
25 Springer, Boston, MA. https://doi.org/10.1007/0-387-30842-3_21, 1984.
- 26 Fossen H.:*Structural Geology*. Cambridge University Press, New York, 524, 2016.
- 27 Frodeman, R.:Geological reasoning: geology as an interpretive and historical science, *Geol. Soc. Am. Bull.* 107,
28 960e968, 1995.
- 29 Galton, A.: Spatial and temporal knowledge representation, *Earth Science Informatics*, pp. 169–187,
30 <https://doi.org/10.1007/s12145-0090027-6>, 2009.
- 31 Geodes-Solutions, <https://geode-solutions.com/opengeode/>, last access: 6 April 2024).
- 32 Giraud, J., Caumon, G., Grose, L., Ogarko, V., and Cupillard, P.: Integration of automatic implicit geological
33 modelling in deterministic geophysical inversion, *Solid Earth*, 15, 63–89, [https://doi.org/10.5194/se-15-63-](https://doi.org/10.5194/se-15-63-2024)
34 [2024](https://doi.org/10.5194/se-15-63-2024), 2024.
- 35 Giraud, J., Lindsay, M., Jessell, M., and Ogarko, V.: Towards plausible lithological classification from geophysical
36 inversion: honouring geological principles in subsurface imaging, *Solid Earth*, 11, 419–436,
37 <https://doi.org/10.5194/se-11-419-2020>, 2020.
- 38 Grose, L., Ailleres, L., Laurent, G., Armit, R., and Jessell, M.: Inversion of geological knowledge for fold geometry,
39 *J. Struct. Geol.*, 119, 1–14, <https://doi.org/10.1016/j.jsg.2018.11.010>, 2019.
- 40 Gleeson, T., Wagener, T., Döll, P., Zipper, S. C., West, C., Wada, Y., Taylor, R., Scanlon, B., Rosolem, R.,
41 Rahman, S., Oshinlaja, N., Maxwell, R., Lo, M.-H., Kim, H., Hill, M., Hartmann, A., Fogg, G., Famiglietti, J.



- 1 S., Ducharne, A., de Graaf, I., Cuthbert, M., Condon, L., Bresciani, E., and Bierkens, M. F. P.: GMD
2 perspective: The quest to improve the evaluation of groundwater representation in continental- to global-scale
3 models, *Geosci. Model Dev.*, 14, 7545–7571, <https://doi.org/10.5194/gmd-14-7545-2021>, 2021.
- 4 Gong, P. and Mu, L.: Error detection through consistency checking, *Geographic Information Sciences*, 6, 188–193,
5 2000.
- 6 Groshong, R. H. Jr.: 3-D Structural Geology a Practical Guide to Quantitative Surface and Subsurface Map
7 Interpretation, (2nd Edition), Springer-Verlag Berlin Heidelberg., ISBN-13, 978-3540310549, 2006.
- 8 Harrap, R.: A Legend Language for Geologic Maps, Precambrian Times, Geological Association of Canada,
9 Precambrian Division Newsletter, Volume 01, Issue 1, Jan./Feb. 2001, p.1, 3-9, 2001.
- 10 Hillier, M. J., Wellmann, F., Brodaric, B., de Kemp, E. A., and Schetselaar, E.: Three-Dimensional Structural
11 Geological Modeling Using Graph Neural Networks, *Mathematical Geosciences*,
12 <https://doi.org/10.1007/s11004-021-09945-x>, 2021.
- 13 Hillier, M. J., Schetselaar, E. M., de Kemp, E. A., and Perron, G.: Three-dimensional modelling of geological
14 surfaces using generalized interpolation with radial basis functions, *Mathematical Geosciences*, 46, 931–953,
15 2014.
- 16 Hillier, M.J., Schetselaar, E. M., de Kemp, E. A.: SURFE implicit code library repository, (Open Source),
17 <https://github.com/MichaelHillier/surfe>, (last access: 6 April 2024), 2021.
- 18 Hinojosa, J.H. and Mickus, K.L.: Foreland basin-a FORTRAN program to model the formation of foreland basins
19 resulting from the flexural deflection of the lithosphere caused by a time-varying distributed load, *Comp.*
20 *Geosci.*, 19(9), 1321-1332, [https://doi.org/10.1016/0098-3004\(93\)90032-Z](https://doi.org/10.1016/0098-3004(93)90032-Z), 1993.
- 21 Hobbs, B., Regenauer-Lieb, K. and Ord, A.: Thermodynamics of Folding in the Middle to Lower Crust, *Geology*, 35
22 (2), 175-178, <https://doi.org/10.1130/G23188A.1>, 2007.
- 23 Jayr, S., Gringarten, E., Tertois, A.-L., Mallet, J.-L. and Dulac, J.-C.: The need for a correct geological modelling
24 support: The advent of the UVT-transform, *First Break*, 26, 73-79, [https://doi.org/10.3997/1365-
25 2397.26.10.28558](https://doi.org/10.3997/1365-2397.26.10.28558), 2008.
- 26 Jessell, M. W.: Noddy: an interactive map creation package, Unpublished MSc Thesis, University of London, 1981.
- 27 Jessell, M., Ogarko, V., de Rose, Y., Lindsay, M., Joshi, R., Piechocka, A., Grose, L., de la Varga, M., Ailleres, L.,
28 and Pirot, G.: Automated geological map deconstruction for 3D model construction using map2loop 1.0
29 and map2model 1.0, *Geosci. Model Dev.*, 14, 5063–5092, <https://doi.org/10.5194/gmd-14-5063-2021>, 2021.
- 30 Jessell, M. W., Aillères, L. and de Kemp, E. A.: Towards an Integrated Inversion of Geoscientific data: what price
31 of Geology? *Tectonophysics*, 490(3-4), 294-306, 2010.
- 32 Jessell, M., Aillères, L., de Kemp, E.A., Lindsay, M., Wellmann, J., Hillier, M., Laurent, G., Carmichael, T., and
33 Martin, R.: Next generation three-dimensional geologic modeling and inversion, *Society of Economic
34 Geologists Special Publication*, 18, 261–272, 2014.
- 35 Kardel, T. and Paul Maquet, P.: Nicolaus Steno: Biography and Original Papers of a 17th Century Scientist, 2013th
36 (Kindle) Edition, Springer; 2013th edition, 739, ISBN-13 978-3642250781, 2012.
- 37 Lajevardi, S. and Deutsch, C.V.: Stochastic regridding of geological models for flow simulation, *Bulletin of*
38 *Canadian Petroleum Geology*, 63 (4): 374–392, <https://doi.org/10.2113/gscpgbull.63.4.374>, 2015.
- 39 Lajaunie, C., Courrioux, G., Manuel, L.: Foliation fields and 3D cartography in geology: principles of a method
40 based on potential interpolation. *Math Geol* 29(4):571–584, <https://doi.org/10.1007/BF02775087>, 1997.



- 1 Le, H.H., Gabriel, P., Gietzel, J. and Schaeben, H.: An object-relational spatio-temporal geoscience data model,
2 *Comp. Geos.*, 57, 104-115, ISSN 0098-3004, <https://doi.org/10.1016/j.cageo.2013.04.014>, 2013.
- 3 Lindsay, M.D., Aillères, L., Jessell, M.W., de Kemp, E.A., Betts, P.G.: Locating and quantifying geological
4 uncertainty in three-dimensional models: Analysis of the Gippsland Basin, southeastern Australia,
5 *Tectonophysics*, 546-547, 10-27, <https://doi.org/10.1016/j.tecto.2012.04.007>, 2012.
- 6 Lyell, (Sir) C.: *Principles of Geology: Or, the Modern Changes of the Earth and Its Inhabitants, Considered As*
7 *Illustrative of Geology, Volume 1, (Original 1833) 2022.*
- 8 Legare Street Press, ISBN-13 978-1015539976, 496, (original 1833) 2022. Mallet, J-L.: Space-Time Mathematical
9 Framework for Sedimentary Geology. *Math. Geol.*, 36, 1-32,
10 <https://doi.org/10.1023/B:MATG.0000016228.75495.7c>, 2004.
- 11 Mallet, J-L., Jacquemin, P. and Cheimanoff, N.: GOCAD project: Geometric modeling of complex geological
12 surfaces, in: SEG Technical Program Expanded Abstracts, 126-128, <https://doi.org/10.1190/1.1889515>, 1989.
- 13 McKay, G. and Harris, J.R.: Comparison of the Data-Driven Random Forests Model and a Knowledge-Driven
14 Method for Mineral Prospectivity Mapping: A Case Study for Gold Deposits Around the Huritz Group and
15 Nueltin Suite, Nunavut, Canada. *Nat. Resour. Res.*, 25, 125-143. <https://doi.org/10.1007/s11053-015-9274-z>,
16 2016.
- 17 McMechan, M.E., Root, K.G., Simony, P.S. and Pattison, D.R.M.: Nailed to the craton: Stratigraphic continuity
18 across the southeastern Canadian Cordillera with tectonic implications for ribbon continent models, *Geology*,
19 49 (1): 101-105, <https://doi.org/10.1130/G48060>, 2021.
- 20 Melnikova, Y., Cordua, K. S., & Mosegaard, K. (2012). History Matching: Towards Geologically Reasonable
21 Models. Abstract from EAGE Integrated Reservoir Modelling: Are we doing it right?, Dubai, United Arab
22 Emirates.
- 23 Michalak, J.: Topological conceptual model of geological relative time scale for geoinformation systems, *Comp.*
24 *Geos.*, 31-7, 865-876, ISSN 0098-3004, <https://doi.org/10.1016/j.cageo.2005.03.001>, 2005.
- 25 Morley, C.K.: Out-of-sequence thrusts, *Tectonics*, 7 (3), 539-561, 1988.
- 26 New South Wales (NSW), Australia, Department of Primary Industries (DPI), History of Geology, February 2007,
27 Primefacts 563, 6,
28 ([https://digs.geoscience.nsw.gov.au/api/download/c6ae94e9dc65f614492646c269ea3731/Primefact_563_Minif](https://digs.geoscience.nsw.gov.au/api/download/c6ae94e9dc65f614492646c269ea3731/Primefact_563_Minifact_60_History_of_geology.pdf)
29 [act_60_History_of_geology.pdf](https://digs.geoscience.nsw.gov.au/api/download/c6ae94e9dc65f614492646c269ea3731/Primefact_563_Minifact_60_History_of_geology.pdf), last access: 20 April 2022).
- 30 Nikoohemat, S., Diakit , A. A., Lehtola, V., Zlatanova, S., and Vosselman, G.: Consistency grammar for 3D indoor
31 model checking, *Transactions in GIS*, pp. 189-212, <https://doi.org/10.1111/tgis.12686>, 2021.
- 32 Pellerin, J., Caumon, G., Julio, C., Mejia-Herrera, P., Botella, A.: Elements for measuring the complexity of 3D
33 structural models: Connectivity and geometry, *Computers & Geosciences*, 76, 130-140, ISSN 0098-3004,
34 <https://doi.org/10.1016/j.cageo.2015.01.002>, 2015.
- 35 Pellerin, J., Botella, A., Bonneau, F., Mazuyer, A., Chauvin, B., L vy, B., Caumon, G.: RINGMesh: A programming
36 library for developing mesh-based geomodeling applications, *Comp. Geos.*, 104, 93-100, ISSN 0098-3004,
37 <https://doi.org/10.1016/j.cageo.2017.03.005>, 2017.
- 38 Perrin, M., Poudret, M., Guiard, N. and Schneider, S.: Chapter 6: Geological Surface Assemblage, In: *Shared Earth*
39 *Modeling, Knowledge driven solutions for building and managing subsurface 3D geological models*, ifp
40 *Energies Nouvelles Publications - TECHNIP, Paris, France*, 115-139, 2013.
- 41 Perrin, M., Morel, O., Mastella, L., and Alexandre, L.: Geological Time Formalization: an improved formal model
42 for describing time successions and their correlation, *Earth Science Informatics*, pp. 81-96,
43 <https://doi.org/10.1007/s12145-011-0080-9>, 2011.



- 1 Pyrcz, M. J., Sech, R. P., Covault, J. A., Willis, B. J., Sylvester, Z., Sun, T., and Garner, D.: Stratigraphic rule-based
2 reservoir modeling, *Bulletin of Canadian Petroleum Geology*, 63, 287–303, 2015.
- 3 Ranalli, G.: A stochastic model for strike-slip faulting. *Math. Geol.*, 12, 399–412,
4 <https://doi.org/10.1007/BF01029423>, 1980.
- 5 Rauch, A., Sartori, M., Rossi, E., Baland, P. and Castellort, S.: Trace Information Extraction (TIE): A new
6 approach to extract structural information from traces in geological maps, *Jour. Struct. Geol.*, 126, 286–300,
7 ISSN 0191-8141, <https://doi.org/10.1016/j.jsg.2019.06.007>, 2019.
- 8 Rothery, D.: *Geology: A Complete Introduction*, Quercus; 1st edition (Feb. 16 2016), ISBN-13, 978-1473601550,
9 384, 2016.
- 10 Schetselaar, E. M. and de Kemp, E. A.: Topological encoding of spatial relationships to support geological
11 modelling in a 3-D GIS environment, *Int. Assoc. for Mathematical Geology XIth International Congress*,
12 *Université de Liège - Belgium*, 2006.
- 13 Shokouhi, P., Kumar, V., Prathipati, S., Hosseini, S.A., Giles, C.L., and Kifer, D.: Physics-informed deep learning
14 for prediction of CO₂ storage site response, *Journal of Contaminant Hydrology*, 241, 103835, ISSN 0169-
15 7722, <https://doi.org/10.1016/j.jconhyd.2021.103835>, 2021.
- 16 Snyder, D. B., Hillier, M. J., Kjarsgaard, B. A. K., de Kemp, E. A. & Craven, J. A.: Lithospheric architecture of the
17 Slave craton, northwest Canada, as determined from an interdisciplinary 3-D model. *Geochemistry*,
18 *Geophysics, Geosystems (G3)*, 15, 1-16. <https://doi.org/10.1002/2013GC005168>, 2014.
- 19 Thapa, P. and McMechan, M.E.: Methodology for portraying 3D structure using ArcGIS: a test case from the
20 southern Canadian Rocky Mountains, British Columbia and Alberta, *Geological Survey of Canada, Open File*
21 8576, 17 p, 2019.
- 22 Thiele, S. T., Jessel, M. W., Lindsay, M., Ogarko, V., Wellmann, J. F., and Pakyuz-Charrier, E.: The topology of
23 geology 1: Topological analysis, *Jour. Struct. Geol.*, 91, 27-38, <https://doi.org/10.1016/j.jsg.2016.08.009>,
24 2016a.
- 25 Thiele, S.T., Jessel, M.W., Lindsay, M., Wellmann, J.F. and Pakyuz-Charrier, E.: The topology of geology 2:
26 Topological uncertainty, *Jour. Struct. Geol.*, 91, 74-87, ISSN 0191-8141,
27 <https://doi.org/10.1016/j.jsg.2016.08.010>, 2016b.
- 28 van Giffen, B. Herhausen, D., Fahse, T.: Overcoming the pitfalls and perils of algorithms: A classification of
29 machine learning biases and mitigation methods, *Journal of Business Research*, 144, 93-106, ISSN 0148-2963,
30 <https://doi.org/10.1016/j.jbusres.2022.01.076>, 2022.
- 31 Van Oosterom, P.: Maintaining consistent topology including historical data in a large spatial database, 13, 327–
32 336, 1997.
- 33 Wellmann, F. and Caumon, G.: 3-D Structural geological models: Concepts, methods, and uncertainties, *Advances*
34 *in Geophysics*, pp. 1–121, <https://doi.org/10.1016/bs.agph.2018.09.001>, 2018.
- 35 Zhan, X., Lu, C. and Hu, G.: A Formal Representation of the Semantics of Structural Geological Models, *Scientific*
36 *Programming*, 2022, 5553774, <https://doi.org/10.1155/2022/5553774>, 2022.
- 37 Zhan, X., Liang, J., Lu, C., and Hu, G.: Semantic Description and Complete Computer Characterization of Structural
38 Geological Models, *Geoscientific Model Development Discussions*, 1–39, 2019.
- 39 Ziggelaar, A.: The age of Earth in Niels Stensen's geology, *Memoir of the Geological Society of America*
40 ISSN:0072-1069, 203, 135-142, 2009.
- 41 Zlatanova, S., Rahman, A. A., and Shi, W.: Topological models and frameworks for 3D spatial objects, *Comp.*
42 *Geos*, 30, 419–428, 2004.