Responses to CC3 (Hoyeon Shi)

Dear Hoyeon Shi,

Thank you for your interest in our work and for providing valuable suggestions. We have provided clarification and discussion regarding your questions, and we have included several significant revisions in the revised manuscript, as detailed in the attached document. The original comments are in black, and our replies are written in blue.

Best regards,
Yi Zhou and other co-authors.

Dear Zhou et al.,

Thank you for sharing the interesting results with the Cryosphere community. I enjoyed reading the manuscript, but I have two concerns, so I'm leaving them here. I would appreciate it if the authors could consider the following comments.

Firstly, we need to inform that we have made some major revisions, and all the modifications can be found in ***Supplementary A1*** at the end.

The first is about using the modal IS2 freeboard, which has already been raised in detail by "Reply on AC1" by Arttu Jutila. Although the authors provide two arguments in L179, the question that must be answered regarding this issue is "How could the modal freeboard be more physically compatible with the hydrostatic balance equation (used for IBD estimation) than the mean freeboard?" I would like to emphasize that my question is not related to the authors' assumption of the log-normal distribution of freeboard. This is because both "Mean" and "Mode" are available for any kind of statistical surface height distribution. What would be the physical meaning of modal height? How much do the parameterizations obtained from the mean and modal freeboards differ? The hydrostatic balance equation describes the balance of snow and ice mass and buoyancy, which are the quantities proportional to physical volume. Considering that the volume is area times height, it sounds more natural to me to multiply mean height than the modal height to area. I would like to hear the authors' opinions on this.
Authors' response: We will add more details in the ***Methods*** to further clarify your question. In this study, our primary aim is to match the IMB array and IS2 measurements to retrieve IBD. All selected buoys were deployed on level ice and showed good agreement with broader transect measurements, and thus we are confident that the buoy deployment strategy is sufficient to represent the level ice at the DN scale. On this basis, the mean ice thickness and snow depth from the buoy array (at least 10 buoys per day) were considered as the mean ice thickness and snow depth of level ice at the DN scale. However, for IS2 (or ALS) measurements, we cannot directly distinguish the total freeboard of level ice, thus we need to rely on the so-called "modal freeboard". By analyzing the frequency distribution of IS2 total freeboard within the DN region, we considered the quasi-frequency peak corresponding to the total freeboard as a proxy of the average total freeboard of level ice. We have validated the feasibility of this approach using AWI Icebird total freeboard data, which has a rigorous ice surface classification **(see our response to CC2 for details)**. Therefore, we actually matched the level ice thickness and snow depth from the buoy array with the total freeboard of level ice from IS2 modal characteristics. This, combined with auxiliary information on snow bulk density obtained from level ice, was used to calculate IBD based on the hydrostatic equilibrium equation.

In addition, a spatial scale correction term has been added to mitigate spatial scale differences between the buoy array and IS2 measurements **(see our response to RC1 for details)**.

Second, the manuscript is missing some context about using bivariate parameters for the IBD parameterization. The authors wrote they examined bivariate parameters, unlike previous studies that focused solely on the univariate parameters. This might mislead readers into thinking that there has been no study that used bivariate parameters to parameterize IBD, which is not true. The two papers already cited in the manuscript, Alexandrov et al. (2010) and Shi et al. (2023), have used the ice freeboard-to-thickness ratio to parameterize the sea ice bulk density. It should be noted that the 'ice draft-to-thickness ratio' is actually exactly the same as 1 – ice freeboard-to-thickness ratio. Furthermore, since Eq. (8) of the manuscript is a first-order linear equation, the form of the parameterization suggested by this study is the same as the one previously suggested by the two papers, i.e.:

IBD = a1 * draft-to-thickness-ratio + a2 = a1 * (1 – freeboard-to-thickness-ratio) + a2 = -a1 * freeboard-to-thickness-ratio + (a1 + a2)

Previous studies interpreted "-a1" as the density difference between floating and submerged parts of sea ice and "a1 + a2" as the density of submerged part of sea ice. Moreover, this parameterization has been implemented in the simultaneous estimation method that consistently estimates snow depth, sea ice thickness, ice freeboard, and IBD (Shi et al., 2023). Although the authors wrote in L358, "The strong linear relationship between the two bivariate parameters and IBD …, supporting previous suggestions (Alexandrov et al., 2010; Shi et al., 2023)", I would recommend authors to provide a more complete context of using bivariate parameters in section 2.2.4.

Nevertheless, I think this study's novelty comes from determining coefficients of parameterization based on the multisource observations rather than using representative values available from the literature. Accordingly, it would be very valuable to our community if the authors could include the following things.

(1) Parameterization of IBD using the ice freeboard-to-thickness ratio, i.e.:

IBD = a1 * freeboard-to-thickness-ratio + a2

(2) Comparison of the determined parameterization equation above and the equations used in the previous studies with physical interpretation of the difference between them.

For instance, in Alexandrov et al. (2010), for multiyear sea ice, a1 is -370 kg m-^{3} (= 550 – 920) and a2 is 920 kg m^{-3}. In Shi et al. (2023), a2 is the same, while a1 is -105 kg m-3 (= 815 – 920) for multiyear sea ice and -45 kg m^{-3} (= 875 – 920) for first-year sea ice.

Authors' response: Thank you for your clarification, and we will include background information on using bivariate parameters for IBD parameterization in the revised manuscript. So far, we have added more detail to the IBD parameterization, and beyond the ice draft-thickness ratio (which shows a strong linear relationship with IBD), we have considered three fitting relationships for the other parameters: quadratic polynomial, exponential, and power equation **(Fig. A1)**. Considering that the draft-to-thickness ratio can be formally converted to the freeboard-to-thickness ratio, we also provided the coefficients for the latter in the revised manuscript. Furthermore, following your suggestion, we compared the new IBD parameterizations with previous studies **(Fig. A2)**. The scope of the comparison was constrained within the valid range of input parameters used in this study. More details regarding the comparisons will be provided in the revised manuscript; however, it is crucial to clarify that the regression coefficients of the ice freeboard-to-thickness parameterization we derived are solely statistical significance, differing fundamentally from the coefficients obtained by A10 and S23 based on the reference IBDs used for weighting.

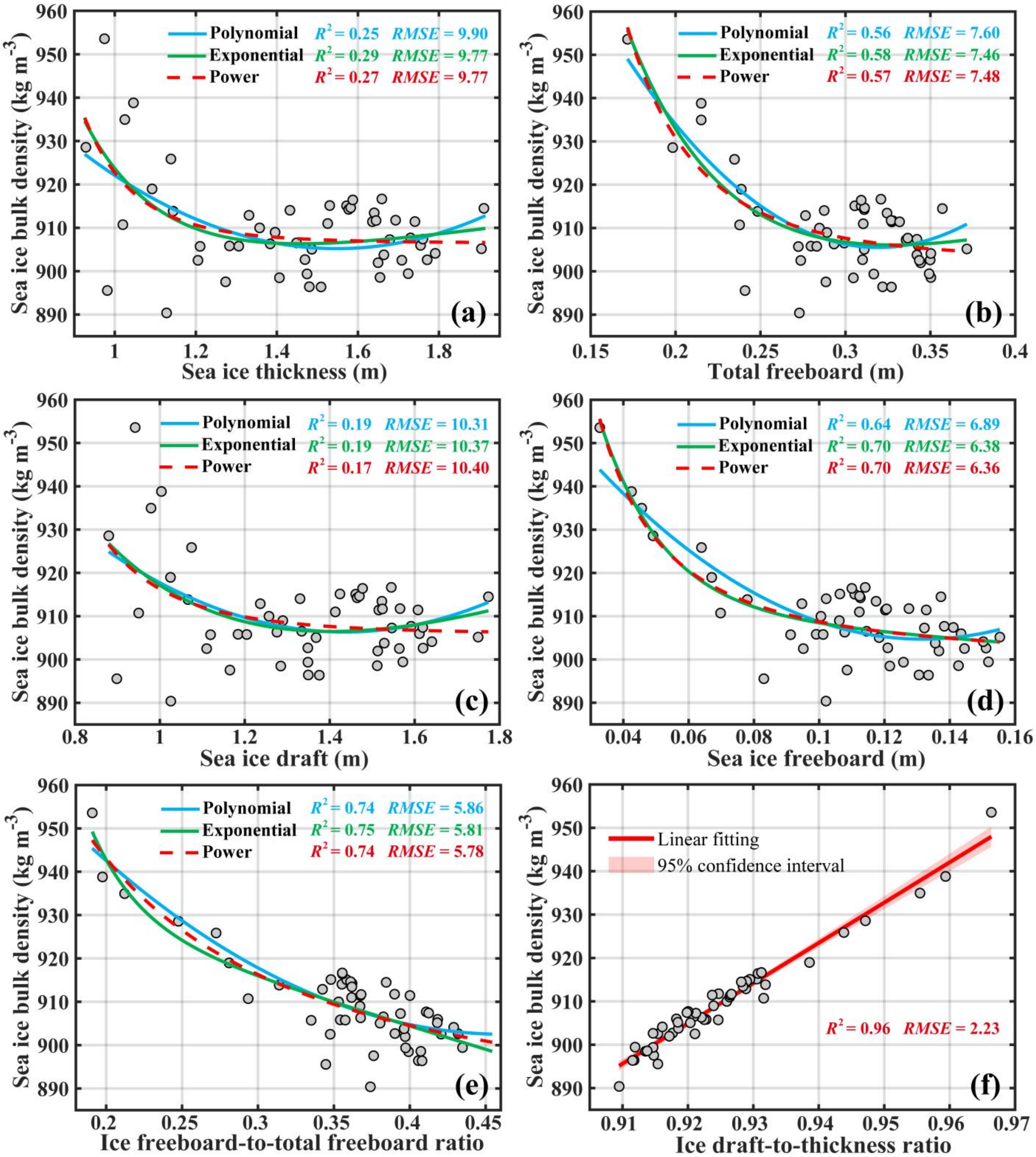

Figure A1. Parameterization of the IBD, including regression models using (a) sea ice thickness, (b) total freeboard, (c) sea ice draft, (d) sea ice freeboard, (e) ice freeboard-to-total freeboard ratio, and (f) ice draft-to-thickness ratio. Each panel shows model fit metrics, including the coefficient of determination (R^2) and the root mean square error (*RMSE*). Note that the statistical *P*-value for all results is less than 0.05 and the unit of *RMSE* is in kg m^{-3}.

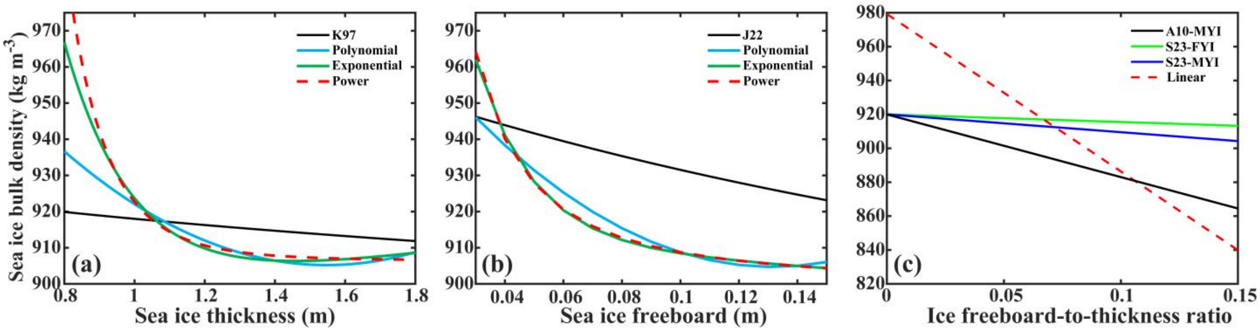

Figure A2. Intercomparison of sea ice bulk density parameterizations. (a) Sea ice thickness-based parameterizations, where the black line represents the results from Kovacs (1997) (K97) and the other colored lines are from this study. (b) Sea ice freeboard-based parameterizations, where the black line represents the results from Jutila et al. (2022) (J22) and the other colored lines are from this study. (c) The parameterizations based on ice freeboard-to-thickness ratio, where the black line represents the MYI results from Alexandrov et al. (2010) (A10), green and blue lines represent the FYI and MYI results from Shi et al. (2023) (S23) ,respectively, and red line is from this study.

Reference.

Alexandrov, V., Sandven, S., Wahlin, J., and Johannessen, O.: The relation between sea ice thickness and freeboard in the Arctic, The Cryosphere, 4, 373-380, 2010.

Jutila, A., Hendricks, S., Ricker, R., von Albedyll, L., Krumpen, T., and Haas, C.: Retrieval and parameterisation of sea-ice bulk density from airborne multi-sensor measurements, The Cryosphere, 16, 259-275, 2022.

Kovacs, A.: Estimating the full-scale flexural and compressive strength of first-year sea ice, Journal of Geophysical Research: Oceans, 102, 8681-8689, 1997.

Shi, H., Lee, S.-M., Sohn, B.-J., Gasiewski, A. J., Meier, W. N., Dybkjær, G., and Kim, S.-W.: Estimation of snow depth, sea ice thickness and bulk density, and ice freeboard in the Arctic winter by combining CryoSat-2, AVHRR, and AMSR measurements, IEEE Transactions on Geoscience and Remote Sensing, 2023. 2023.

Supplementary A1

[Data]

a) IS2 Freeboard Data Updated to ATL10 version 6.

Kwok et al. (2023), ATLAS/ICESat-2 L3A Sea Ice Freeboard, Version 6. [Data Set]. NSIDC.(https://doi.org/10.5067/ATLAS/ATL10.006).

b) Airborne Laser Scanning (ALS) Data: Added L-site scale data during the MOSAiC freezing season.

Hutter et al. (2023), Gridded segments from helicopter-borne laser scanner during MOSAiC. [PANGAEA](https://doi.org/10.1594/PANGAEA.950339).

c) AWI IceBird Multi-Sensor Sea Ice Parameters Updated to the Latest Version

- Jutila et al. (2024), Airborne sea ice parameters during the IceBird Winter 2019 campaign in the Arctic Ocean, Version 2. [PANGAEA](https://doi.org/10.1594/PANGAEA.966057).

- Jutila et al. (2024),Airborne sea ice parameters during the PAMARCMIP2017 campaign in the Arctic Ocean, Version 2. [PANGAEA](https://doi.org/10.1594/PANGAEA.966009).

d) Core-Based Ice Density Data: Added data on sea ice density from the *Sea Ice Physics Group* during MOSAiC legs 1 to 4.

- Oggier et al. (2023), First-year sea-ice salinity, temperature, density, oxygen and hydrogen isotope composition from the main coring site (MCS-FYI) during MOSAiC legs 1 to 4 in 2019/2020. [PANGAEA](https://doi.org/10.1594/PANGAEA.956732).

- Oggier et al. (2023), Second-year sea-ice salinity, temperature, density, oxygen and hydrogen isotope composition from the main coring site (MCS-SYI) during MOSAiC legs 1 to 4 in 2019/2020. [PANGAEA](https://doi.org/10.1594/PANGAEA.959830).

[Method]

a) IS2 Modal Freeboard Calculation: In the revised manuscript, we have retained the original resolution of IS2 ATL10 v6 and no longer perform the 150-segment averaging. Additionally, we no longer use log-normal fitting to estimate modal freeboard. Instead, we directly use the average of the freeboard values corresponding to the five highest frequencies in the freeboard distribution to obtain the modal freeboard (standard deviation is used as the uncertainty for the quasi-peak region of the total freeboard distribution). The purpose of this modification is to preserve the original distribution characteristics of the data as much as possible, without imposing fitting constraints or making resolution adjustments. Moreover, we have adopted the same approach to obtain the modal ALS freeboard.

b) Modal Value Feasibility for Level Ice: The AWI Icebird dataset was utilized to assess the feasibility of our approach, which features rigorously defined ice surface classification labels. We obtained the modal freeboard from the total freeboard distribution that includes both level and rough ice, and compared it with the average total freeboard of level ice.

c) Spatial Scale Correction: We introduced a spatial scale correction term to better align buoy array data with IS2/ALS modal freeboards.

d) Added Uncertainty for IS2 Modal Freeboard: We added uncertainty to the IS2 modal freeboard by calculating the mean difference (~0.0125 m) between IS2 and reference ALS modal freeboard.

[Results]

a) Sea Ice Bulk Density (IBD) Recalculation: All IBD results have been recalculated and re-evaluated following substantial revisions.

b) Enhanced IBD Parameterizations: Additional details have been included in the IBD parameterizations to improve clarity and accuracy.

c) IBD Results at Different Scales: Besides the DN scale, IBD results have now been extended to include the L-site scale.

d) High-Precision Ice Core Density: Ice core density that we used were obtained using the high-precision hydrostatic weighing method.

e) Expanded Discussion on IBD Uncertainty: More comprehensive discussions have been added regarding the uncertainty of IBD, its seasonal variations, spatial heterogeneity, limitations, and potential applications.