

The paper is much improved, in my opinion, and the equations are now clearer. The authors have carefully addressed reviewer comments. In general, I dislike requesting additional work after a second round of reviews, but I do believe some issues need addressing before your paper can conclude accurately on the persistence of FTLE. I also think that with a bit more work, your paper will be a better contribution to our understanding of the robustness and persistence of FTLE/LCS. As it is right now, I feel your conclusions could be misleading (due to overlooking a possibility I explain below). I do believe my suggestions will represent a moderate amount of work.

Some other issues could (and should) have been corrected through a more careful review by the authors and by using free software for grammar/spelling checking. I begin with the main issue.

Main issue:

In Figure 7b, variance diminishes as the averaging period increases. Your interpretation is that: "Meanwhile, the spatial variance of Ft continues diminishing, signifying that FTLE fields are more robust over many members than persistent over long time periods." You also mention: "The decay of variance as more FTLE fields are included in an average (Fig. 7a vs. Fig. 7b) indicates that FTLE features are more robust than persistent." However, figure 8h gives us a clue of what could be happening: FTLE evolves over time by changing positions (as one would expect in an unsteady flow) so that the time-average of these FTLE results in a more homogenous field (less variance). In other words, if we were to look at the FTLE fields that are averaged, we would likely see persistent FTLE features that, over time, move slightly in space, e.g., by meandering and other types of small changes in location. This is especially true at locations with a slope that tends to anchor the mean flow. The average would then capture a smeared field. Instead of distinct FTLE ridges localized at one location (as in 8g), we see diluted/smeared FTLE over a greater area. However, does a transport barrier that meanders not prevent trajectories from crossing the barrier just because it meanders? Can we say that a transport barrier that meanders is not persistent just because there is a temporal evolution to the exact location of the barrier, even if the meandering is relatively small, and it happens about a fixed location? I feel there is a strong possibility that time averaging LCS/FTLE is not the best (or only) way to determine persistence due to the nature of unsteady geophysical flows. However, the work you have done is still important to understand this. I believe it will be simple enough to assess persistence in a different way: first, ensemble-average the velocity at each time, then compute FTLE using the ensemble-averaged velocity, and finally, time average the FTLE over periods of increasing length as you already did. Presumably, the ensemble averaging of the velocity will remove the meanders and leave behind the persistent flow features. Now assess the persistence of the FTLE without being thrown off by small changes in location. I am not advocating for one method over the other. All I am saying is that without testing both methods, there is a high likelihood that we are left with an incomplete picture and that conclusions solely based on the incomplete picture may be misleading. After careful thought, I do believe that the topic of LCS persistence is a nuanced one. By comparing both methods of time-averaging FTLE, we will gain better insight into their persistence, and the user can decide which method suits best their needs. The method you tested is: you compute FTLE from each ensemble member, then ensemble-average FTLE, and then average FTLE in time over increasing windows. The method I suggest is: first, ensemble-average the velocity, then compute FTLE, and then time average FTLE over increasing windows. I do believe that with these additions,

your paper will be a better contribution to our understanding of the robustness and persistence of LCS/FTLE. I also fear that your current conclusions that FTLE is not as persistent may be misleading, as explained above with an example based on meandering. You do seem to suggest that first averaging the velocity is not a good idea (although it is unclear what type of averaging you refer to, more on that later). But I argue that without first computing the ensemble average velocity, and then estimating persistence, we are left wondering what exactly is the meaning of the lack of persistence that your report. It could be an efficient barrier persisting in time but not showing up in the averages except as spatially smeared, due to small spatial excursions over time. I argue we need both types of time averages to discern, and truly understand.

This is a very nice suggestion. In fact, similar arguments crossed our mind when we initially started working on the manuscript, but we decided then that we wanted to keep the time and ensemble dimensions separate. However, after taking the reviewer comment into account, we think this is a very good idea that should be investigated further.

We have therefore computed FTLE fields following the reviewer's suggestion, that is by ensemble-averaging velocity fields first and then examining time averages over the resulting FTLE fields. A paragraph describing this approach is added in lines 166-172. A new figure has been added (Fig. 7) which shows such time averages over December, January and February. Curves showing how the spatial variance of the new FTLE fields respond to time averaging have also been added to Fig. 8b. Text describing these new results is added in lines 252-261, 275-276 and 356-359.

As suggested by the reviewer, ensemble-averaging the velocity field removes the most unpredictable and time variable flow features. Thus the resulting FTLE fields contain less chaotic features, and the resulting time-averaged FTLE fields are less "noisy", and highlight ridges which are likely to be persistent.

Other issues:

In the abstract, you mention:

"Generally, Lagrangian trajectories as well as FTLE analysis inherit uncertainty from the underlying ocean model, bearing substantial uncertainties as a result of chaotic and turbulent flow fields."

Trajectories can indeed suffer from large uncertainty in different realizations of a velocity. Even a small spatial difference in the location of a repelling LCS will cause a large difference for a trajectory initiated at the same location and time (you can see an example of this in the Wikipedia entry for Lagrangian Coherent Structure where they compare a real saddle-point type structure with the same structure from a model that places the structure slightly shifted from the real location). However, many of us have noted that LCS tend to be robust to small changes in the velocity field (with no "substantial uncertainties"), and this paper aims to investigate the robustness of LCS. Thus, I believe you are stating that FTLE inherits substantial uncertainty without much support. Indeed, later in the abstract, you mention that you find FTLE to be robust, contradicting the statement that FTLE inherits "substantial uncertainty". I noticed you cite other papers by Balasuriya. Perhaps this is where you take the view that FTLE can inherit uncertainty. However, as mentioned in my previous review, his work relies heavily on adding a stochastic component to the velocity to study uncertainty in

LCS. This has two problems: 1) the dynamical systems approach we are discussing (FTLE/LCS) is strictly valid for a deterministic velocity (wrong tool for the problem) and 2) a stochastic component is not representative of the uncertainty we are interested in (wrong problem). Please consider this and account for it in your paper.

*I would also suggest changing (as this is part of what you are trying to find out):
In addition, velocity fields and resulting FTLE evolve rapidly ...*

To

In addition, velocity fields and resulting FTLE could evolve rapidly ...

You will be better positioned to discuss the evolution of FTLE by comparing the two methods of time averaging FTLE, the one you did and the one I suggest.

Thank you for spotting this. In the abstract we are indeed first comparing FTLE to Lagrangian trajectories, saying that both are uncertain, without much basis for it. We have done some changes in the abstract and are more careful with the wording.

In 25 of the revised manuscript: LCS don't suggest transport barriers they identify them rigorously (at least with the theory of Haller and collaborators). FTLE do suggest transport barriers as they may or may not be LCS.

Wording has been changed in line 25

In 39, do you mean unsteady instead of unstable? Unstable is not a good adjective in this context. The word unstable is used for different things depending on whether you refer to LCS (unstable manifolds), any vector field as a solution to an ODE (unstable vector field), or flow instability.

Yes, we meant to write "unsteady" here, this was an error.

45, the method by Duran et al does not detect LCS, they detect persistent attracting structures that they call climatological LCS. However, these structures are not hyperbolic LCS. For example, they are not material lines, nor are they rigorous transport barriers, although they tend to identify persistent barriers. This comment is also relevant to other places in the papers where cLCS are referred to as LCS.

Thank you for pointing this out. We changed the reference in line 45. The paragraph discussing cLCS (lines 337-345) has been updated to reflect the comment.

In equation 1, the velocity also depends explicitly on the time.

The equation has been updated.

Equations 6 and 7 could be clearer, I would suggest dropping the t_0 and t subscripts and superscripts (you can mention they are understood to be there after you define FTLE), using an index for the sum and express the spatial and temporal dependence of the FTLE fields (the temporal mean will not have a dependence on time, but the ensemble mean will). Also, it is unlikely N will be the same in both averages, use N and, say, M to indicate this.

These averages need to be clearly defined because there are at least two ways of obtaining ensemble averaged FTLE as discussed above.

We state that the subscripts are dropped for simplicity in line 141. Equations 6 and 7, as well as the text in lines 157-161 have been updated to reflect the reviewers comment.

163-166 I find the wording unclear, you seem to suggest computing a time average and then computing FTLE from the time-averaged velocity? This is confusing because you can't integrate a time-averaged velocity unless you repeat the same velocity at each time as if it were a steady velocity. This averaging needs to be clarified, and I would suggest that time averaging the velocity is questionable as you want to consider some time evolution of the flow, as you correctly suggest, even if this time evolution happens after ensemble averaging the velocity, as suggested above.

We replaced this paragraph with a paragraph about ensemble averaging velocity fields, then computing FTLE, then conducting the time-average of FTLE fields (as suggested above).

Figure 5 caption: you don't explain what c and d are.

Caption has been fixed.

Figure 8, If we are comparing FLTE from different sources, why not plot them all with the same colorscale so the strengths of the FTLE can be compared? Including a colorbar and units gives you additional physical information that has value, as you do in other figures.

Color schemes in Figure 8 (now Figure 9) have been changed, so that all subfigures now follow the same color scheme. A colorbar has also been added, and the subfigures should be more comparable now.

This sentence in 257–259 is unclear: “depends on which members are included in the average for few members, which might affect the results as there is a possibility that any randomly selected member might deviate largely from other members.”

We decided to remove this sentence from the manuscript as it was confusing and didn't add much to the text.

392 you mention Serra et al 2020 and velocity uncertainty; that paper discussed the robustness of LCS (OECS to be precise), in the context of operational use, and velocity uncertainty. This is very relevant to your topic so I suggest adding a bit of text to your discussion commenting on their findings and what that means in terms of your findings. They show (through a different method) that LCS/OECS are robust to uncertainty in the velocity. I think their discussion on this topic is in the main paper, but perhaps it is also in their supplementary information.

Thank you for bringing this to our attention. We agree that their findings are highly relevant to our paper, and have added a paragraph about it to our discussion (lines 426-432).