

Dear reviewer, we appreciate your comments. We provide a detailed answers below in the form of bullet points.

Reviewer comment: [The manuscript deals with a classical 1D analysis of the unsaturated flow stability during a wetting event.](#)

- It is important to note that the manuscript is not focused on 1D analysis, but 2D analysis instead. Specifically, we show how 2D flow varies with different inflows changing from diffusion flow to fingering flow and back to diffusion flow as the infiltration rate increases (viz results sections 3.1 – 3.5 and discussion section 4).

In the case of 1D flow, there exist a few models that are able to capture the transition between diffusion and finger-like behavior for various infiltration rates, for instance [Cueto-Felgueroso, Juanes, 2009]. However, the 2D case is significantly different because not only this transition but also finger spacing and finger width are addressed [DiCarlo, 2009]. This makes the analysis much more complex and complicated. One of the main evaluating factors according to DiCarlo is that the model “can produce predictions of the 2-D and 3-D preferential flow in terms of finger widths and finger spacings.” In our manuscript, we have demonstrated that the semi-continuum model is capable of this. Moreover, to the best of our knowledge, no such model has yet been available to simulate this complex 2D behavior.

Let us note that all simulations performed in the manuscript are in 2D. The exception is section *3.6 Wetting front instability*, where we analyze the connection between 1D and 2D flow in unsaturated porous media. This section is included to demonstrate that we are consistent with experimental observations, so that the saturation overshoot in 1D is closely related to the preferential flow in 2D. Again, to the best of our knowledge, such a connection has not been demonstrated by any model so far.

To make this as clear as possible, we will highlight it in the manuscript in more detail. And we also plan to change the title of the manuscript to make it clear that we are dealing with 2D simulations.

Reviewer comment: [The numerical treatment of the problem at stake, although quite standard and therefore with no new insight, appears to be correct \(I went through it, and I didn’t find any error\).](#)

- The numerical discretization of the governing equation is indeed standard with one major exception. The discretization of the Prandtl-type hysteresis operator is not common in flow modeling as the discretization is dependent on the size of the used mesh (blocks). However, this is not a key part of the manuscript as the semi-continuum model has been already published, e.g., in [Vodák et al., 2022] and [Kmec et al., 2023].

Reviewer comment: [Instead, the English usage requires a very solid and sounding proofreading.](#)

- English will be proofread for the revised manuscript.

Reviewer comment: [Besides this \(marginal\) aspect, the main issue which I see with the manus is of methodological nature. In particular, my skepticism is two-fold. First, accounting for the gravity solely \(thus neglecting the impact of retention\) may be unrealistic especially if one is interested \(as it is usually happens in the applications\) on the “onset” of the stability vsinstability”.](#)

- The semi-continuum model describes the flow of water in unsaturated porous medium. Water is characterized by saturation in the equation, since saturation is defined as the ratio of the total water volume to the pore volume. This is a standard approach in modeling multiphase flow [DiCarlo, 2009].

Note that the gravity is essential in forming the finger flow and is therefore included in the semi-continuum model. However, retention is not neglected at all; the water retention is controlled mostly by the pressure-saturation relation (the water retention curve) defined by equations Eq. (2) and Eq. (3). Moreover, according to Eq. (6), the gravity term takes a role only in the case of vertical fluxes, while the term related to the retention curve is included for horizontal fluxes as well. Again, this approach is standard to model water retention in porous medium, for example using the Richards’ equation [Simunek, 1994].

Reviewer comment: [Second, the authors have carried out a long and intensive analysis of the flow rates which make \(or not\) stable the flow, but what and where is the stability analysis?](#)

- There is probably a misunderstanding in the used terminology and we apologize for that. The stability refers to diffusion-like flow where no saturation overshoot is observed, and the instability refers to preferential flow (or finger-like flow) that is accompanied by the saturation overshoot. This is standard terminology used in the community [DiCarlo, 2013]. However, to make it as clear as possible, we will explain it the manuscript and change the text accordingly. Also, we plan to change the title of the manuscript to: *Modeling 2D gravity-driven flow in unsaturated porous media for different infiltration rates*.

A short part of the manuscript deals with the instability of the wetting front (section 3.6 *Wetting front instability*). As mentioned in the response to the first reviewer’s comment, in this section we analyze the connection between saturation overshoot in 1D and preferential flow in 2D. However, this is not the main part of the manuscript because we focus on 2D preferential flow and show that the semi-continuum model correctly captures flow experiments that no model has captured before.

Reviewer comment: [The very new and innovative insight could have been a Turing analysis of the stable/unstable flow patterns in order to highlight which ones are those parameters \(and perhaps the infiltration rate is the most important one\) that regulate such a stability. Instead, the manuscript, as it is, is nothing more a numerical analyses \(followed by an experimental benchmark\), quite similar to many others, already existing in the literature.](#)

- Thank you for your suggestion to include analysis using Turing instability and Turing pattern concept. However, the manuscript addresses different topic as the stability/instability has different meaning (see the response to the previous comment). It deals with 2D preferential flow as a function of applied influx. Specifically, we show that the model is able to predict 2D preferential flow in terms of finger widths and finger spacings.

According to “quite similar to many others, already existing in the literature”: We conjecture that there are no other models that are able to correctly capture the transition from diffusion to finger-like and back to diffusion flow as the infiltration flux increases. In our humble opinion, the most promising model is a nonlocal model proposed by [Beljadid et al., 2020], however, the flow is preferential for very low fluxes which is not consistent with experiments. This is not the case for the semi-continuum model presented in our manuscript. The model correctly captures the influx dependence even for very low fluxes and, in addition, the finger widths and finger spacings are consistent with experimental observations, as shown in section 3.4 *Finger width as function of influx*.

However, if there is another model dealing with the same topic (i.e., 2D preferential flow as a function of applied influx), we would like to ask the reviewer for a reference, where this issue is properly addressed. We would very much appreciate that.

References

- [DiCarlo, 2009], D. A. DiCarlo, Can continuum extensions to multiphase flow models describe preferential flow? *Vadose Zone J.*, DOI: 10.2136/vzj2009.0099
- [Simunek, 1994], J. Šimunek and D. L. Suarez, Two-dimensional transport model for variably saturated porous media with major ion chemistry, *Water Resour. Res.*, DOI: 10.1029/93WR03347
- [DiCarlo, 2013], D. A. DiCarlo, Stability of gravity-driven multiphase flow in porous media: 40 Years of advancements, *Water Resour. Res.*, DOI: 10.1002/wrcr.20359
- [Cueto-Felgueroso, Juanes, 2009], L. Cueto-Felgueroso and R. Juanes, A phase field model of unsaturated flow, *Water Resour. Res.*, DOI: 10.1029/2009WR007945
- [Vodák et al., 2022], R. Vodák and T. Furst and M. Šír and J. Kmec, The difference between semi-continuum model and Richards’ equation for unsaturated porous media flow, *Scient. Rep.*, DOI: 10.1038/s41598-022-11437-9

[Kmec et al., 2023], J. Kmec and M. Šír and T. Furst and R. Vodák, Semi-continuum modeling of unsaturated porous media flow to explain Bauters' paradox, *Hydrol. E. Sys. Sci.*, DOI: 10.5194/hess-27-1279-2023

[Beljadid et al., 2020], A. Beljadid and L. Cueto-Felgueroso and R. Juanes, A continuum model of unstable infiltration in porous media endowed with an entropy function, *Adv. Water Resour.*, DOI: 10.1016/j.advwatres.2020.103684