In this response letter, our responses are labeled in blue. Revised sentences in the updated manuscript are shown in *italic purple text*. All line numbers correspond to the revised manuscript with tracked changes, where deletions are shown in red and additions are shown in blue.

This paper is well written, with good explanations of the OEM methodology and analysis that supports the conclusions. One could argue that the Optimal Estimation methodology is well known, but the paper has practical examples that accompany the explanations.

Having said that, not much new or unexpected conclusions are in this paper, but it can find a place in an Atmospheric Measurements and Techniques paper if some fixes are made.

We thank the reviewer for the valuable comments that improve the quality of our manuscript. We have revised the manuscript accordingly.

In terms of the new or unexpected conclusions in our manuscript, our intention is to introduce the role and ability of new technology in hyperspectral microwave instruments to retrieve temperature and water vapor vertical profiles. We have included a sentence on Lines 72-73: *"With the development of HiSRAMS, we aim to demonstrate the capabilities of new technology in hyperspectral microwave instruments for retrieving temperature and water vapor vertical profiles under both clear-sky and all-sky conditions."*.

1) Line 470 : it is very hard to tell if the results presented truly are statistical N >> 1, or if only one case is done for the instruments alone on the ground, and one other case is done for the HISRAMS/AERI synergy. Your conclusions are too general if only one case is considered, especially since what you find is not surprising.

(AERI and HISRAMS together give "better" results (closer to truth) than one instrument alone, and a top/bottom sandwich brings a good deal of information together).

In our case study, N equals 1. The conclusion remains valid when determining the Degree of Freedom of the Signal (DFS) and the posterior uncertainty, as $\mathbf{K}$, $\mathbf{S_a}$, and $\mathbf{S_e}$ usually do not change significantly between cases. However, the comparison of the retrieved results with the truth is based solely on the presented case study.

We have revised the sentence in the Conclusions and discussion section on Lines 514-517: *"Additionally, the retrieval comparison in this study relies on limited samples from a single campaign* ***which only provides one case study for each retrieval configuration****, thus bounding the usefulness of the error statistics and comprehensiveness of this assessment. Specifically,* ***only a single radiosonde*** *was launched during the field campaign, which may have induced temporal and spatial variability in the truth profile."*.

2) Similarly, are you really doing an clear-sky retrieval. How do you know there are no clouds? Your paper gets confusing at the beginning when you mention "clear sky" in the title and then talk about the use of HISRAMS in allsky conditions. Can you clarify?

Thanks to the new technology of hyperspectral microwave radiometers, their primary advantage is the ability to penetrate clouds, which helps improve all-sky temperature and water vapor profiling. However, as this is a new technology, it is important to first test the instrument under well-controlled conditions, for which we have selected clear-sky environments. Thus, in the Introduction section, we have revised the sentence on Lines 73-75: *"While the primary advantage of microwave radiometers lies in their ability to retrieve in cloudy-sky conditions, this study focuses initially on clear-sky retrievals* ***under well-controlled conditions to ensure the instrument's performance.****"*.

In terms of determining whether the sky condition is clear, we have presented the radiosonde profile along with the clear-sky radiative closure analysis results in Liu et al. (2024). The personnel who collected the data also confirmed that no visible clouds were present that day. Absence of clouds below 6.8 km was also confirmed with airborne in-situ probes. The possibility of sub-visible cirrus is discussed in Liu et al. (2024), and it is considered extremely low.

Liu, L., Bliankinshtein, N., Huang, Y., Gyakum, J. R., Gabriel, P. M., Xu, S., and Wolde, M.: Radiative closure tests of collocated hyperspectral microwave and infrared radiometers, Atmospheric Measurement Techniques, 17, 2219-2233, 2024.

3) Figure 2,6 : Please show mean(observations - calculations) and std(observations - calculations) for both instruments. You could be getting great results while having poor spectral fits (ie the bias and std. dev are larger than the NeDT of the instruments).

In particular could you indicate the surface channels of the HISRAMS when showing the biases?

Since we have only a single case study for different geometric configurations, we presented the observed versus simulated spectra for various case studies in Figures S3-1 and S3-2. In most channels, the differences between observations and final retrieved simulations fall within the 3σ uncertainty range. However, exceptions include some spike channels in the ground-based AERI retrieval and the weak absorption channels (around 52 GHz) in the ground-based HiSRAMS retrieval. Liu et al. (2024) analyzed the radiative closure for both HiSRAMS and AERI, noting a systematic bias around 52 GHz in HiSRAMS. In ground-based HiSRAMS zenith-pointing measurements, the surface channels correspond to strong absorption channels located between 54 and 58 GHz within the oxygen band.

We have included this analysis in the Supplement Information.

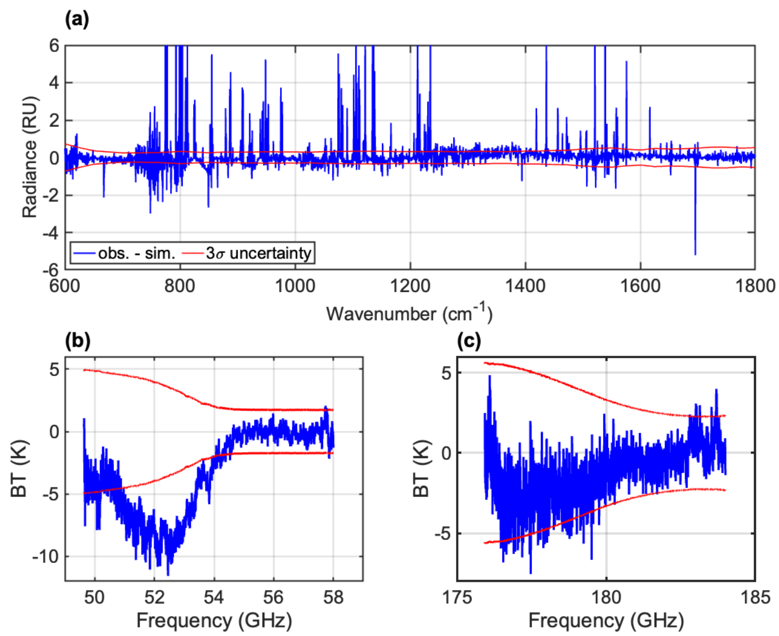

Figure S3-1: Radiance or brightness temperature differences between observations and the final retrieved simulation for ground-based retrievals: (a) AERI; (b) HiSRAMS oxygen band; (c) HiSRAMS water vapor band. The 1σ uncertainties are determined from the square root of the diagonal components of $\boldsymbol{S_e}$.

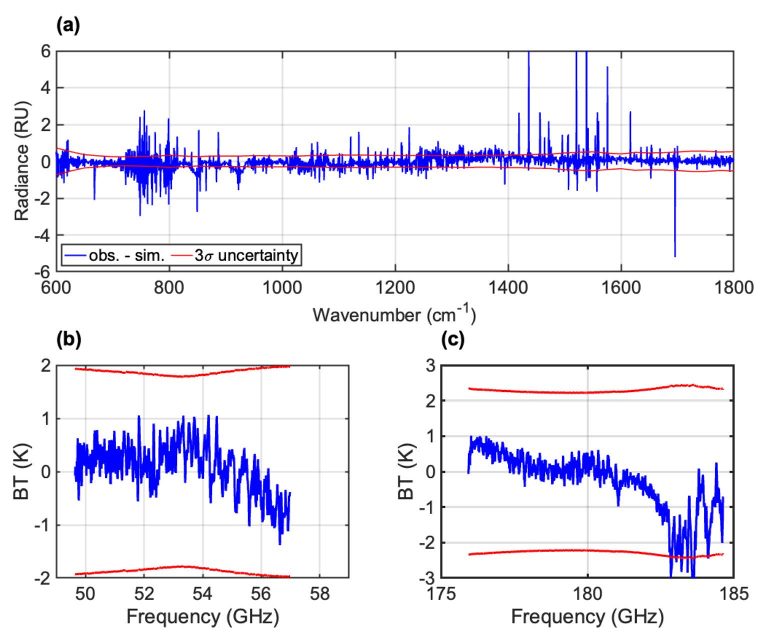

Figure S3-2: Radiance or brightness temperature differences between observations and the final retrieved simulation for joint retrievals: (a) AERI; (b) HiSRAMS oxygen band; (c) HiSRAMS water vapor band. The 1σ uncertainties are determined from the square root of the diagonal components of $\boldsymbol{S_e}$.

Liu, L., Bliankinshtein, N., Huang, Y., Gyakum, J. R., Gabriel, P. M., Xu, S., and Wolde, M.: Radiative closure tests of collocated hyperspectral microwave and infrared radiometers, Atmospheric Measurement Techniques, 17, 2219-2233, 2024.

4) The paper could be much better organized. For example instead of "sandwich" you could use the more traditional "synergy" term.

The term “synergy” refers to the combination of different instruments. “Sandwich,” on the other hand, implies the combination of both zenith-pointing and nadir-pointing measurements from different instruments simultaneously. Thus, we have decided to retain the use of the term “sandwich” measurements.

5) Figures S5 - S8 are ..?

Figures S1 to S7 are provided in the supplemental document, which can be accessed at https://doi.org/10.5194/egusphere-2024-1045-supplement.

6) Lines 155-162, and lines 174-180, should be combined together rather than being separated. Then again line 189 the n_{level} is mentioned.

We have combines these two paragraphs together with the description of the matrices for joint retrievals on Lines 173-189: “In this study, we retrieve temperature and water vapor vertical profiles simultaneously using single instruments (AERI or HiSRAMS) and joint instruments (AERI and HiSRAMS) respectively. Thus, $\boldsymbol{x}$ equals to $\begin{bmatrix} \boldsymbol{x}_T \\ \boldsymbol{x}_q \end{bmatrix}$ with a dimension of $\mathbf{38 \times 2 = 76}$. For all retrieval cases, the dimensions of the matrices $\boldsymbol{S_a}$, $\boldsymbol{S}$, and $\boldsymbol{A}$ are based solely on the dimension of the vertical level ($\boldsymbol{n_{level}} \times \boldsymbol{n_{level}}$), maintaining a consistent structure with the upper-left sub-matrix for temperature and the lower-right sub-matrix for water vapor. This structure allows us to separate the information of temperature and water vapor. Because HiSRAMS is an airborne instrument, its observational capabilities can be limited by altitude, affecting $\boldsymbol{n_{level}}$ for different case studies when it is nadir-pointing. In order to test the full potential of AERI and HiSRAMS to retrieve temperature and water vapor concentration profiles, all the instrumental channels are kept, with the result that $\boldsymbol{n_{AERI}} = \mathbf{2490}$ and $\boldsymbol{n_{HiSRAMS}} = \mathbf{2850}$ (including the measurements of both spectrometers of HiSRAMS). When retrieving the temperature and water vapor vertical profiles using either AERI or HiSRAMS alone, the dimensions of $\boldsymbol{S_e}$ and $\boldsymbol{K}$ are $\boldsymbol{n_{instrument}} \times \boldsymbol{n_{instrument}}$ and $\boldsymbol{n_{instrument}} \times \boldsymbol{n_{level}}$, respectively, where ‘instrument’ refers to either AERI or HiSRAMS. For joint retrieval:

$$\boldsymbol{y_{joint}} = \begin{bmatrix} \boldsymbol{y_{AERI}} \\ \boldsymbol{y_{HiSRAMS}} \end{bmatrix} \quad (8)$$

$$\boldsymbol{S_{e,joint}} = \begin{bmatrix} \boldsymbol{S_{e,AERI}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{S_{e,\,HiSRAMS}} \end{bmatrix} \quad (9)$$

$$\boldsymbol{K}_{joint} = \begin{bmatrix} \boldsymbol{K}_{AERI} \\ \boldsymbol{K}_{HiSRAMS} \end{bmatrix} \quad (10)$$

The dimensions of $\boldsymbol{y}_{joint}$, $\boldsymbol{S}_{e,joint}$, and $\boldsymbol{K}_{joint}$ are $(\boldsymbol{n}_{AERI} + \boldsymbol{n}_{HiSRAMS}) \times \mathbf{1}$, $(\boldsymbol{n}_{AERI} + \boldsymbol{n}_{HiSRAMS}) \times (\boldsymbol{n}_{AERI} + \boldsymbol{n}_{HiSRAMS})$, and $(\boldsymbol{n}_{AERI} + \boldsymbol{n}_{HiSRAMS}) \times \boldsymbol{n}_{level}$ respectively.".

7) Similarly lines 434 to 439 could be moved to the conclusion

This paragraph summarizes Section 5: Joint airborne HiSRAMS and ground-based AERI retrievals. We intend to provide summary paragraphs for both Section 4 (Lines 356-360) and Section 5 (Lines 478-483). In the Conclusion and Discussion section, we have also briefly summarized our work, findings, and the limitations of the analysis.