Evaluating adsorption isotherm models for determining the partitioning of ammonium between soil and soil-pore water in environmental soil samples
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Abstract: Ammonium in soil pore water is thought to participate in bidirectional exchange with the atmosphere; however, common soil nutrient analysis methods determine the bulk quantity of ammonium associated with the soil particles, rather than determining the aqueous ammonium concentration. Previous works have applied the Langmuir and Freundlich isotherm equations to ammonium-enriched soils to estimate partitioning, but this may not be representative of conditions in natural, unmanaged soils. In this work, environmental soil samples were collected from greenspaces in Toronto and used to evaluate several commonly used adsorption isotherm equations, including the Langmuir, Freundlich, Temkin and Toth equations, to determine their applicability in lightly managed and non-fertilized soils. We then compare ammonia emission potentials (a quantity predicting the propensity of ammonia to volatilize from a liquid reservoir) calculated using a conventional nutrient analysis method to that modelled using the Temkin and Langmuir equations, and demonstrate that conventional approaches may overestimate emission potentials from soils by a factor of 5 – 12.
Key words: ammonium soil adsorption; emission potential;

1. Introduction:

1.1. Contextualizing the significance of ammonium partitioning in soils

Globally, emissions of reduced nitrogen compounds (NH₃) make up as much as half of the global atmospheric reactive nitrogen sources (Flechard et al., 2013). Of the NH₃ budget, approximately two thirds of emissions are related to anthropogenic agricultural activities, with natural sources being responsible for only 15 – 20% of emissions (Bouwman et al., 1997; Sutton et al., 2008, 2013; Van Aardenne et al., 2001). Following emission, NH₃ tends to partition into fine particulate matter or deposit via wet or dry deposition on a timescale of hours to days. NH₃ is understood to engage in bidirectional exchange throughout the earth system, with NH₃ depositing, or volatilizing depending on local environmental conditions (Farquhar et al., 1980; Flechard et al., 1999, 2013; Nemitz et al., 2000; Sutton et al., 1995). The bidirectional exchange of NH₃ between soils and the atmosphere has been considered important to its overall budget, particularly in remote areas, but research on the mechanics of NH₃ partitioning in soils between adsorbed (inaccessible) and aqueous (accessible) NH₃ often focuses on fertilized croplands.
with substantial concentrations of ammonia present (Venterea et al., 2015; Vogeler et al., 2011). However, cropland is estimated to make up less than 15% of the Earth’s land area, while unfertilized or irregularly fertilized natural, semi-natural or pastoral land reflects nearly three quarters of terrestrial surfaces (Ellis et al., 2020). The short atmospheric lifetime of NH₃ makes it important to understand the exchange of NH₃ over all types of surfaces, despite agricultural cropland being the most globally significant source.

1.2. Importance of developing an ammonium adsorption partitioning model

Ammonium in soils is thought to be partitioned between NH₄⁺(ads) adsorbed to soil particles and NH₄⁺(aq) dissolved in soil pore water. Because soils tend to exist with both permanent and pH-dependent negative charges (Bache, 1976), NH₄⁺ ions compete for adsorption against other cations in solution. The total quantity of cations that can adsorb to the soil particles is termed the cation exchange capacity (CEC, typically reported as centimoles positive charge/kg). Traditional methods for determining ammonium in soil—intended for nutrient analysis in which the total ammonia nitrogen is more important than the partitioning between NH₄⁺(ads) and NH₄⁺(aq)—use high concentration salt solutions to displace all cations from the soil. As a result, these approaches do not distinguish between NH₄⁺(aq) and NH₄⁺(ads) (Li et al., 2012) and therefore likely overestimate the emission potential of soils. In this manuscript, we explore a variety of adsorption isotherm models with the goal of identifying a simple approach to relate the quantity of ammonium in a soil sample able to participate in bidirectional exchange with the total amount of ammonium, as a function of other readily measurable quantities.

1.3. Adsorption Isotherm equations:

The adsorption behavior of molecules to surfaces is complex and dependent on the properties of both the surface and the adsorbed molecules. Numerous adsorption equations have been proposed, based on both theoretical and empirical models; however, as the partitioning of ammonium between soil pore water and soil particles is complex and influenced by many other simultaneous equilibria, we consider each of the examined equations as being empirically determined only, rather than based on a theoretical treatment of the system. Previous studies have sought to develop an adsorption model for NH₄⁺ partitioning soils based on the Langmuir (Alnsour, 2020; Venterea et al., 2015) and Freundlich equations (Vogeler et al., 2011), but have focused on croplands and other agricultural soils. In this work, in addition to the Langmuir and Freundlich equations, we also investigate the application of the Temkin and Toth adsorption equations to NH₄⁺ partitioning in soils. Each of these equations (as formulated in Table 1) represents the adsorbed NH₄⁺ concentration (S, mg kg⁻¹) in terms of the NH₄⁺ concentration in solution (C, mg L⁻¹). Except for the Freundlich equation, each of these equations incorporate a saturation point or maximum adsorption capacity (S_max, mg kg⁻¹), which this work treats as an experimentally determined value equivalent to the CEC, converted to mg of NH₄⁺ kg⁻¹ soil.

**Table 1: Adsorption isotherm equations applied in this work and their parameters**

<table>
<thead>
<tr>
<th>Adsorption Isotherm Model</th>
<th>Equation</th>
<th>Units</th>
<th>Reference</th>
</tr>
</thead>
</table>


Langmuir  
(1) \[ S = \frac{S_{\text{max}}K_{L}C}{1 + K_{L}C} \]  
\( S_{\text{max}} \) (mg kg\(^{-1}\)),  
\( K_{L} \) (L kg\(^{-1}\))  

Freundlich  
(2) \[ S = k_{F}C^{n_{F}} \]  
\( k_{F} \),  
\( n_{F} \) (dimensionless)  
(Freundlich, 1909)

Temkin  
(3) \[ S = q_{T}\ln(1 + K_{T}C) \]  
\( q_{T} \) (product of \( S_{\text{max}} \) (mg kg\(^{-1}\)) and  
\( R_{T} \) (dimensionless)),  
\( K_{T} \) (L mg\(^{-1}\))  
(Temkin & Pyzhev, 1940)

Toth  
(4) \[ S = \frac{bC}{(K_{T} + C)^{\frac{1}{n_{T}}}} \]  
\( b \) (product of \( S_{\text{max}} \) (mg kg\(^{-1}\)) and a  
dimensionless scaling factor),  
\( K_{T} \) (mg L\(^{-1}\)),  
\( n_{T} \) (dimensionless)  
(Tóth, 1995)

a The Temkin model is also given as \( S = \frac{b}{n_{T}} \ln(K_{T}C) \), see (Chu, 2021) for this formulation

2. Methods:

2.1. Soil Collection:

Soil samples were collected from several greenspace (parks, urban forest, roadside-sites, etc.) locations across Toronto. Samples were collected only on days preceded by at least two days without precipitation and were collected by inserting a 7.5 cm internal diameter steel tube into the ground to a depth of 5 – 10 cm and recovering a soil core by removing the tube from the ground. The soil cores were mixed, and sieved immediately, and transported back to the lab for analysis. Samples analyzed for NH\(_3\) content were always analyzed immediately to avoid potential artefacts from freezing, samples analyzed for CEC or adsorption curves were frozen for storage prior to analysis. Soil samples were collected from eight locations in Fall 2021 as a training set for developing the model. A subsequent 16 soil samples from across Toronto were collected in Spring/Summer 2023 to be used to evaluate the effectiveness of applying the model to uncharacterized soils, and to determine the impact on soil emission potentials.

2.2. Soil analysis:

2.2.1. Cation Exchange Capacity determination

The CEC was determined using the inductively-coupled plasma optical emission spectroscopy (ICP-OES) cation sum method (Bache, 1976). Briefly, 1 g of soil was measured out and mixed with 25 mL of 1 M NH\(_4\)Cl, shaken, and refrigerated for 36 - 48 hours to settle. The supernatant was filtered using 0.2 um syringe filters, diluted 50-fold using volumetric glassware and analyzed for the common soil-associated cations, Na\(^{+}\), K\(^{+}\), Mg\(^{2+}\), and Ca\(^{2+}\). The ICP-OES was calibrated using a commercially available mixed standard of 6 cations (Li\(^{+}\), Na\(^{+}\), NH\(_4^{+}\), K\(^{+}\), Mg\(^{2+}\), Ca\(^{2+}\)) (Thermo Fisher Scientific). Another soil-associated cation that can contribute to CEC is Al\(^{3+}\), our initial measurements screened for Al, but we did not detect it in solution. The CEC was determined for all 24 collected samples and used to select three of the soil samples from the 16 collected in 2023 to be used as a test set for the developed model.
2.2.2. Adsorption curve characterization

The determination of the adsorption behavior of NH$_4^+$ was performed using an approach similar to that described by Venterea et al. (2015). Briefly, a series of batch equilibrium samples were prepared by mixing 1 g of soil with 25 mL aliquots of NH$_4$Cl solutions with concentrations ranging from 2.5 – 1000 mM. The samples were shaken, refrigerated for 36 – 48 hours to settle, filtered with 0.2 um syringe filters, diluted 50-fold with volumetric glassware, and analyzed using ICP-OES. The quantity of NH$_4^+$ adsorbed onto each soil was then inferred based on the displaced Na$^+$, Mg$^{2+}$, Ca$^{2+}$ and K$^+$ ions measured in solution. Adsorption curves were determined for the original eight soil samples in the training set, as well as the three selected soil samples for the test set.

2.2.3. pH and NH$_4^+$ determination

pH and NH$_4^+$ content was determined for the 16 soil samples collected in 2023. pH was determined for each soil sample using a typical approach of mixing soil with DIW in a 1:1 ratio, and then measuring the pH of the slurry by immersing a pH electrode (Hach) until a stable pH reading was obtained. To determine the pH using an alternative extraction solution, this process was repeated using a 0.01 M CaCl$_2$ solution.

NH$_4^+$ was extracted from the soil using a batch equilibrium method using both a 2 M KCl extraction solution and a DIW solution. In both cases, 2.5 g of soil was mixed with 25 mL of the extraction solution, shaken, and refrigerated for 36 hours to allow suspended solids to settle out of solution. Afterward, the supernatant was filtered using 0.2 µm PES membrane syringe filters. The soil NH$_4^+$ was quantified using the indophenol-blue salicylate method (Kempers & Zweers, 1986). Briefly, two reagent solutions, A and B were prepared: Reagent A consisted of a solution of 1 M sodium salicylate and 100 mg L$^{-1}$ sodium nitroprusside, while Reagent B consisted of a solution of 1 M NaOH and 0.12% by volume of 5% available chloride NaOCl. Soil extracts were prepared for analysis by adding 0.6 mL of reagent A to 2 mL of soil extract, followed by the addition of 1.4 mL of reagent B. The mixtures were then stored for 2 hours in the dark for color development, and then quantified using UV-VIS spectrometry (Lambda 365, Perkin-Elmer) at 649 nm.

2.3. Data analysis:

2.3.1. Characterizing adsorption parameters using a training and test set

Curve-fitting was done in R using the nls function to fit our experimental data for the eight samples in our training set to the Langmuir, Freundlich, Temkin, and Toth equations. As $R^2$ is a poor criterion for non-linear models (Spiess & Neumeyer, 2010), the residual standard error (RSE) was used to evaluate the goodness of fit, where the RSE is the square root of the sum of the squared residuals divided by the degrees of freedom, expressed as a percentage of the geometric mean response variable (Eq. (5)).

$$RSE \% = \sqrt{\frac{\Sigma(y-y)^2}{\exp\left(-\Sigma \ln(y)\right)}}$$
Average fitting parameters for each equation were determined in two ways: 1) by fitting each equation to the adsorption curve of each sample, and averaging the resulting fitting parameters, and 2) by pooling all the experimental data, standardizing each adsorption curve by the maximum adsorption achieved (i.e., all curves went from 0 to 1), and then fitting each equation from Table 1 to those curves.

To validate the effectiveness of these equations when applied to uncharacterized soil samples, we selected three soil samples from the 16 soil samples collected in 2023 to form a test set, these samples were selected by choosing the soil samples with the lowest (10.9) and highest CECs (37.2), and a soil with an average CEC (25.3). The test set was characterized in the same way as the training set, and was then modelled using three approaches:

i. Using the average CEC for all the soil samples of 25 cmole kg\(^{-1}\) (\(S_{\text{max}}\) of 4500 mg kg\(^{-1}\)), and the training set parameters.

ii. Using the measured CEC for each soil sample to calculate \(S_{\text{max}}\), and the training set parameters.

iii. Using the fitting algorithm as described above to determine the least squares fit for each equation to the experimental data.

### 2.3.2. Emission potential determination

The emission potential of ammonia is a quantity calculated as the ratio of aqueous NH\(_4^+\) to H\(^+\) (Eq. (6)).

\[
(6) \quad \Gamma = \frac{[\text{NH}_4^+]}{[\text{H}^+]} \tag{6}
\]

Commonly, for soils this would be calculated using the total NH\(_4^+\) (determined using a salt solution extraction) and the pH measured using an extraction with deionized water. We denote this as \(\Gamma_{\text{STD}}\), corresponding to \(\frac{[\text{NH}_4^+]_{\text{salt}}}{[\text{H}^+]_{\text{DIW}}}\). The pH may also be measured in a (less concentrated) salt solution, which we denote as \(\Gamma_{\text{SALT}}\), corresponding to \(\frac{[\text{NH}_4^+]_{\text{salt}}}{[\text{H}^+]_{\text{salt}}}\). Similarly, though we are not aware of this as a common method, NH\(_4^+\) could be determined using a DIW extraction solution, resulting in a third parameterization of the emission potential as \(\Gamma_{\text{DIW}}\), representing \(\frac{[\text{NH}_4^+]_{\text{DIW}}}{[\text{H}^+]_{\text{DIW}}}\). Lastly, by applying one of the adsorption isotherm models, the total soil NH\(_4^+\) can be partitioned into NH\(_4^+\)\(_{\text{ads}}\) (S) and NH\(_4^+\)\(_{\text{aq}}\) (C), and the emission potential can be calculated using only the NH\(_4^+\) in solution (C). These versions of the emission potential are denoted as \(\Gamma\)-sub-equation-name (e.g., as \(\Gamma_{\text{Langmuir}}, \Gamma_{\text{Temkin}}\) etc).

### 3. Results:

#### 3.1. Performance of adsorption isotherm equations applied to an environmental soil training and test set:

We evaluated the ability of the Langmuir, Freundlich, Temkin and Toth equations to model the partitioning of NH\(_4^+\) between adsorbed and aqueous forms. We did this both by fitting the adsorption curve for each soil separately, and by standardizing each adsorption curve to its maximum adsorption, and then pooling the data from each soil adsorption experiment and fitting.
the resulting data. The adsorption curves and fitting parameters for the pooled data approach are shown in Fig. 1 and Table 2, the equivalents for the separate fit approach are shown in Fig. A1 and Table A1. The fitting parameters determined with these two approaches are approximately equivalent to one another and could be used interchangeably (with the exception of the Toth equation parameters, which will be discussed further below).

While the Langmuir (Alnsour, 2020; Guo et al., 2022; Venterea et al., 2015) and Freundlich (Vogeler et al., 2011) equations have been previously reported as being effective at modelling NH₄⁺ partitioning in soils, we found them to be the least effective of the equations we examined, both over- and under-estimating the adsorbed NH₄⁺ concentrations (Fig. 1) with consistently high residuals (Table 2: 24% RSE (Langmuir), 15% RSE (Freundlich)). The Temkin and Toth equations better fit the experimental data, and have smaller RSE’s of 12% (Temkin) and 9.0% (Toth). The Toth and Temkin equations have even lower residuals for the individually fit curves (A1, A2), 3.0 – 16.1% (Temkin), and 0.9 – 6.5% (Toth).

![Figure 1: Curve-fitting comparison between the Langmuir (red, solid), Freundlich (blue, – – –), Temkin (green, ‧ ‧ ‧), and Toth (orange, ‧ ‧ ‧) equations. The curves are plotted using the experimental data from all eight soil adsorption experiments, standardizing the y-axis by the maximum adsorption achieved during each experiment.](image)

**Figure 1: Curve-fitting comparison between the Langmuir (red, solid), Freundlich (blue, – – –), Temkin (green, ‧ ‧ ‧), and Toth (orange, ‧ ‧ ‧) equations. The curves are plotted using the experimental data from all eight soil adsorption experiments, standardizing the y-axis by the maximum adsorption achieved during each experiment.**

| Table 2: Comparison of residual standard errors and fitting parameters of the pooled fitting parameters |
|---|---|---|---|---|---|---|
| Equation | Number of fitted parameters | RSE | Parameter 1 | Mean ± Standard error | Parameter 2 | Mean ± SE | Exponential Factor |

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With our objective being to evaluate how well each equation can fit soils without going through the full characterization procedure, we analyzed the adsorption curves of the low-CEC, medium-CEC and high-CEC soils in our test set using: i) a “typical” CEC of 25, and the fitting parameters from Table 2, ii) the measured CEC and the fitting parameters from Table 2, and iii) by fitting the equations using the least-squares fitting algorithm. Using the first approach, we found that none of the equations could reasonably fit the experimental data when using an incorrect CEC and that each of the equations fit the experimental data reasonably well using the average parameters and the correct CEC (Fig. 2). The average RSE under these conditions were 31% (Langmuir), 21% (Freundlich), 21% (Temkin), and 14% (Toth) (a full summary is given in Table A2). However, repeating this analysis using the average parameters from the separately fit equations (Table A1) results in an average RSE (for ii) of 31% (Langmuir), 28% (Freundlich), 25% (Temkin), and 36% (Toth).

As was noted above, the Toth equation was the only equation for which the two approaches of calculating average fitting parameters resulted in significantly different parameters. Most likely the fitting algorithm is over-fitting the data due to the Toth equation having an additional parameter, making the Toth equation unreliable to generalize to uncharacterized soils. Additionally, while the Freundlich equation performs reasonably in this analysis using the pooled data fitting parameters, pooling the data from individual adsorption curves created an implicit dependence of $K_F$ on $S_{max}$ (i.e. the saturation of adsorption at high concentrations), which is inconsistent with the behaviour of the Freundlich equation, which does not saturate at high concentrations. However, without a dependence on $S_{max}$, the Freundlich equation performs quite poorly at modelling uncharacterized soils. Consequently, we find that the Langmuir and Temkin equations are most suitable for use evaluating NH$_4^+$ adsorption from uncharacterized soil samples.
3.2. Determining aqueous NH$_4^+$ concentrations using the Langmuir and Temkin equations

To relate the total ammonium measured ($m_{NH4}$) to the adsorbed portion (S) and the aqueous portion (C), we define Eq. (7):

$$m_{NH4} = S + \frac{wC}{\rho}$$

Where $m_{NH4}$ is the total mass of ammonium per kg soil (mg kg$^{-1}$), $w$ the volumetric moisture content of the soil (L water L$^{-1}$ soil), and $\rho$ is the bulk density of the dry soil (kg L$^{-1}$). By substituting the Langmuir (1) or Temkin (3) equations into Eq. (7), $m_{NH4}$ can be expressed in terms of C:

$$m_{NH4} = \frac{s_{max}k_Lc}{1+kLC} + \frac{wC}{\rho}$$

$$m_{NH4} = qT\ln(1 + K_TC) + \frac{wC}{\rho}$$

To solve for C, we inverted these equations using Wolfram Mathematica, yielding Eq. (10) (Langmuir) and Eq. (11) (Temkin).

$$C = \frac{-s_{max}k_L(1+kLC-m_{NH4})^2}{\sqrt{4k_LZs_{max}m_{NH4}(1+kLC)}}$$
(11) \[ C = \frac{-Z + K_T q_T W \left( e^{\frac{Z}{K_T q_T}} - \frac{m_{NH4}}{K_T q_T} Z \right)}{K_T Z}, \text{ where } W(x) \text{ is the Lambert W function, and } Z = \frac{w}{\rho} \]

If it is assumed that \( m_{NH4} \approx S \), then \( \frac{wC}{\rho} \approx 0 \), and these equations can be simplified to Eq. (12) and Eq. (13) respectively.

(12) \[ C = \frac{m_{NH4}}{K_L (S_{max} - m_{NH4})} \]

(13) \[ C = \frac{e^{-\frac{m_{NH4}}{K_T q_T} - 1}}{K_T} \]

We tested this assumption and found that for our unfertilized soil samples, the simplified equations have a positive bias of only 0.57 – 1.5%, and that for most soils fertilized with the equivalent of up to 300 kg N ha\(^{-1}\), the simplified equations should have a positive bias of less than 5%. Consequently, we used the simplified equations for our analysis.

### 3.3. Determining environmental soil emission potentials

Soil emission potentials were then calculated using the approaches described in Sect. 2.3.3, corresponding to the ‘standard’ approach, two approaches based on matching the \( NH_4^+ \) and pH extraction solutions, and two approaches based on applying the Langmuir and Temkin equations to the measured extract concentrations and pH values. For the Langmuir calculations, the fit parameters were \( K_L = 9.29 \times 10^{-4} \text{ L kg}^{-1} \), and \( S_{max} \) calculated as the equivalent of the CEC in units of mg kg\(^{-1}\). For the Temkin calculations, the fit parameters were \( K_T = 1.33 \times 10^{-2} \text{ L mg}^{-1} \), and \( q_T \) (mg kg\(^{-1}\)) = 0.180 \( S_{max} \). Among the approaches, \( \Gamma_{STD} \) results in the highest estimate for the emission potential (17,000 ± 19,000), followed by \( \Gamma_{Langmuir} \) and \( \Gamma_{SALT} \) (3300 – 3530), and \( \Gamma_{Temkin} \) and \( \Gamma_{DIW} \) (1370 – 1450) (Table 3). Emission potential is linearly related to equilibrium vapor pressure, at a temperature of 15°C, these emission potentials correspond to equilibrium concentrations of 4 – 50 ppb. Although CEC is inversely related to the proportion of \( NH_4^+ \) present in the aqueous phase, the emission potentials increase exponentially as a function of the CEC due to the strong positive correlation between soil pH and CEC (Fig. 3).

### Table 3: Comparison of emission potentials determined using the standard, Langmuir, Temkin, salt extraction and DIW extraction methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Equation</th>
<th>Average Emission Potential (mol mol(^{-1})) (( n = 16 ))</th>
<th>Equilibrium vapor pressure at 15°C (ppb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma_{STD} )</td>
<td>[ [NH_4^+]<em>{salt} \left[ H^+ \right]</em>{DIW} ]</td>
<td>17,000 ± 19,000</td>
<td>50 ± 55</td>
</tr>
<tr>
<td>( \Gamma_{SALT} )</td>
<td>[ [NH_4^+]<em>{salt} \left[ H^+ \right]</em>{salt} ]</td>
<td>3300 ± 3100</td>
<td>9.6 ± 9.0</td>
</tr>
<tr>
<td>( \Gamma_{DIW} )</td>
<td>[ [NH_4^+]<em>{DIW} \left[ H^+ \right]</em>{DIW} ]</td>
<td>1450 ± 1900</td>
<td>4.2 ± 5.5</td>
</tr>
<tr>
<td>( \Gamma_{Temkin} )</td>
<td>[ \frac{[NH_4^+]<em>{salt}}{k_L (S</em>{max} - [NH_4^+]<em>{salt})} \left[ H^+ \right]</em>{DIW} ]</td>
<td>1370 ± 1300</td>
<td>4.0 ± 3.8</td>
</tr>
</tbody>
</table>
Figure 3: Soil emission potentials using the standard, salt extraction, Temkin, Langmuir and DIW extraction approaches, arranged by CEC.

4. Discussion

The conventional approach for calculating emission potentials in soils lacks a solid theoretical foundation, and ought to be applied with caution. Empirically, this approach has performed poorly, with several studies reporting that calculated values of $\Gamma_{STD}$ are unrealistically high and do not correspond well to measured or modelled emission fluxes (Cooter et al., 2010; Flechard et al., 2013; Neftel et al., 1998; Nemitz et al., 2001; Xu et al., 2022). Our assessment that the conventional method overestimates the soil emission potential by at least a factor of 5 is similar to findings reported by Nemitz et al., (2001) and Cooter et al.,...
(2010) who reported needing to reduce their soil emission potentials by a factor of 6.66 and 2 – 3 respectively for their modelled predictions to match their flux observations. We believe that our empirical treatment of this system with the Langmuir and Temkin equations provides a more reasonable approach to estimate the soil emission potential. Additionally, in our view, the ‘like-with-like’ extraction approach (e.g. $\Gamma_{SALT}$ or $\Gamma_{DIW}$) has a more sound theoretical basis than the conventional approach involving dissimilar extraction solutions. A few recent studies have reported on a similar approach using a Langmuir adsorption model (Alnsour, 2020; Guo et al., 2022), as well as by matching the $\text{NH}_4^+$ and pH extraction solutions (Wu et al., 2023), showing that these methods can be feasibly implemented into an environmental sampling campaign. Venterea et al., (2015) reported that soil ammonium partitioning could be effectively modelled using a modified version of the Langmuir isotherm equation of the form $S = \frac{S_{\text{max}}C}{K + C}$, where $K$ (mg L$^{-1}$) is the aqueous concentration at which $S = \frac{1}{2} S_{\text{max}}$. The Venterea equation is equivalent to the Langmuir isotherm when $K = \frac{1}{K_L}$; for the parameters determined in this study, $\frac{1}{2}$ the saturation capacity is reached at exactly $C = \frac{1}{K_L} = K$, indicating that for our analysis the Langmuir and Venterea equations are equivalent.

How does this approach compare to the implementation of bidirectional exchange of $\text{NH}_3$ in chemical transport models? Widely used models such as GEOS-CHEM and the EPA’s CMAQ model soil emission potentials mechanistically, rather than being based on emission potentials prescribed using land-use categories (Bash et al., 2013; Pleim et al., 2013; Pleim, Ran, Appel, Shephard, et al., 2019; Zhu et al., 2015). In both models, the exchangeable soil ammonia has been parameterized as the volumetric molar concentration of ammonia in the top 1 – 5 cm divided by the volumetric water content in the soil, or $[\text{NH}_4] = \frac{NH_4 \text{ mol m}^{-3}}{\theta \text{ m}^3 \text{m}^{-3}}$ (Massad et al., 2010). In this parameterization, the entire soil ammonia content of the top layer of soil is treated as being dissolved into the soil water, thus a lower water content would result in a higher concentration of ammonia, and consequently a higher emission potential. (The parameter used by these models for the resistance to ammonia emissions from soil is inversely proportional to soil moisture, such that the actual emission of ammonia from soil would still be proportional to soil water content (Pleim et al., 2013).) In our model of the system, most ammonium is present adsorbed to soils, with the aqueous concentration controlled by the equilibrium between adsorbed and aqueous ammonium, increasing the soil water content would thus allow more ammonium to enter the aqueous phase to maintain the equilibrium concentration. (Increased water content could also result in nutrient runoff, reducing the soil ammonium content, or increased mineralization, increasing the soil ammonium content.) In the subsequent development of the bidirectional model coupled with an agricultural ecosystem model, Pleim et al., (2019) noted that reducing the ammonia available for emission by implementing the Langmuir adsorption isotherm approach proposed by Venterea et al. (2015) appeared to lead to unexpectedly low fractions of ammonia available for exchange. It may be that for recently fertilized soils, the applied fertilizer is not in contact with sufficient soil for this equilibrium relation to apply; and more research may be needed to understand the limitations and applications of both approaches. An updated version of CMAQ (v5.2.1) uses a Langmuir-derived approach described in Venterea et al. (2015) to predict $\text{NH}_3$ bi-directional exchange and Kelly et al., (2019) explores its indirect impact on PM2.5 composition across the U.S.
5. Conclusion

This work evaluated the Langmuir, Freundlich, Temkin, and Toth adsorption isotherm equations as applied to environmental soil samples and NH$_4^+$ partitioning. We determined that the Toth equation was the most effective of these equations at fitting soil adsorption curves, but that the Langmuir and Temkin equations were more effective at predicting the adsorption behaviour of soil samples when only the soil CEC was characterized. Applying this method to a series of environmental soil samples, we determined that the conventional method for directly measuring soil emission potentials may overestimate them by a factor of 5 – 12 (relative to using the Langmuir/Temkin equations respectively). Comparing these empirical equations with an alternative approach for determining emission potential based on extracting NH$_4^+$ and pH with the same extraction solutions (i.e. DIW/DIW or Salt/Salt) showed that the adsorption equation-based approach is in good agreement with the equivalent-ratio based method. Of these four alternative methods, we suggest that the Temkin equation performs better at modelling the NH$_4^+$ partitioning between soil and solution than the Langmuir equation.

Appendix A

Figure A1: Curve-fitting comparison between the Langmuir (red, solid), Freundlich (blue, – – –), Temkin (green, ‧ ‧ ‧), and Toth (orange, ‧ ‧ ‧) equations for soil samples collected from a) King’s College Circle, b) the University of Toronto Scarborough forest, c) High Park, and d) Queen’s Park.
Table A1: Comparison of the residual standard errors, and the range of fitted parameters for adsorption curves fit to the un-pooled data.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Number of fitted parameters</th>
<th>RSE</th>
<th>Parameter 1</th>
<th>Mean ± sd</th>
<th>Parameter 2</th>
<th>Mean ± sd</th>
<th>Exponential Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Langmuir</td>
<td>1</td>
<td>17.5 – 25.7%</td>
<td>$K_L$</td>
<td>$9.33E-4 ± 2.0E-4$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Freundlich</td>
<td>2</td>
<td>5.72 – 17.6%</td>
<td>$K_F$</td>
<td>$197 ± 60 (S_{max}*[0.053 ± 0.012])$</td>
<td>-</td>
<td>-</td>
<td>$0.3064 ± 0.023$</td>
</tr>
<tr>
<td>Temkin</td>
<td>2</td>
<td>3.0 – 16.1%</td>
<td>$K_T$</td>
<td>$1.34E-2 ± 5.1E-3$</td>
<td>$q_T$</td>
<td>$S_{max}*[0.183 ± 0.013]$</td>
<td>-</td>
</tr>
<tr>
<td>Toth</td>
<td>3</td>
<td>0.9 – 6.5%</td>
<td>$K_{To}$</td>
<td>$3.75 ± 1.3$</td>
<td>b</td>
<td>$9960 ± 5590 (S_{max}*[2.42±0.72])$</td>
<td>$0.276 ± 0.054$</td>
</tr>
</tbody>
</table>

Table A2: Residual standard errors when applying average fit parameters from the pooled and (separately fit) training data to the test set soil samples, for i) a CEC of 25; ii) the experimentally measured CEC; iii) when the data is fit using the fitting algorithm

<table>
<thead>
<tr>
<th>Sample Location</th>
<th>CEC (cmole kg⁻¹)</th>
<th>NH₄⁺ (mg kg⁻¹)</th>
<th>pH</th>
<th>RSE (%) using estimated fit parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Langmuir</td>
</tr>
<tr>
<td>High Park</td>
<td>10.95</td>
<td>2.906</td>
<td>7.04</td>
<td>280 (285)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>26 (26)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>Corktown</td>
<td>25.3</td>
<td>3.552</td>
<td>7.82</td>
<td>33 (33)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>34 (34)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>Riverdale Park East</td>
<td>37.25</td>
<td>2.835</td>
<td>7.42</td>
<td>64 (63)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>35 (35)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>31</td>
</tr>
<tr>
<td>Geometric Mean RSE</td>
<td>Estimated</td>
<td>84 (84)</td>
<td>80 (85)</td>
<td>77 (81)</td>
</tr>
<tr>
<td></td>
<td>Estimated with measured CEC</td>
<td>31 (31)</td>
<td>21 (28)</td>
<td>21 (25)</td>
</tr>
<tr>
<td></td>
<td>Fitted Equation</td>
<td>28.3</td>
<td>14.8</td>
<td>17.0</td>
</tr>
</tbody>
</table>

Author contribution statement

MGD: Investigation, Formal analysis, Writing – Original Draft; KY: Investigation, Methodology; JGM: Supervision, Funding acquisition, Writing – Review & Editing.
**Competing interests statement**

The authors declare that they have no conflict of interest.

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References


