

## Supplementary Information 4

### Results of the Crystal Orientation Fabrics using the Laue X-ray method for the deep section of the Dome Fuji ice core

First row: The crystal orientation fabric is viewed along the beam propagation direction during dielectric measurements. The green triangle indicates the vertical direction, while the red dot represents the  $c$ -axis and the blue dot represents the  $a$ -axis. This projection is obtained by rotating the crystal orientation fabric data from thin sections used for Laue diffraction by  $90^\circ$ . The depth and the total number of crystal grains measured within the thin section are shown.

Second row: The crystal orientation fabric is viewed from the direction normal to the maximum inclination of the internal layer, rotated around the vertical axis. The green triangle again marks the vertical direction. The  $c$ -axis is most concentrated at the edge of this diagram. When electromagnetic waves are directed from this angle, the dielectric anisotropy is at its maximum. However, in practice, the waves are directed from the angle shown in the first row. As a result, the  $c$ -axis concentration deviates from the diagram's edge, causing the measured dielectric anisotropy to be less than the maximum possible value. This discrepancy becomes more noticeable at depths greater than 2700 meters.

Third row: The  $c$ -axis concentration is centered in the diagram. The green triangle still indicates the vertical direction. The gap between the vertical direction and the  $c$ -axis concentration increases with depth. At depths greater than 2975 meters, the number of measurements is too small to accurately determine the  $c$ -axis concentration. Letter symbols are used to explore the angular relationships between crystal grains.

Fourth row: The normalized density  $\rho$  of the  $a$ -axis along the girdle plane of the  $a$ -axis is displayed.  $\theta$  (degrees) refers to the angle from the top of the diagram.

