The SSWs could cause the dramatic change in the temperature, winds and components in the middle atmosphere, which has been widely reported in literature. In contrast, the neutral density evolution during the SSW is less reported. However, the density is one of the most important parameters for the atmosphere drag force calculation in the design of aircraft design. This research reports on the density evolution during the middle atmosphere during the 2021 major SSW event using the observation and simulation. The satellites data show a rapid increase of over 50% in the mesospheric density at high latitudes around the onset date during the SSW. The global view shows that the influenced area extends to the the middle latitudes. Beijing lidar observed obvious density disturbances during the SSW event. The simulation using SD-WACCM demonstrates that the observed enhanced density is primarily attributed to the altered planetary waves and residual circulation during the SSW event. The density variation at altitude layers during the SSW is different from a simple inference by the temperature at certain pressure levels. This study is interesting and the results are sound. However, there are still some very minor concerns that should be considered. Therefore, I recommend a minor revision.

We thank you for your review and helpful comments.

Major comments:

The authors emphasize the importance of the BD circulation for the density change during the SSW, and say that the BDC transport denser air from lower latitudes to higher latitudes. However, the climatological distribution of the air density is not shown in the paper, which is a core plot. Based on Figure 3, I do not see very clearly the contrast of the air density between lower and higher latitudes. Only showing the pressure, the key message is not clearly present.

In the original manuscript, Figure 6d has shown that the temporal evolution of atmospheric density at 56 km. The density over the equator is higher than density over high-latitude in December before the onset, while the density over the equator is higher than density over the equator during the SSW. However, since this figure is a simulation result, we adopt the reviewer's suggestion to add a global distribution corresponding in the revised manuscript, which gives the relative deviation of the global density with the global mean density (Figure 1). In the climatological situation, the density over the equator is higher than the density of the other regions at 20-25 km, and the density over the Southern Hemisphere is overall higher than the density over the Northern Hemisphere at 25-90 km. The global distribution on 20 December 2020 is generally consistent with climatic winter feature. On 4 January 2021, the atmospheric density over the Arctic region at 30-85 km has increased by more than 20% larger than the global mean density, and this increase is relatively significant compared to the climatic Standard deviations. On 7 January 2021, the atmospheric density over the Arctic region at 50-90 km quickly returns to that situation before the onset, while atmospheric density at the 30-50 km is still higher. The maximum increase located at 46 km is also more significant compared to the climatic Standard deviations. This added figure gives the contrast of the air density between lower and higher latitudes. The Figure 2 in the original manuscript demonstrates the contrast between the air density distributions during the SSW and the climatic Standard deviations.

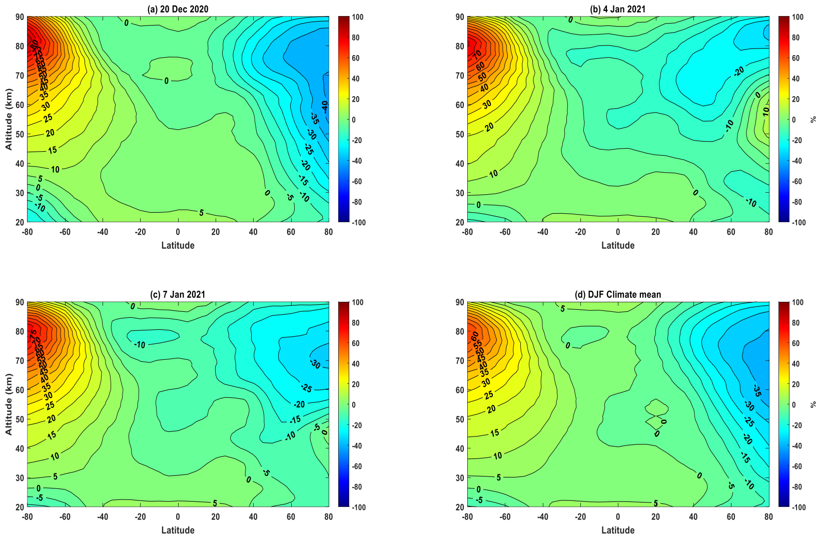

Figure 1: Relative deviations between the zonal mean density and the global mean density: (a) November 20, 2020; (b) January 4, 2021; and (c) January 7, 2021 from Aura/MLS data and (d) climatic winter average obtained for the period from 2004 to 2021.

Some jargons should be defined in the main text or in the figure captions. The authors claim that they show the deviation for the air density relative to the climatology. But the legend shows the relative change with a unit %. So where can the readers find the definition for the air density deviation (or relative deviation)?

Since the atmospheric density decreases exponentially with altitude, relative density deviation was used to represent the density change. We have added the definition for the relative density deviation δ at each altitude as:

$$\delta = \frac{\rho_o - \rho_r}{\rho_r} \times 100\%$$

where ρ_o is the observed density and ρ_r is the reference density. We have also revised the other similar details.

The authors only emphasize the dynamics importance for the change of the air density. But actually, the ideal state equation also explains the change of the air density in some circumstances. P=rho*R*T. You might find that the P and the T increase suddenly during the SSW, so which increases faster? If the P increases faster than the T, the increase in rho is also true, and the ideal state equation can also explain the change in the air density. I suggest to include a comparison between the thermodynamics and dynamics.

In the original manuscript, we have discussed that the temperature cooling magnitude of only approximately 20 K (8%) at 58 km could not cause the density enhancement of more than 50% recorded in the observational data. Figure 2 show the temporal evolution of the temperature and density at the isobaric levels and the corresponding approximate height is indicated in the right ordinate. This isobaric-level form temperature result is commonly given at present, such as Figure 1 in Manney et al. (2009), Figure 1 in Chandran and Collins (2014), Figure 4 in Kodera et al. (2016) and Figure1 in Lu et al. (2021). The accurate density information is hardly Most previous studies only give the atmospheric temperature change (Figure 2a), not the density and pressure, which unable to provide accurate density information to the aircraft designers and the aircraft industry. Since the atmospheric drag calculated on flight experiments should considers the density at a given altitude, the altitude results in our work are very meaningful. This large density increase should cause more attention in the calculation of atmospheric drag. The above discussion will be added in the revised manuscript.

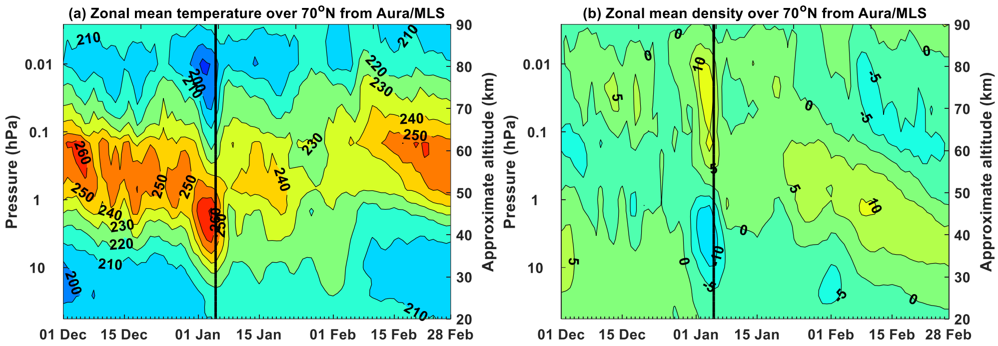

Figure 2: Temporal evolution of the temperature and density from December 1, 2020, to February 28, 2021. (a) Zonal temperature and (d) the density deviation between that during the SSW event and the climatology over 70 °N from the MLS data. The vertical black lines indicate the onset date.

Minor comments:

Line 23: Please be more specific for the "upper air model". I do not understand.

The "upper air model" means the reference atmosphere model used in the design of aircraft (Hale et al., 2002). We have added a description in the revised manuscript, as "standard upper air models, such as the 1976 US reference atmosphere".

Line 32: These references are too old and some most recent publications for the Southern Hemisphere SSW should be mentioned (doi: 10.1029/2020JD032723)

Thank you for the useful recommended manuscript. It is added in the revised manuscript.

Line 54: Please clarify the necessity of the density investigation.

It is reported that a dramatically varying actual atmospheric density often contributes to a negative angle of attack bias, which can have adverse thermal consequences.

Line 98: When introducing Beijing lidar, consider providing the detection principle or method.

We have added the detection principle: "The lidar emits pulsed laser, which elastically collides with atmospheric molecules. The received backward Rayleigh scattering signal can be captured by the detector of the lidar system and is used to calculate the atmospheric density and temperature."

Figure 3: The negative values are all too weak. Please modify the legend colors to match the figure well.

Thank you for your advice. The Figure 3 has been modified in the revised manuscript.

Line 230: Revised the sentence "the lidar density data constitute a directed parameter".

We have revised this sentence: "Furthermore, as the lidar density data is a directed parameter by Beijing lidar system, the consistency between lidar observations with the Aura/MLS observations confirms the computational methods described in Sect. 2. "

Figure 5: Here is not the PW1/2, but the wave amplitude? (see Line 234)

The reviewer is right. We have revised it.

Line 267: I can not see the climatological density contrast between the lower latitudes and higher latitudes.

We have added a global climatological density distribution (Figure 1d). In climatic winter average situation, the density over the equator is higher than the density over higher latitudes at 20-25 km, and the density over the Southern Hemisphere is overall higher than the density over the Northern Hemisphere at 25-90 km. The feature on 20 December 2020 is generally consistent with the climatology.

Line 300: The sharp increase in air density is observed during the 2021 SSW. Whether the similar variations have occurred during other SSWs and what is the difference?

According to the classification based on the stratospheric response (Kodera et al., 2016), 2022 major SSW is more of a mixed type of warming event. We investigated two other typical style SSW. The 2006 major SSW was a typical displacement type event triggered by the PW1 (Chandran and Collins, 2014), while the 2018 major SSW was a split type event by PW2 (Rao et al., 2018). The sudden positive anomalies of neutral air density are also observed around the onset date in the 2018 and 2006 major SSW. In 2018, the maximum value of 49.54% at 48 km occurs on 15 February lags 4 days after the onset date. In 2006, the maximum value of 51.44% at 61 km occurs on 11 January, 11 days before the onset date.

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