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On dissipation time scales of the basic second-order moments: the effect on the Energy and Flux-Budget (EFB) turbulence closure for stably stratified turbulence

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Abstract. The dissipation rates of the basic turbulent second-order moments are the key parameters controlling turbulence energetics and spectra, turbulent fluxes of momentum and heat, and playing a vital role in turbulence modelling. In this paper, we use the results of Direct Numerical Simulations (DNS) to evaluate dissipation rates of the basic turbulent second-order moments and revise the energy and flux-budget turbulence closure model for stably stratified turbulence. We delve into the theoretical implications of this approach and substantiate our closure hypotheses through DNS data. We also show why the concept of down-gradient turbulent transport becomes incomplete when applied to the vertical turbulent flux of potential temperature under very stable stratification. We reveal essential feedback between turbulent kinetic energy, the vertical flux of buoyancy and turbulent potential energy, which is responsible for maintaining shear-produced stably stratified turbulence up to extreme static stability.

1 Introduction

Turbulence and associated turbulent transport have been studied theoretically, experimentally, observationally and numerically during decades [see books by Batchelor (1953); Monin and Yaglom (1971, 2013); Tennekes and Lumley (1972); Frisch (1995); Pope (2000); Davidson (2013); Rogachevskii (2021), and references therein], but some important questions remain. This is particularly true in applications to atmospheric physics and geophysics where Reynolds and Peclet numbers are very large so

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that the governing equations are strongly nonlinear. The classical Kolmogorov's theory (Kolmogorov 1941a,b; 1942; 1991) has been formulated only for neutrally stratified homogeneous and isotropic turbulence.

In atmospheric boundary layers, temperature stratification causes turbulence to become anisotropic and inhomogeneous making some assumptions underlying Kolmogorov's theory questionable. Numerous alternative turbulence closure theories [see reviews by Weng and Taylor (2003); Umlauf and Burchard (2005); Mahrt (2014)] have been formulated using the budget equations not only for turbulent kinetic energy (TKE), but also for turbulent potential energy (TPE) (see, e.g., Holloway, 1986; Ostrovsky and Troitskaya, 1987; Dalaudier and Sidi, 1987; Hunt et al., 1988; Canuto and Minotti, 1993; Schumann and Gerz, 1995; Hanazaki and Hunt, 1996; Keller and van Atta, 2000; Canuto et al., 2001; Stretch et al., 2001; Cheng et al., 2002; 2002; Hanazaki and Hunt, 2004; Rehmann and Hwang, 2005; Umlauf, 2005). The budget equations for all three energies, TKE, TPE and total turbulent energy (TTE), were considered by Canuto and Minotti (1993), Elperin et al. (2002, 2006), Zilitinkevich et al. (2007), and Canuto et al. (2008).

The energy and flux budget (EFB) turbulence closure theory which is based on the budget equations for the densities of TKE, TPE and turbulent fluxes of momentum and heat, was developed for stably stratified atmospheric flows (Zilitinkevich et al., 2007, 2008, 2009, 2013; Kleeorin et al. 2019), for surface layers in atmospheric convective turbulence (Rogachevskii et al. 2022) and for passive scalar transport (Kleeorin et al. 2021). The EFB closure theory has shown that strong atmospheric stably stratified turbulence is maintained by large-scale shear (mean wind) for any stratification, and the "critical" Richardson number, considered many years as a threshold between the turbulent and laminar states of the flow, actually separates two turbulent regimes: the strong turbulence typical of atmospheric boundary layers and the weak three-dimensional turbulence typical of the free atmosphere and characterized by a strong decrease in the heat transfer in comparison to the momentum transfer.

Some other turbulent closure models (Mauritsen et al. 2007, Canuto et al., 2008, Sukoriansky and Galperin, 2008, Li et al. 2016) do not imply the critical Richardson number, so shear-generated turbulent mixing may persist for any stratification. In particular, Mauritsen et al. (2007) have developed a turbulent closure based on the budget equation for TTE (instead of TKE) and different observational findings to take into account the mean flow stability. They used this turbulent closure model to study the turbulent transfer of heat and momentum under very stable stratification. In their model, whereas the turbulent heat flux tends toward zero beyond a certain stability limit, the turbulent stress stays finite. However, the model by Mauritsen et al. (2007) has not used the budget equation for TPE and the vertical heat flux.

L'vov et al. (2008) have performed detailed analyses of the budget equations for the Reynolds stresses in the turbulent boundary layer (relevant to the strong turbulence regime) taking explicitly into consideration the dissipative effect in the horizontal heat flux budget equation, in contrast to the EFB "effective-dissipation approximation". However, the theory by L'vov et al. (2008) still contains the critical gradient Richardson number for the existence of the shear-produced turbulence.

Sukoriansky and Galperin (2008) apply a quasi-normal scale elimination theory that is similar to the renormalization group analysis. Sukoriansky and Galperin (2008) do not use the budget equations for TKE, TPE and TTE in their analysis. This theory correctly describes the dependence of the turbulent Prandtl number versus the gradient Richardson number and does

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not imply the critical gradient Richardson number for the existence of turbulence. However, this approach does not allow obtaining detailed Richardson number dependences of the other non-dimensional parameters, like the ratio between TPE and TTE, dimensionless turbulent flux of momentum or dimensionless vertical turbulent flux of potential temperature. Their background non-stratified shear-produced turbulence is assumed to be isotropic and homogeneous. Canuto et al. (2008) have generalized their previous model (see Cheng et al., 2002) introducing the new parameterization for the buoyancy time scale to accommodate the existence of stably stratified shear-produced turbulence at arbitrary Richardson numbers.

Li et al. (2016) have developed the co-spectral budget (CSB) closure approach which is formulated in the Fourier space and integrated across all turbulent scales to obtain flow variables in physical space. CSB models allow turbulence to exist at any gradient Richardson number, however, CSB yields different (from EFB) predictions for the vertical anisotropy versus Richardson number.

All state-of-the-art turbulent closures follow the so-called Kolmogorov hypothesis: all dissipation time scales of turbulent second-order moments are assumed to be proportional to each other, which at first glance looks reasonable but, in fact, hypothetical for stably stratified conditions. Our Direct Numerical Simulations (DNS) results are limited to gradient Richardson numbers up to Ri $= 0.2$, but despite this constraint, we aim to disprove this proportionality and instead propose that the stability dependency is inherent in the ratios of dissipation time scales.

2 Problem setting and basic equations

We consider plane-parallel, stably stratified dry-air flow and employ the familiar budget equations underlying turbulence-closure theory (e.g., Kleeorin et al. 2021; Zilitinkevich et al., 2013; Kaimal and Fennigan, 1994; Canuto et al., 2008) for the Reynolds stress, $\tau_{ij} = \langle u_i u_j \rangle$, the potential temperature flux, $F_i = \langle \theta u_i \rangle$, and the intensity of potential temperature fluctuations, $E_\theta = \langle \theta^2 \rangle / 2$:

$$\frac{D\tau_{ij}}{Dt} + \frac{\partial}{\partial x_k}\Phi_{ijk}^{(\tau)} = -\tau_{ik}\frac{\partial U_j}{\partial x_k} - \tau_{jk}\frac{\partial U_i}{\partial x_k} - \left[\varepsilon_{ij}^{(\tau)} - \beta\left(F_j\delta_{i3} + F_i\delta_{j3}\right) - Q_{ij}\right], \quad (1)$$

$$\frac{DF_i}{Dt} + \frac{\partial}{\partial x_j}\Phi_{ij}^{(F)} = \beta\delta_{i3}\langle\theta^2\rangle - \frac{1}{\rho_0}\langle\theta\frac{\partial p}{\partial x_i}\rangle - \tau_{ij}\frac{\partial\Theta}{\partial z} - F_j\frac{\partial U_i}{\partial x_j} - \varepsilon_i^{(F)}, \quad (2)$$

$$\frac{DE_\theta}{Dt} + \frac{\partial}{\partial x_i}\Phi_i^{(\theta)} = -F_z\frac{\partial\Theta}{\partial x_j} - \varepsilon_\theta. \quad (3)$$

Here, $x_1 = x$ and $x_2 = y$ are horizontal coordinates, $x_3 = z$ is the vertical coordinate; t is time; $\mathbf{U} = (U_1, U_2, U_3) = (U, V, W)$ is the vector of mean wind velocity; $\mathbf{u} = (u_1, u_2, u_3) = (u, v, w)$ is the vector of velocity fluctuations; $\mathbf{\Theta} = T(P_0/P)^{1-1/\gamma}$ is mean potential temperature (expressed through absolute temperature, T , and pressure, P); T_0, P_0 and ρ_0 are reference values of temperature, pressure and density, respectively; $\gamma = c_p/c_v = 1.41$ is the specific heats ratio; θ and p are fluctuations of potential temperature and pressure; $D/Dt = \partial/\partial t + U_k\partial/\partial x_k$ is the operator of full derivative over time t; angle brackets denote averaging; $\beta = g/T_0$ is the buoyancy parameter; g is the acceleration due to gravity; δ_{ij} is the unit tensor ($\delta_{ij} = 1$ for

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$i = j$ and $\delta_{ij} = 0$ for $i \neq j$); $\Phi_{ijk}^{(\tau)}$, $\Phi_{ij}^{(F)}$ and $\Phi_{i}^{(\theta)}$ are the third-order moments, which define turbulent transport~~s~~ of the second-order moments under consideration:

$$\Phi_{ijk}^{(\tau)} = \langle u_i u_j u_k \rangle + \frac{1}{\rho_0}\left(\langle p u_i \rangle \delta_{jk} + \langle p u_j \rangle \delta_{ik}\right) - \nu \left(\langle u_i \frac{\partial u_j}{\partial x_k} \rangle + \langle u_j \frac{\partial u_i}{\partial x_k} \rangle\right), \tag{4}$$

$$\Phi_{ij}^{(F)} = \langle u_i u_j \theta \rangle - \nu \langle \theta \frac{\partial u_i}{\partial x_j} \rangle - \kappa \langle u_i \frac{\partial \theta}{\partial x_j} \rangle, \tag{5}$$

$$\Phi_{i}^{(\theta)} = \frac{1}{2}\langle \theta^2 u_i \rangle - \frac{\kappa}{2}\frac{\partial}{\partial x_i}\langle \theta^2 \rangle; \tag{6}$$

Q_{ij} terms represent the correlations between fluctuations of pressure and strain-rate tensor, which control the interaction between the Reynolds stress components:

$$Q_{ij} = \frac{1}{\rho_0}\langle p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \rangle. \tag{7}$$

Here, $\varepsilon_{ij}^{(\tau)}$, $\varepsilon_{i}^{(F)}$ and ε_θ are the second-order moments dissipation rate terms:

$$\varepsilon_{ij}^{(\tau)} = 2\nu \langle \frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k} \rangle, \tag{8}$$

$$\varepsilon_{i}^{(F)} = (\nu + \kappa) \langle \frac{\partial u_i}{\partial x_j}\frac{\partial \theta}{\partial x_j} \rangle, \tag{9}$$

$$\varepsilon_\theta = \kappa \langle \left(\frac{\partial \theta}{\partial x_j}\right)^2 \rangle, \tag{10}$$

where ν is kinematic viscosity and κ is thermal conductivity.

The budgets of TKE components, $E_i = \langle u_i^2 \rangle / 2$ ($i = 1,2,3$), are expressed by Eq. (1) for $i = j$, summing them up yields the familiar TKE budget equation:

$$\frac{DE_K}{Dt} + \frac{\partial}{\partial z}\left(\frac{1}{2}\langle u_i^2 w \rangle + \frac{1}{\rho_0}\langle p w \rangle - \frac{\nu}{2}\frac{\partial \langle u_i^2 \rangle}{\partial z}\right) = -\boldsymbol{\tau} \cdot \frac{\partial \mathbf{U}}{\partial z} + \beta F_z - \varepsilon_K, \tag{11}$$

where $E_K = \sum E_i$ is TKE and $\varepsilon_K = \sum \varepsilon_{ii}^{(\tau)} / 2$ is the TKE dissipation rate. The sum of the terms Q_{ii} (the trace of the tensor Q_{ij}) is equal to zero because of the incompressibility constraint on the flow velocity field, $\partial u_i / \partial x_i = 0$, i.e. Q_{ij} only redistribute energy between TKE components.

Likewise, ε_θ is the dissipation rate of the intensity of potential temperature fluctuations, E_θ; and $\varepsilon_i^{(F)}$ are the dissipation rates of the three components of the turbulent flux of potential temperature, F_i.

Following Kolmogorov (1941, 1942), the dissipation rates ε_K and ε_θ are taken proportional to the dissipating quantities divided by corresponding time scales,

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$$\varepsilon_K = \frac{E_K}{t_K},\ \varepsilon_\theta = \frac{E_\theta}{t_\theta}, \tag{12}$$

where t_K is the TKE dissipation time scale and t_θ is the dissipation time scale of E_θ. Here, the formulation of the dissipation rates is not hypothetical: it merely expresses one unknown (dissipation rate) through another (dissipation time scale).

In this study, we consider the EFB model in its simplest, algebraic form, neglecting non-steady terms in all budget equations and neglecting divergence of the fluxes of TKE, TPE and fluxes of F_z (determined by third-order moments). This approach is reasonable because, e.g., the characteristic times of variations of the second moments are much larger than the turbulent time scales for large Reynolds and Peclet numbers. We also assume that the terms related to the divergence of the fluxes of TKE and TPE for stably stratified turbulence are much smaller than the rates of production and dissipation in budget equations (3) and (11). In this case, the TKE budget equation, Eq. (11), and the budget equation for E_θ, Eq. (3), become

$$0 = -\tau\frac{\partial U}{\partial z} + \beta F_z - \varepsilon_K, \tag{13}$$

$$0 = -F_z\frac{\partial \Theta}{\partial z} - \varepsilon_\theta. \tag{14}$$

E_θ determines TPE:

$$E_P = \frac{\beta E_\theta}{\partial\Theta/\partial z}, \tag{15}$$

so that Eq. (14) becomes

$$0 = -\beta F_z - \varepsilon_P, \tag{16}$$

where $\varepsilon_P = E_P/t_\theta$ is TPE dissipation time scale.

The first term on the right-hand side (r.h.s.) of Eq. (13), $-\tau\,\partial U/\partial z$, is the rate of the TKE production, while the second term, βF_z, is the buoyancy which in stably stratified flow causes decay of TKE, i.e., it results in conversion of TKE into TPE. The ratio of these terms is the flux Richardson number:

$$\mathrm{Ri}_f \equiv -\frac{\beta F_z}{\tau\partial U/\partial z}, \tag{17}$$

and this dimensionless parameter characterises the effect of stratification on turbulence.

Taking into account Eq. (17), the steady-state versions of TKE and TPE budget equations, Eqs. (13) and (14), can be rewritten as

$$E_K = \tau\frac{\partial U}{\partial z}\left(1 - \mathrm{Ri}_f\right)t_K, \tag{18}$$

$$E_P = \tau\frac{\partial U}{\partial z}\mathrm{Ri}_f t_\theta. \tag{19}$$

Thus, the ratio of TPE to TKE is:

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$$\frac{E_P}{E_K} = \frac{\mathrm{Ri}_f}{1-\mathrm{Ri}_f}\frac{t_\theta}{t_K}. \tag{20}$$

Zilitinkevich et al. (2013) suggested the following relation linking Ri_f with another stratification parameter, z/L:

$$\mathrm{Ri}_f = \frac{kz/L}{1+kR_\infty^{-1}z/L}, \qquad \frac{z}{L} = \frac{R_\infty}{k}\frac{\mathrm{Ri}_f}{R_\infty-\mathrm{Ri}_f}, \tag{21}$$

where $L = -\tau^{3/2}/\beta F_z$ is the Obukhov length-scale, $k = 0.4$ is the von Kármán constant, and $R_\infty = 0.2$ is the maximum value of the flux Richardson number.

On the r.h.s. of Eq. (20), there is an unknown ratio of two dissipation time scales, t_θ/t_K. The Kolmogorov hypothesis suggests that it is a universal constant. We do not imply this assumption, but instead investigate a possible stability dependency of dissipation time scales ratios and improve the EFB turbulence closure model accounting for it.

3 Methods and data used for empirical validation

For the purpose of our study, we performed a series of DNS of stably stratified turbulent plane Couette flow. In Couette flow, the total (turbulent plus molecular) vertical fluxes of momentum and potential temperature are constant (i.e., they are independent of the height), which, in particular, assures a very certain fixed value of the Obukhov length scale, L. We recall that all our derivations are relevant to the well-developed turbulence regime where molecular transports are negligible compared to turbulent transports so that turbulent fluxes practically coincide with total fluxes. This is the case in our DNS, except for the narrow near-wall viscous-turbulent flow-transition layers. Data from these layers, obviously irrelevant to the turbulence regime we consider, are shown by grey points in the figures and are ignored in fitting procedures.

Numerical simulation of stably stratified turbulent Couette flow was performed using the unified DNS-, LES- and RANS-code developed at the Moscow State University (MSU) and the Institute of Numerical Mathematics (INM) of the Russian Academy of Science (see, Mortikov, 2016; Mortikov et al., 2019; Bhattacharjee et al., 2022; Debolskiy et al., 2023; Gladskikh et al., 2023) designed for high-resolution simulations on modern-day HPC systems. The DNS part of the code solves the finite-difference approximation of the incompressible Navier-Stokes system of equations under the Boussinesq approximation. Conservative schemes on the staggered grid (Morinishi et al., 1998; Vasilyev, 2000) of 4th-order accuracy are used in horizontal directions and in the vertical direction the spatial approximation is restricted to 2nd-order accuracy with near-wall grid resolution refinement sufficient to resolve near-wall viscous region. The projection method (Brown et al., 2001) is used for the time-advancement of momentum equations coupled with the incompressibility condition, while the multigrid method is applied to solve the Poisson equation to ensure that the velocity is divergence-free at each time step. For the Couette flow periodic boundary conditions are used in the horizontal directions, and no-slip/no-penetration conditions are set on the channel walls for the velocity. The stable stratification is maintained by prescribed Dirichlet boundary conditions on the potential temperature. In all experiments, the value of molecular Prandtl number (ratio of kinematic viscosity and thermal diffusivity of

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the fluid) was fixed at 0.7. The simulations were performed for a wide range of Reynolds numbers, Re, defined by the wall velocity difference, channel height and kinematic viscosity: from 5200 up to 120 000. All experiments were carried out using the resources of MSU and CSC HPC centers. For the maximum Re values achieved the numerical grid consisted of more than 2×10^8 cells and the calculations used about 10 000 CPU cores.

For a fixed value of Reynolds number, we considered a series of experiments: starting from neutral conditions (no imposed stability), the stable stratification was increased gradually in each subsequent experiment, eventually resulting in flow laminarization. For each stability condition, the turbulent flow was allowed sufficient time to develop and reach statistical steady-state conditions (e.g., total momentum flux is constant and TKE balance is in steady state), while all the terms in the second-order moments budget equations, Eqs. (1)-(3), were evaluated in a manner consistent with the finite-difference approximation resulting in negligible residual. This allowed us to study the features of shear-produced stably stratified turbulence up to extreme static stability explicitly resolving all dissipation times scales of turbulent second-order moments.

4 Novel formulation for the steady-state regime of turbulence

In the steady-state, Eq. (1) for the vertical component of the turbulent flux of momentum, τ, becomes

$$0 = -2E_z \frac{\partial U}{\partial z} - [\varepsilon_\tau - \beta F_x - Q_{13}]. \tag{22}$$

Following Zilitinkevich et al. (2007) we define the sum of all terms in square brackets on the r.h.s. of Eq. (22) as the “effective dissipation”:

$$\varepsilon_\tau^{(eff)} = \varepsilon_\tau - \beta F_x - Q_{13} \equiv \frac{\tau}{t_\tau}. \tag{23}$$

Thus, Eq. (22) becomes

$$0 = -2E_z \frac{\partial U}{\partial z} - \frac{\tau}{t_\tau}, \tag{24}$$

yielding the well-known down-gradient formulation of the vertical turbulent flux of momentum:

$$\tau = -K_M \frac{\partial U}{\partial z}, \; K_M = 2A_z E_K t_\tau, \tag{25}$$

where $A_z \equiv E_z/E_K$ is the vertical share of TKE (the vertical anisotropy parameter).

Substituting Eq. (25) into Eq. (18), we obtain

$$\left(\frac{\tau}{E_K}\right)^2 = \frac{2A_z}{1-\mathrm{Ri}_f} \frac{t_\tau}{t_K}. \tag{26}$$

In Eq. (26) all the variables are exactly resolved numerically in DNS making a detailed investigation on t_τ/t_K possible. Figure 1 demonstrates t_τ/t_K to be a function of the stratification parameter z/L rather than a constant. We propose to approximate this function with a ratio of two first-order polynomials:

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$$\frac{t_\tau}{t_K} = \frac{C_1^{\tau K} z/L + C_2^{\tau K}}{z/L + C_3^{\tau K}}. \quad (27)$$

Here, the dimensionless empirical constants are obtained from the best fit of Eq. (27) to DNS data: $C_1^{\tau K} = 0.08$, $C_2^{\tau K} = 0.4$, $C_3^{\tau K} = 2$. The fitting is done using a rational regression model.

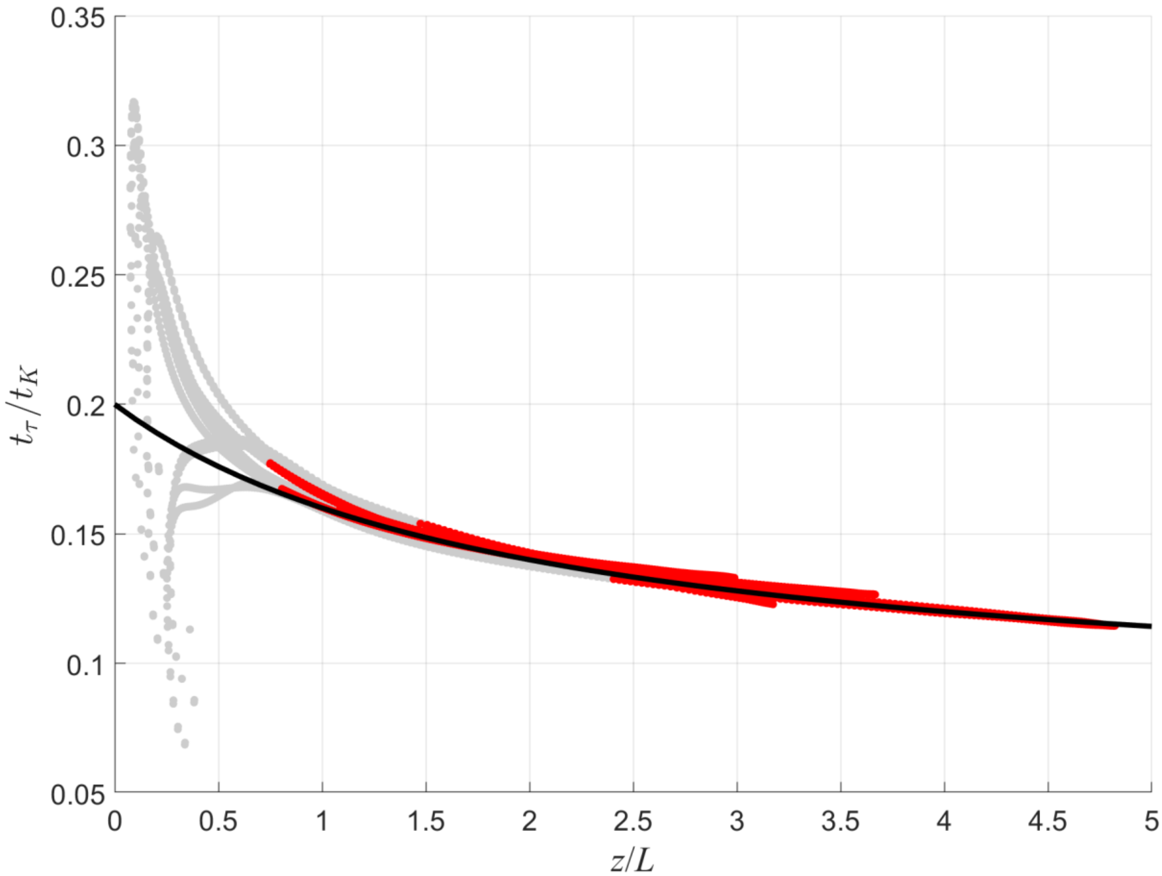

Figure 1: The ratio of the effective dissipation time scale of τ and the dissipation time scale of TKE, t_τ/t_K, versus z/L. Empirical data used for the calibration are obtained in DNS experiments employing the MSU/INM unified code (red dots). Dark grey dots belong to the viscous sub-layer (very narrow near-surface layer essentially affected by molecular viscosity): $0 < z < 50\nu/\tau^{1/2}$. The black solid line shows Eq. (27) with empirical constants $C_1^{\tau K} = 0.08$, $C_2^{\tau K} = 0.4$ and $C_3^{\tau K} = 2$, obtained from the best fit of Eq. (27) to DNS data in the turbulent layer: $z > 50\nu/\tau^{1/2}$.

Proceeding to the vertical flux of potential temperature, F_z, we derive its budget equation from Eq. (2):

$$\frac{\partial}{\partial z}\Phi_{33}^{(F)} = \beta\langle\theta^2\rangle - \frac{1}{\rho_0}\langle\theta\frac{\partial p}{\partial z}\rangle - 2E_z\frac{\partial\Theta}{\partial z} - \varepsilon_F. \quad (28)$$

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DNS modelling showed $\frac{\partial}{\partial z}\Phi_{33}^{(F)}$ term to be of the same order of magnitude as ε_F and of the same sign, so we introduce the 'effective dissipation rate' $\varepsilon_F^{(eff)}$:

$$\varepsilon_F^{(eff)} = \varepsilon_F + \frac{\partial}{\partial z}\Phi_{33}^{(F)} \equiv \frac{F_z}{t_F}. \tag{29}$$

215 Consequently, Eq. (28) reduces to

$$0 = \beta\langle\theta^2\rangle - \frac{1}{\rho_0}\langle\theta\frac{\partial p}{\partial z}\rangle - 2E_z\frac{\partial\Theta}{\partial z} - \frac{F_z}{t_F}. \tag{30}$$

Traditionally, the pressure term was either assumed to be negligible or declared to be proportional to $\beta\langle\theta^2\rangle$ term. Unfortunately, our DNS data proved it to be neither negligible nor proportional to any other term in the budget equation, Eq. (30). Instead, we found it to be well approximated by a linear combination of production and transport terms of Eq. (30) (see
220 Fig. 2):

$$\frac{1}{\rho_0}\langle\theta\frac{\partial p}{\partial z}\rangle = C_\theta\beta\langle\theta^2\rangle + C_\nabla 2E_z\frac{\partial\Theta}{\partial z}. \tag{31}$$

The dimensionless constants $C_\theta = 0.76$ and $C_\nabla = 0.78$ are obtained from the best fit of Eq. (31) to DNS data.

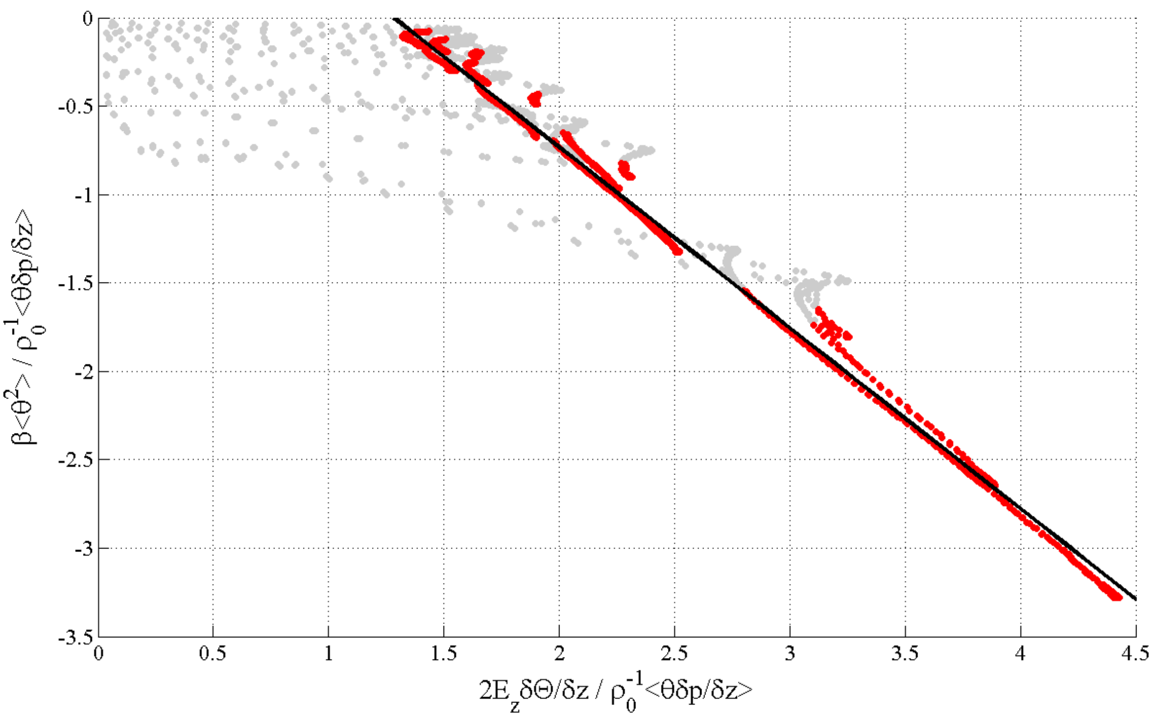

Figure 2: Comparison of $\beta\langle\theta^2\rangle/\frac{1}{\rho_0}\langle\theta\frac{\partial p}{\partial z}\rangle$ and $2E_z\frac{\partial\Theta}{\partial z}/\frac{1}{\rho_0}\langle\theta\frac{\partial p}{\partial z}\rangle$ after the same DNS for stably stratified Couette flow (red dots). The
225 **black solid line corresponds to the linear combination Eq. (31) with $C_\theta = 0.76$ and $C_\nabla = 0.78$.**

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Substituting Eq. (31) into Eq. (30), we rewrite the budget equation as

$$0 = (1 - C_\theta)\beta\langle\theta^2\rangle - (1 + C_\nabla)2E_z\frac{\partial\Theta}{\partial z} - \frac{F_z}{t_F}. \tag{32}$$

Substituting Eq. (15) for $\langle\theta^2\rangle$ into Eq. (32) allows expressing F_z thought familiar temperature-gradient expression:

$$F_z = -K_H\frac{\partial\theta}{\partial z},\; K_H = \left[(1 + C_\nabla) - (1 - C_\theta)\frac{E_P}{E_K}\frac{1}{A_z}\right]2A_zE_Kt_F. \tag{33}$$

Then substituting Eq. (33) into Eq. (14) gives

$$\frac{F_z{}^2}{E_\theta E_K} = 2\left[(1 + C_\nabla)A_z - (1 - C_\theta)\frac{E_P}{E_K}\right]\frac{t_F}{t_\theta}. \tag{34}$$

Similarly to t_τ/t_K approximation (27), we approximate t_F/t_θ as a universal function of z/L (see Fig. 3):

$$\frac{t_F}{t_\theta} = \frac{C_1^{F\theta}z/L + C_2^{F\theta}}{z/L + C_3^{F\theta}}. \tag{35}$$

Here, the dimensionless empirical constants are obtained from the best fit of Eq. (35) to DNS data just like before: $C_1^{F\theta} = 0.015$, $C_2^{F\theta} = 0.7$, $C_3^{F\theta} = 2.7$.

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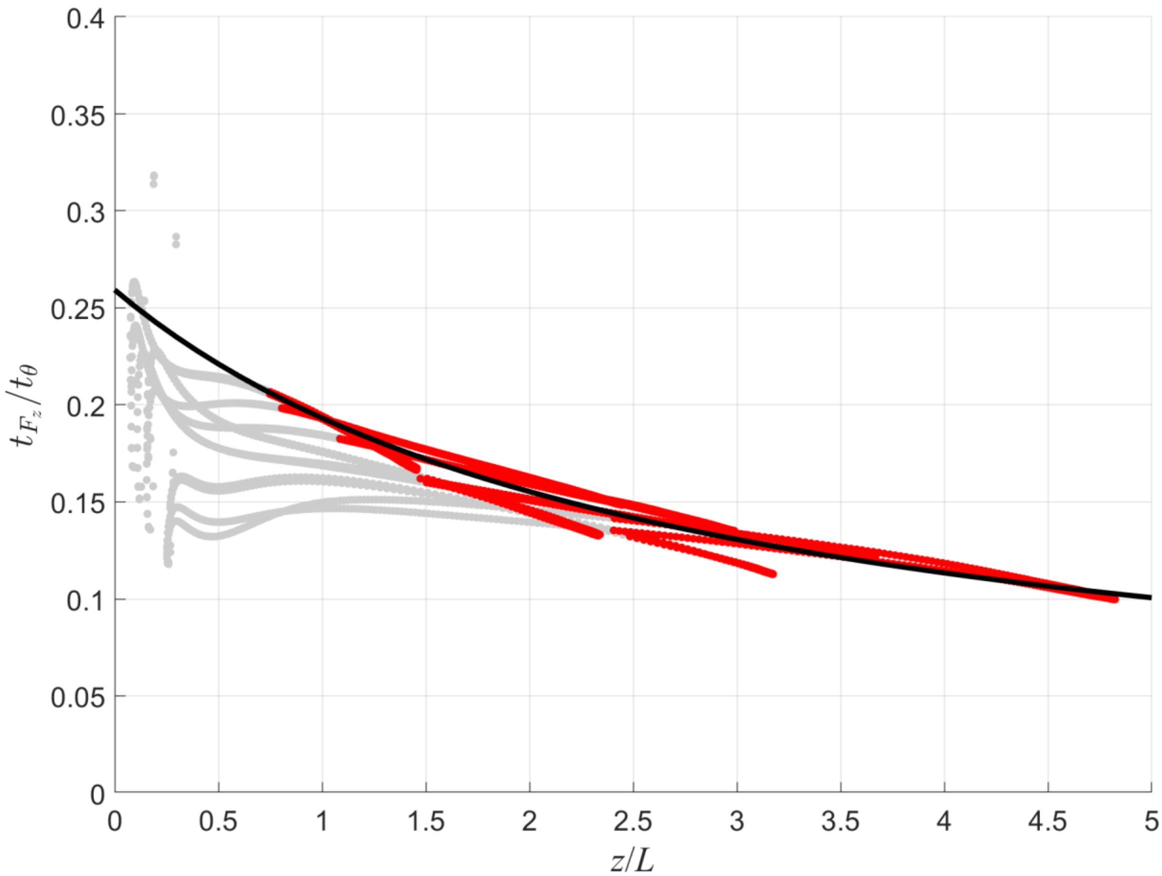

Figure 3: The ratio of the effective dissipation time scale of F_z and dissipation time scale of $\langle\theta^2\rangle$, t_F/t_θ, versus z/L. Empirical data are from the same sources as in Fig. 1. The black solid line shows Eq. (35) with empirical constants $C_1^{F\theta}=0.015$, $C_2^{F\theta}=0.7$ and
240 **$C_3^{F\theta}=2.7$, obtained from the best fit of Eq. (35) to DNS data in the turbulent layer: $z>50\nu/\tau^{1/2}$.**

The turbulent Prandtl number, defined as $\mathrm{Pr}_T = K_M/K_H$, is given by

$$\mathrm{Pr}_T = \frac{t_\tau}{t_F} \Big/ \left[(1+C_\nabla)-(1-C_\theta)\frac{E_P}{A_z E_K}\right]. \quad (36)$$

As shown, e.g., by Zilitinkevich et al. (2013), $\mathrm{Pr}_T|_{(z/L=0)}=0.8$ and $\mathrm{Pr}_T|_{(z/L\to\infty)}\to Ri/R_\infty$.

It leads to the following equations:

245 $$\left.\frac{t_\tau}{t_F}\right|_{(z/L=0)} = (1+C_\nabla)\mathrm{Pr}_T|_{(z/L=0)} \approx 1.4. \quad (37)$$

$$\left[(1+C_\nabla)-(1-C_\theta)\left.\left(\frac{E_P}{A_z E_K}\right)\right|_{(z/L\to\infty)}\right]=0. \quad (38)$$

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To proceed further, it is important to point out that we currently lack any additional information or constraints regarding the energetics of $z/L \to \infty$ asymptotic regime. Therefore, to close our system of equations, we have to make certain assumptions. Based on the DNS data available, we assume that the vertical share of TKE, A_Z, either remains constant or undergoes minimal changes as the stratification increases (see Fig. 4). The available data suggest an average value of $A_Z = 0.17$. Consequently, the asymptotic value of the TPE to TKE ratio would be $\left(\frac{E_P}{E_K}\right)\Big|_{(z/L\to\infty)} \approx 1.26$, corresponding to extremely strong stratification. If future modelling results or natural observations reliably indicate a different value for this asymptote, it would imply that assuming a constant A_Z is an oversimplified approximation. In such a case, a parameterization for A_Z would need to be introduced. However, since we currently lack evidence to support any alternative scenarios, we have chosen the simplest option available.

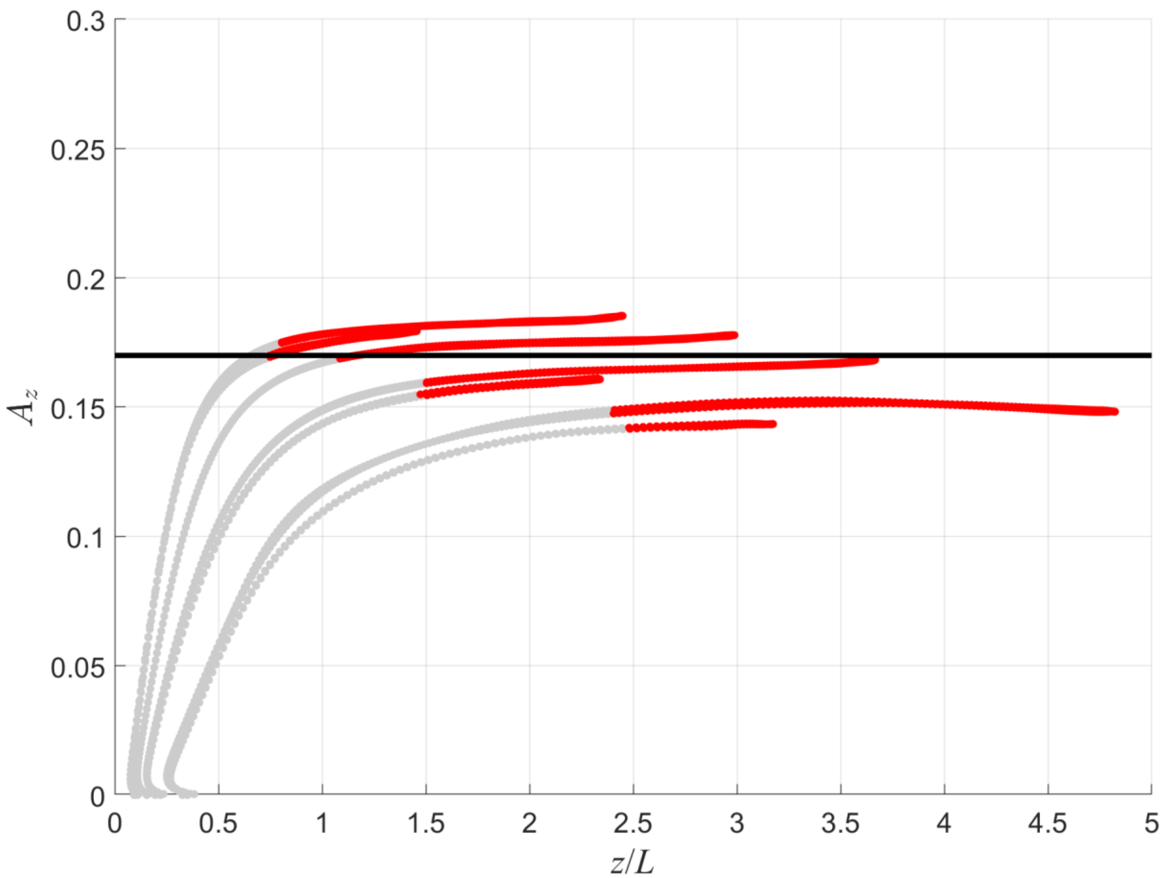

Figure 4: The vertical share of TKE A_Z, versus stratification parameter z/L. Empirical data are from the same sources as in Fig. 1. The black solid line corresponds to $A_Z = 0.17$, which is an average value of A_Z in the turbulent layer, $z > 50\nu/\tau^{1/2}$.

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Now we may revisit the ratio between the dissipation time scale of TKE, t_K and dissipation time scale of $\langle\theta^2\rangle$, t_θ:

$$\frac{t_K}{t_\theta} = \frac{t_\tau}{t_F}\frac{t_F}{t_\theta} \Big/ \frac{t_\tau}{t_K}, \tag{39}$$

where t_τ/t_K and t_F/t_θ are defined by Eqs. (27) and (35).

We approximate t_K/t_θ with a ratio of two first-order polynomials as before,

$$\frac{t_K}{t_\theta} = \frac{C_1^{K\theta} z/L + C_2^{K\theta} C_3^{K\theta}}{z/L + C_3^{K\theta}}. \tag{40}$$

Here we have only one unknown dimensionless empirical constant, $C_3^{K\theta}$, since we know that $C_1^{K\theta} = (t_K/t_\theta)|_{(z/L\to\infty)} \approx 0.2$ and $C_2^{K\theta} = (t_K/t_\theta)|_{(z/L=0)} \approx 1.85$ from Eqs. (37) and (38). The best fit to DNS data gives $C_3^{K\theta} = 11$ (see Fig. 5).

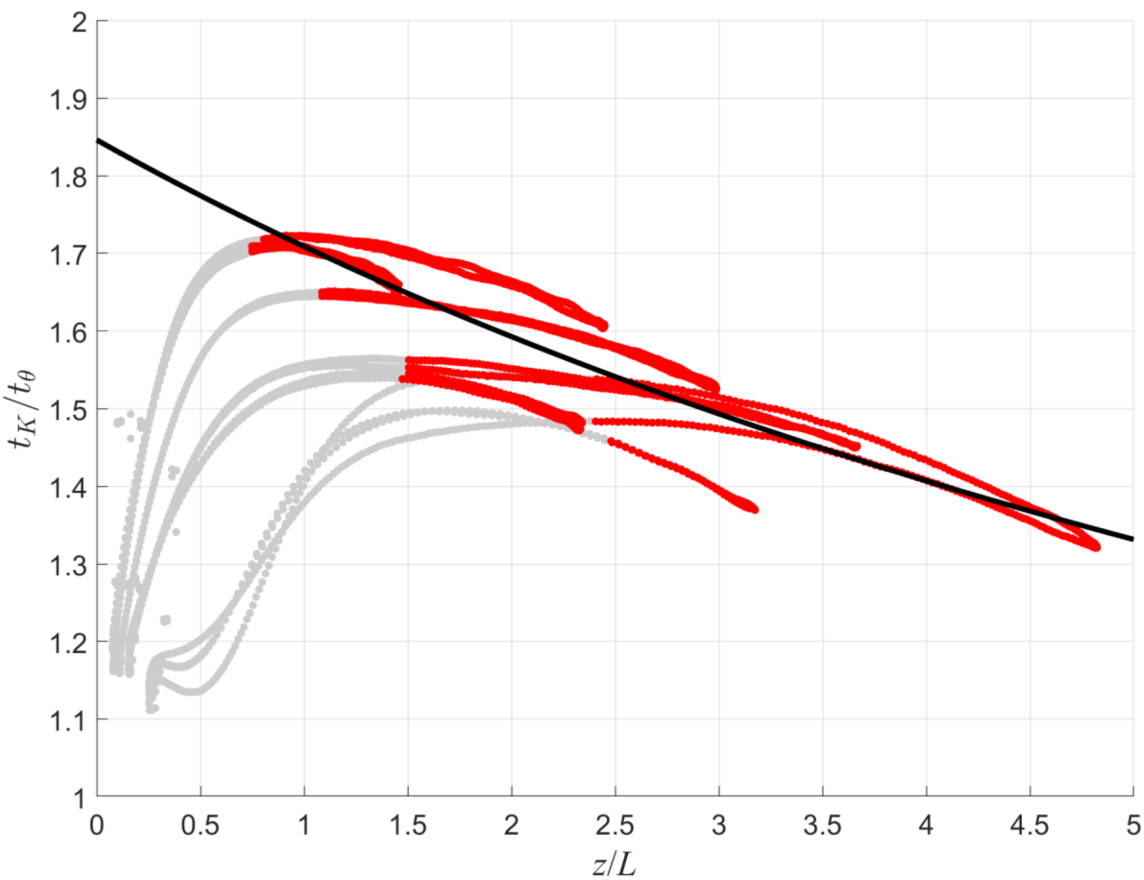

Figure 5: The ratio of TKE and $\langle\theta^2\rangle$ dissipation time scales, t_K/t_θ, versus z/L. Empirical data are from the same sources as in Fig. 1. The black solid line shows Eq. (40) with empirical constant $C_3^{K\theta} = 11$ obtained from the best fit of Eq. (40) to DNS data in the turbulent layer: $z > 50\nu/\tau^{1/2}$.

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With the inclusion of Eq. (40), our turbulence closure is now complete, allowing us to proceed with the validation process using independent energetic dimensionless ratios and DNS results. Figure 6 provides empirical evidence supporting the stability dependencies given by Eqs. (27) and (35).

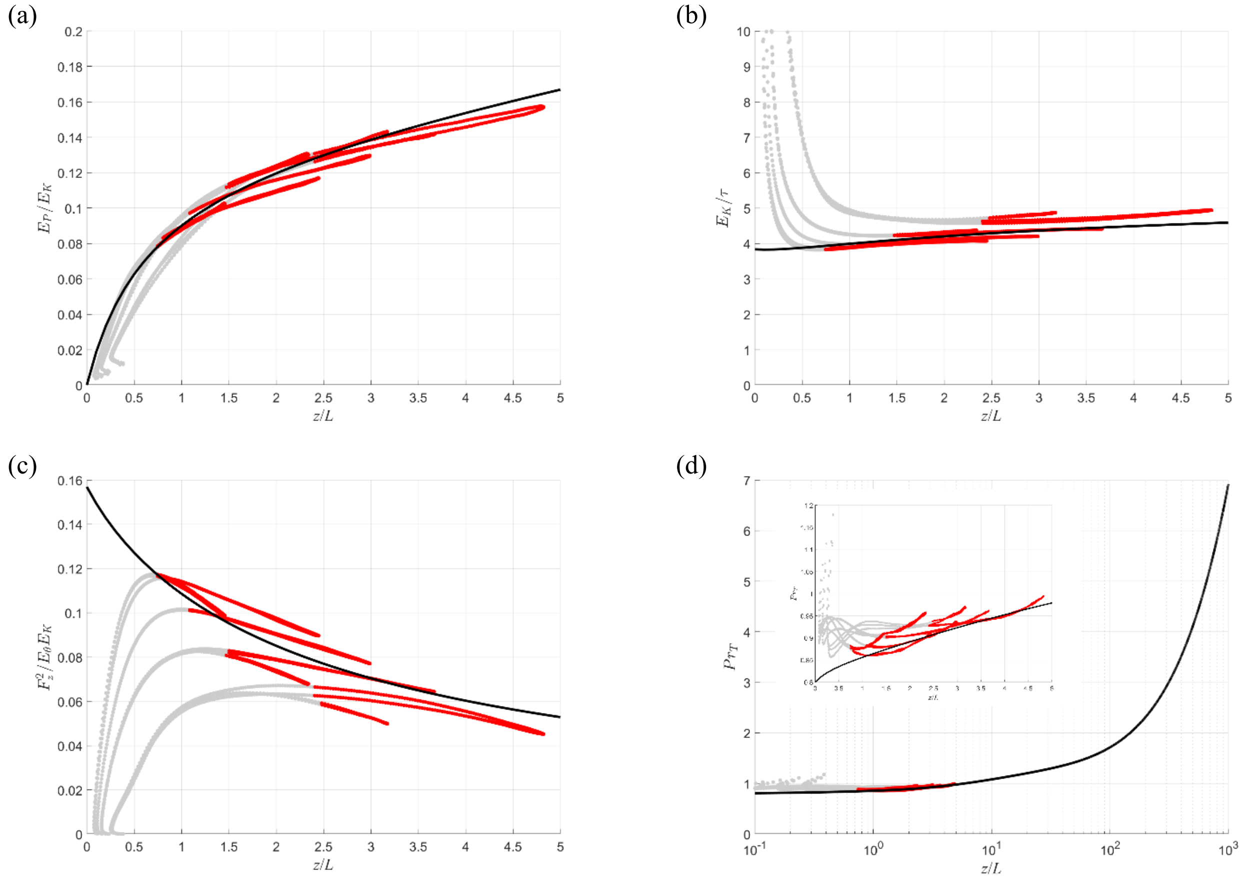

Figure 6: Resulting energetic dimensionless ratios. Panel (a) shows the TPE to TKE ratio, E_P/E_K, versus z/L. The black solid line (Eq. 20) shows a good agreement with the DNS data in the turbulent layer: $z > 50\nu/\tau^{1/2}$. Panel (b) shows the squared dimensionless turbulent flux of momentum, $(\tau/E_K)^2$, versus z/L. The black solid line (Eq. 26) fits the DNS data in the turbulent layer: $z > 50\nu/\tau^{1/2}$ very well. Panel (c) shows the squared dimensionless turbulent flux of potential temperature, ${F_z}^2/E_\theta E_K$, versus z/L. The black solid line (Eq. 34) shows an agreement with the DNS data in the turbulent layer: $z > 50\nu/\tau^{1/2}$. Panel (d) shows the turbulent Prandtl number, Pr_T, versus z/L. The black solid line (Eq. 36) shows a good agreement with the DNS data in the turbulent layer: $z > 50\nu/\tau^{1/2}$. Empirical data are from the same sources as in Fig. 1. There has been no fitting here.

For practical reasons, most operational numerical weather prediction and climate models parameterize these dimensionless ratios as functions of the gradient Richardson number rather than z/L. This preference arises from the fact that the gradient Richardson number is defined by mean quantities only, e.g., square of buoyancy and shear frequencies, which in practice imposes lesser computational restrictions on the model's time step. Since $\mathrm{Ri} = \mathrm{Pr}_T \, \mathrm{Ri}_f$ and both Pr_T and Ri_f are known functions of z/L, Ri is also a known function of z/L. Unfortunately, solving this dependency explicitly every time step at every

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grid point might be computationally expensive (it is a polynomial equation of the 5th degree), so we propose to use yet another approximation. Zilitinkevich et al. (2013) demonstrated that in near-neutral stratification Pr_T can be treated as constant, meaning that $\mathrm{Ri}_f \sim \mathrm{Ri}$, while in the strong-turbulence regime Ri_f is limited by its maximum value of 0.2. We propose to link these regimes through the following interpolation:

$$\mathrm{Ri}_f = \left(\frac{1}{(a\mathrm{Ri})^n} + \frac{1}{(R_\infty)^n}\right)^{-1/n}, \tag{41}$$

where a and n are fitting constants. Fig. 7 shows the best fit with $a = 1.2$ and $n = 5.5$. The relative error for this approximation does not exceed 5% and allows to considerably cut down the computational expenses.

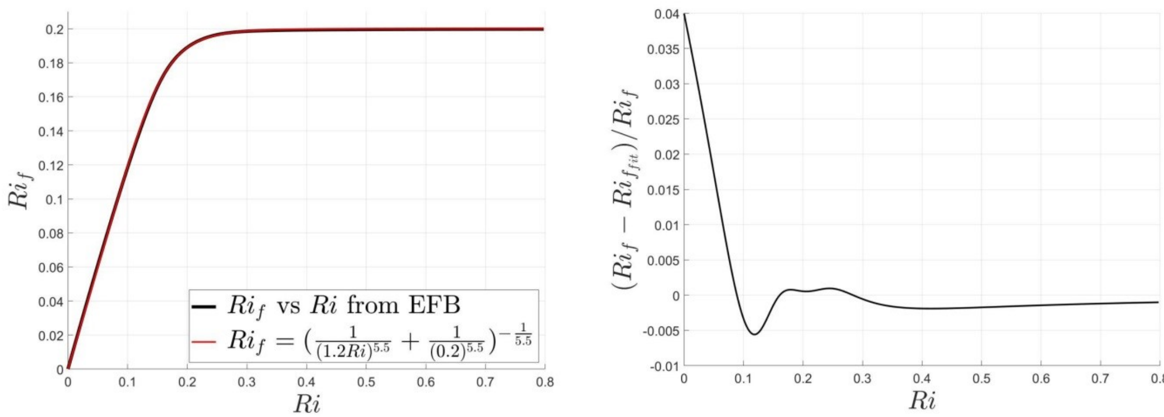

Figure 7: Proposed Ri_f vs Ri approximation, Eq. (41), compared to the exact solution (panel a) and relative error of this approximation as a function of gradient Richardson number, Ri (panel b).

5 Concluding remarks

For many years, our understanding of dissipation rates for turbulent second-order moments has been hindered by a lack of direct observations in fully controlled conditions, particularly in very stable stratification. To address this limitation, we conducted DNS (Direct Numerical Simulation) of stably stratified Couette flows. This allowed us to show that the ratios of dissipation time scales depend on the static stability (e.g., characterized by the gradient Richardson number), contrary to the traditional assumption of them being proportional to one master scale.

Subsequently, we proposed the empirical approximations for these, which serve as simple universal functions of stability parameters across a range of stratifications from neutral to extremely stable conditions. This allowed us to correct the EFB turbulent closure accounting for dissipation time scales shown to be inherent to the basic second-order moments. This approach follows the methodology initially introduced by Zilitinkevich et al. (2007, 2013, 2019). As a result, the revised formulations for eddy viscosity and eddy conductivity reveal greater physical consistency in strongly stratified conditions, thereby enhancing the representation of turbulence in numerical weather prediction and climate modelling.

It is important to note that our DNS experiments were limited to gradient Richardson numbers up to $\mathrm{Ri} = 0.2$. Any data reliably indicating different asymptotic values of the time scale dimensionless ratios or demonstrating their different dependency on the static stability would pose the need for readjusting the proposed parameterization.

Moving forward, the most challenging step will be to explicitly explore the transitional region between traditional weakly-stratified turbulence and extremely stable stratification, where the behaviour of the turbulent Prandtl number shifts from nearly constant to linear one with respect to the gradient Richardson number. Investigating this phenomenon would require unprecedented computational resources for DNS or specialized in-situ or laboratory experiments.

Code and data availability

The DNS code and datasets generated during and/or analysed during the current study are available from the corresponding author upon reasonable request.

Author contribution

EK conceptualised the paper, performed data analysis, wrote the initial text, and prepared the figures. EM contributed to the conceptualisation of the study, developed the DNS code, and performed the numerical simulations. AG contributed to the conceptualisation of the study and code development. NK and IR contributed to the conceptualisation of the study and assisted with literature overview and manuscript editing.

Competing interest

The authors declare that they have no conflict of interest.

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