REVIEW: Improved basal drag of the West Antarctic Ice Sheet from L-curve analysis of inverse models utilizing subglacial hydrology simulations

The authors perform an L-curve analysis on a suite of basal drag inversions applied to a model domain that encompasses much of WAIS. L-curve analysis allows them to examine the balance between cost function terms when different degrees of regularization are utilized in the inversion, represented in the manuscript as a range of λ values, where λ is the weight associated with the regularization term in the inversion cost function. Inversions are analyzed with respect to the trade off between components of the cost function: J_{obs}, associated with velocity misfit compared to observations, and J_{reg}, associated with regularization. The best performing inversion is chosen based on the value of λ which minimizes the cost function without either component of the cost function growing too large. This is done analytically by finding the maximum curvature of the L-curve.

Suites of inversions are performed under the assumption of linear (m=1) and nonlinear sliding (m=3), and using three treatments of effective pressure: the exclusion of effective pressure (Weertman), effective pressure approximated based on ice geometry (Budd with N_{op}), and effective pressure calculated from a subglacial hydrology model (Budd with N_{CUAS}). Comparisons are drawn across the resulting 6 experimental groups allowing for analysis of the λ values which produce the best inversions when different physics are applied in the model parameterization. Additionally, the authors break their large model domain into 6 smaller subdomains that incorporate distinct geographic characteristics and perform the same analysis. This provides practical insights into how the inversion algorithm functions when applied to various portions of the ice sheet, and illustrates some complications of inverting for basal drag across large portions of the ice sheet without specialized treatment to certain areas.

Overall Statement:

On balance, there is a lot of very practical and useful information in this paper. The authors provide a great deal of detail with respect to the methods and L-curve analysis, which makes this work a resource for anyone interested in incorporating a more rigorous analysis to the inversion steps in their modeling procedure. This type of practical resource is not so often published in the glacier modeling community, although it perhaps should be to improve the reproducibility of modeling outputs and provide greater access to learning new methods/improving existing ones. Additionally, the main results of this work provide easy to implement improvements to basal drag inversions across a large portion of the Antarctic Ice Sheet.

My main critiques of this work pertain to the organization of the paper. I believe that the readability of the manuscript could be largely improved by reducing the length and simplifying the language, particularly in the Results section where there seems to be quite a bit of repetition and bouncing back and forth between ideas. That being said, I think the analysis of this work is sound and interesting and merits publication.

General Comments:

The Results section of this paper is 16 pages. While I appreciate the attention to detail, it would benefit from some simplification and reorganization. It is currently divided into subsections, but discussions pertaining to other parts of the study are present in each of the subsections. This causes quite a bit of repetition throughout the Results. I would suggest tightening up each of the subsections to highlight the main result and simplify the message. Comparison of the main results is done quite nicely later in the paper. It doesn't need to be addressed here as well. Readability of the Results section could be further increased by either condensing the description of L-curve analysis and focusing on giving clear interpretations of the figures, or by segmenting off some of these details into a supplemental document.

The text on many of the figures is quite small, making the subplots difficult to interpret. I would suggest making some of the smaller figures larger in general; specifically figures 7, 9, 10, 12, and 16. Some of the markers on subplots are also quite difficult to see. Figures 8 and 15 both illustrate the information being presented very nicely.

The Discussion and Conclusions sections are well written and clearly move through the main conclusions of the paper. However, the connection to subglacial lakes seems a bit out of sync to the rest of the paper, and I am not entirely convinced of the argument being made. It seems to me that correlation between τ_b and observed lake location is due to the use of N in the friction inversion. I think this section could probably be removed or condensed, but right now is somewhat distracting from the main results of this work.

I would be curious to know how these results might differ when using different forms of friction laws. Weertman and Budd are both power laws, but a regularized Coulomb friction law may produce significantly different behaviors, especially when comparing the performance of the inversion across subdomains. I understand that doing the full analysis using additional friction laws is a large amount of work, and I think the results presented here warrant publication without additional modeling. However, if the authors have a sense of how these results might translate to other friction laws, it would be an interesting point of discussion.

Specific Comments:

Abstract

Line 2-5: This sentence is overly complex. I would suggest pairing it down as it is just context.

Line 10: It reveals that Pine Island Glacier being → Pine Island Glacier is

Line 11: “best” and “worst” feel like odd descriptors here without more context for the interpretation of the L-curves. Consider changing or adding a bit more detail.

Line 12: Remove the comma

Line 19: “a larger part of the spatial basal drag coefficient structure” is somewhat confusing, consider rephrasing.

Line 20: “Allowing the inverted drag…” This is an incomplete sentence, combine with previous or remove.

Line 21: “reflect actual variations in basal properties” implies comparison to observations. If this is what you intended it should be stated more explicitly, but I don’t believe that was part of the study.

Introduction

Line 30-31: “The ongoing continuous…” This sentence is awkward and reads as an incomplete thought, consider rephrasing.

Lines 35-45: This paragraph jumps from discussing friction to subglacial hydrology to basal drag inversions. It feels a bit jumbled. I suggest reorganizing to discuss basal friction, the difficulty of parameterizing it, and inversions together first. Then introduce subglacial hydrology as an additional complication in determining basal friction.

Line 58: “However, it would be possible…” Suggest removing this sentence. It is not necessary to make your point.

Line 61-70: This paragraph is good, but it should come earlier. Here you are introducing concepts that are referenced with less context in the previous paragraph as though it is the first time they are being brought up.

The first two paragraphs of the introduction are somewhat disorganized and confusing in the way they introduce the main concepts of this work. But the rest of the introduction flows quite well, is easy to follow, and sets up the objectives of the paper nicely.

Methods

Line 118: Remove the comma.

Line 156: “model that models” is repetitive; this sentence should be rephrased.

Line 219: “we set it to ice pressure pi and neglect”: I assume that “it” is in reference here to effective pressure, but this sentence would be clearer if that was stated explicitly.

Line 237: To improve this instability of the problem, it is beneficial to regularize it → To improve the instability of the inverse problem, it is beneficial to add regularization.

In general, frequent use of pronouns in technical writing adds confusion. When you can be specific in what you are referencing, it is good to do so.

Line 244-246: Description of λ range should be simplified. The distinction of different ranges being used is not necessary here and adds confusion because it is not yet stated when or why each range is applied.

Line 246-249: Not sure that this is necessary.

There is a lot of really useful detail provided in this section that is often left out of modeling papers. I appreciate the inclusion of helpful tips that others in this field might find useful. But I do think that some reorganization could improve the readability and flow of ideas.

Results

Line 313: How are outliers defined?

Line 315-317: Confusing sentence, rephrase.

Line 319: The information about smoothing and convergence criteria in this paragraph is really useful!

Line 330: Maybe point to two subplot panels here that highlight the distinction being made by a very flat L-curve

Line 335: curve fitting procedure is → curve fitting procedure, is

Line 338: “cost curvature” is a bit confusing. I suggest rephrasing to “curvature of the cost function” at least the first time it is mentioned.

Line 340: In this method, you are picking λ_{best} to be the λ corresponding to the absolute maximum curvature of the L-curve, if I am understanding correctly. If so, I would say so rather than describing the small side peak in the curvature figures.

Line 344: Suggest removing reference to results that are not included in this work or published elsewhere.

Line 345: How are you defining good results? Is this in reference to the λ_{best} range being small? If so, can you provide some citations to other work to back up this claim?

Line 374: Fig. 8b-i → Fig. 8b

Line 379: Remove comma

Line 392: Fig. 8c → Fig. 8d

Line 398: Conspicuous is that the subdomain → It is conspicuous that the subdomain

Line 399: This sentence does not follow from the previous one.

Line 401: I suggest separating the discussion of different geographies and discussion of different friction laws. These last few paragraphs could be moved into the section dedicated to linear vs non-linear sliding.

Line 404: I suggest rephrasing this sentence for clarity.

Line 408: Is there really no recognizable impact? I'm not sure that I agree. There are few outliers with linear sliding, but the different regions have quite different shaped L-curves with different λ_{best} values. Please elaborate to make this point more clear.

Table 1: It would be helpful to have the λ_{best} values included here, or in their own table. These values are referenced quite a lot throughout the paper, but they are not listed anywhere.

Line 433: You state that λ_{best} for the N_{CUAS} run is located between the two other runs, but this is not clear on the figure. Please provide λ_{best} for all three runs to make this point.

Figures 9 and 10: Subplots are very difficult to read. Text is too small and outlier markers are difficult to see. I would also suggest using colors that will contrast in grayscale.

Line 452: Unclear what "accounting for less structure" means.

Figure 11: This is super helpful!

Line 494: parallel to → alongside

Line 497/Figure 12: Why not show this on the figure too?

Line 499-507: The point being made here is a bit unclear.

Figure 15: Great figure!

Section 3.4: In this section, I'm still wondering if λ_{best} is actually the best choice of λ, especially since λ_{min} has less error from velocity misfit while still incorporating regularization. I think this section would benefit from reiterating the justification of the choice of λ_{best} directly.

Discussion

Line 557: "the problem of outliers for too small λ values is due to the fact of non-convexity" I'm unclear about what this means.

Line 562-565: Unclear about how HO is being applied- aren't your experiments using SSA inversions? Where does the HO model come in?

Line 618: could also caused → could also be caused

Line 654: As a inclusion → As an inclusion

The discussion is nicely constructed and overall very clear.

Conclusions

Great well written summary!