Response to the Community Comment (CC1)

In-Flight Estimation of Instrument Spectral Response Functions Using Sparse Representations

Jihanne El Haouari, Jean-Michel Gaucel, Christelle Pittet, Jean-Yves Tourneret, and Herwig Wendt

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We would like to thank the Editor and the referees and community members for their careful and thoughtful comments about our paper. We really appreciate the time and effort that has gone into their reviews. All the comments have been considered as detailed below. A detailed answer to the comments can be found in the following pages.

Response to Community Comment (CC1)

Comment 1

The use of an iterative and dictionary-based based approach for estimating ISRFs is a rather original solution.

Response:

We thank the community member for his/her positive appreciation of our work.

Comment 2

The fact that the most simple method, SVD + OMP, eventually leads to the best performance is a very good, yet somehow surprising, news, and could be further commented: has this to do with a particular choice of the hyper-parameter in (8) or with a lack of discrimination of the L1 norm constraint?

Response:

There is, indeed, no theoretical reason for the method SVD + OMP to lead to superior performance when compared to working with the ℓ_1 norm penalty. In our study, the hyperparameters for the algorithms have been chosen in order to achieve the best results. However, the OMP and LASSO algorithms address two distinct problems. Indeed, when the OMP algorithm provides an approximate solution to the problem with the ℓ_0 penalty, the LASSO algorithm solves the problem using the ℓ_1 penalty, hence gives the solution of an alternative, different problem. Certains limitations of the LASSO algorithm have been highlighted in numerous publications, including [1], and may be at the origin of our observations.

Comment 3

The iterations between dictionary estimates and sparse approximation represent an important aspect of the study, and could be further described.

Response:

Thank you for pointing this out. The following pseudo-codes have been added as an Appendix A in the revision of the manuscript:

- A1. The generation of the matrix of theoretical spectrum.
- A2. The construction of the dictionary.
- A3. The description of the OMP algorithm.
- A4. The description of the LASSO algorithm (for which the MATLAB *lasso* function is used).
- A5. The description of the K-SVD algorithm, which is based on the algorithm defined in [2].

Algorithm 1 Generation of the theoretical spectrum matrix.

Input: Theoretical spectrum $\boldsymbol{r}$, wavelengths of the theoretical spectrum $\boldsymbol{\lambda}_r$, wavelengths associated with the measured spectrum $\boldsymbol{\lambda}$, wavelength associated with the ISRF $\boldsymbol{\Delta}$
Output: Theoretical spectrum matrix for all wavelengths $\boldsymbol{R}$.

1: **for** $l = 1, ..., N_\lambda$ **do**
2: $\quad \lambda_l = \boldsymbol{\lambda}(l)$
3: $\quad \boldsymbol{\lambda}_{\text{resp}} = \lambda_l + \boldsymbol{\Delta}$
4: $\quad \boldsymbol{R}(l,:) = \text{interp}(\boldsymbol{\lambda}_r, \boldsymbol{r}, \boldsymbol{\lambda}_{\text{resp}})$
5: **end for**
6: **return** $\boldsymbol{R}$

Algorithm 2 Construction of the dictionary.

Input: Matrix of selected ISRFs $\boldsymbol{I}$, size of the dictionary N_{obs}
Output: Dictionary of ISRFs $\boldsymbol{\Phi}$.

1: $[\boldsymbol{U}, \boldsymbol{\Gamma}, \boldsymbol{V}^*] = \text{SVD}(\boldsymbol{I})$
2: $\boldsymbol{\Phi} = \boldsymbol{V}(:, 1 : N_{\text{obs}})$
3: **return** $\boldsymbol{\Phi}$

Algorithm 3 Orthogonal Matching ¨Pursuit (OMP) algorithm.

Input: Measured spectrum $\boldsymbol{s}_l$, theoretical spectrum matrix $\boldsymbol{R}_l$, dictionary of ISRFs $\boldsymbol{\Phi}$, sparsity parameter K
Output: Sparse vector $\boldsymbol{\alpha}_l$.

1: $\boldsymbol{\Psi}_l = \boldsymbol{R}_l\boldsymbol{\Phi}$
2: $\boldsymbol{U}_1 = \boldsymbol{s}_l$
3: **for** $k = 1, ..., K$ **do**
4: $\quad$ Find $\Psi_{\gamma_k} \in \boldsymbol{\Psi}_l$ that maximize the scalar product $|\langle \boldsymbol{U}_k, \Psi_{\gamma_k}/||\Psi_{\gamma_k}||\rangle|$
5: $\quad$ Find $[\alpha_{\gamma_1}, ..., \alpha_{\gamma_k}] \in \boldsymbol{\alpha}_l$ that solves $\arg\min_{\boldsymbol{\alpha}} ||\boldsymbol{U}_k - \sum_{k'=1}^{k} \alpha_{\gamma_{k'}} \Psi_{\gamma_{k'}}||_2^2$
6: $\quad \boldsymbol{U}_{k+1} = \boldsymbol{s}_l - \sum_{k'=1}^{k} \alpha_{\gamma_{k'}} \Psi_{\gamma_{k'}}$
7: **end for**
8: **return** $\boldsymbol{\alpha}_l$

Algorithm 4 LASSO algorithm.

Input: Measured spectrum $\boldsymbol{s}_l$, theoretical spectrum matrix $\boldsymbol{R}_l$, dictionary of ISRFs $\boldsymbol{\Phi}$, sparsity parameter K, mininimum value of the LASSO sparsity parameter $\mu_{\min}$, maximum value of the LASSO sparsity parameter $\mu_{\max}$

Output: Sparse vector $\boldsymbol{\alpha}_l$.

1: $\boldsymbol{\Psi}_l = \boldsymbol{R}_l\boldsymbol{\Phi}$
2: $\boldsymbol{\alpha}_{\text{resp}} = lasso(\boldsymbol{\Psi}_l, \boldsymbol{s}_l, \text{'lambda'}, \mu_{\max}, \text{'Alpha'}, 1)$
3: **while** sparsity($\boldsymbol{\alpha}_{\text{resp}}$) $\neq K$ **do**
4: $\quad \mu = \frac{\mu_{\min}+\mu_{\max}}{2}$
5: $\quad \boldsymbol{\alpha}_{\text{resp}} = lasso(\boldsymbol{\Psi}_l, \boldsymbol{s}_l, \text{'lambda'}, \mu, \text{'Alpha'}, 1)$
6: $\quad$ **if** sparsity($\boldsymbol{\alpha}_{\text{resp}}$) $< K$ **then**
7: $\quad\quad \mu_{\max} = \mu$
8: $\quad$ **else**
9: $\quad\quad \mu_{\min} = \mu$
10: $\quad$ **end if**
11: **end while**
12: Find the non-zero components in $\boldsymbol{\alpha}_{\text{resp}}$ to form the vector $[\gamma_1, ..., \gamma_K]$
13: Re-estimate the non-zero sparse coefficients: Find $[\alpha_{\gamma_1}, ..., \alpha_{\gamma_k}] \in \boldsymbol{\alpha}_l$ that solves $\arg\min_{\boldsymbol{\alpha}} ||\boldsymbol{s}_l - \sum_{k'=1}^{k} \alpha_{\gamma_{k'}} \Psi_{\gamma_{k'}}||_2^2$
14: **return** $\boldsymbol{\alpha}_l$

Algorithm 5 Construction of the dictionary using the K-SVD algorithm.

Input: Matrix of selected ISRFs $\boldsymbol{I}$, number of selected ISRFs L, size of the dictionary N_{obs}, Dictionary $\boldsymbol{\Phi}$ obtained using SVD in Algorithm (2), sparsity parameter K

Output: New dictionary of ISRFs $\boldsymbol{\Phi}$.

1: **while** not convergence **do**
2: $\quad$ Sparse coding step: $\boldsymbol{x}_l = \text{OMP}(\boldsymbol{I}_l, \boldsymbol{\Phi}, K)\ \forall\ l = 1, ..., L$
3: $\quad$ Dictionary update:
4: $\quad$ **for** $j = 1, ..., N_{\text{obs}}$ **do**
5: $\quad\quad$ Define the group of examples that uses the j-th colum of the dictionary j, $w_j = \{l | 1 \leq l \leq N, \boldsymbol{x}_T^j(l) \neq 0\}$
6: $\quad\quad$ Compute the overall representation error matrix, $\boldsymbol{E}_j = \boldsymbol{I} - \sum_{i\neq j} \boldsymbol{\phi}_i \boldsymbol{x}_T^i$
7: $\quad\quad$ Build $\boldsymbol{E}_j^R$ from $\boldsymbol{E}_j$ using the columns corresponding to w_j
8: $\quad\quad$ SVD decomposition $[\boldsymbol{U}, \boldsymbol{\Gamma}, \boldsymbol{V}^*] = \text{SVD}(\boldsymbol{E}_j^R)$
9: $\quad\quad$ Update the dictionary column $\boldsymbol{\phi}_j$ as the first column of $\boldsymbol{U}$ and the vector $\boldsymbol{x}_R^j$ as the first column of $\boldsymbol{V}\boldsymbol{\Gamma}(1,1)$.
10: $\quad$ **end for**
11: **end while**
12: **return** $\boldsymbol{\Phi}$

Comment 4

The adaptation to calibration errors, and temporal drifts of the feature represent high potential perspectives for this work.

Response:

Thank you for this important suggestion. We are indeed planning to study how our method could be adapted to calibrations errors, and temporal drifts, and will elaborate on this in the Conclusions and Perspectives of the article.

References

[1] R. Tibshirani, "Regression shrinkage and selection via the lasso," *Journal of the Royal Statistical Society (Series B)*, vol. 58, pp. 267–288, 1996.

[2] M. Aharon, M. Elad, and A. Bruckstein, "K-SVD: An algorithm for designing overcomplete dictionaries for sparse representation," *IEEE Trans. Signal Process.*, vol. 54, no. 11, pp. 4311–4322, 2006.