Dear Editor

My co-authors and I are very grateful for your positive recommendation on the manuscript (Manuscript Number: EGUSPHERE-2023-2710) submitted to 'Hydrology and Earth System Sciences'.

We highly appreciate your insightful comments. We have carefully read the reviewers report, and we have made corrections in the revised manuscript accordingly. Here are our responses to each of the questions in the reviewers' report, which are reproduced in italics. The newly added text has been marked in blue.

Response to reviewer #1:

1. *While the authors have made commendable efforts to make their paper more accessible for HESS, based on the earlier reviews, my initial doubts from the first revision about whether this article fits the scope of HESS are not really taken away. Now the authors mention in L90-95 the 'pedagogical aspect' of the work, but that also requires some form of relatedness to the field of the learner (the HESS community). The authors mention the "fundamental problem [...] starting with one-step ahead prediction" and then give no context for this problem, nor any mention of nowcasting studies, or any physics-based rainfall model study.*

   We have noted your concern regarding the manuscript's suitability for HESS. To address this, we have revised the following paragraphs to the introduction to better frame our work in the context of hydrological applications and to emphasize its pedagogical value for the HESS community.

   The main purpose of this study is to provide a reliable one-step-ahead rainfall prediction for hydrological applications, particularly urban flood forecasting and water resource management. This addresses the fundamental challenge in operational hydrology where accurate short-term precipitation forecasts are essential for timely flood warnings and infrastructure management. In order to achieve this objective, it is essential to fully extract the underlying patterns of rainfall time series while preserving their multiscale intermittency structure - a critical requirement for hydrological modeling where extreme events often dominate system response. An additional crucial point is to develop prediction models with a satisfactory level of accuracy for practical implementation in operational hydrological systems. According to the aforementioned two factors, this study implements a hybrid approach known as VMD-RNN, which combines different Recurrent Networks models (RNN) with Variational Mode Decomposition (VMD) to take into account the multiscale nature of precipitation. Moreover, a scaling technique is used to optimize the width of decomposition.

   The effectiveness and reliability of the employed VMD-RNN approach are extensively validated by applying this method to forecast the following step's rainfall in both daily and hourly resolution, representing different temporal scales relevant to hydrological practice.

Furthermore, a comparison study is carried out to further demonstrate the superiority of the adopted VMD-RNN model, in comparison to the baseline method, the pure Long Short-Term Memory model (LSTM) model without decomposition, and linear regression method. In addition, a Universal Multifractal (UM) technique is used to confirm the ability of the predicted time series to accurately describe rainfall variability, ensuring that the predicted series maintain the multifractal properties essential for accurate hydrological modeling and flood risk assessment.

Given the growing usage of deep learning in hydrological research, it is important to bridge the knowledge gap for readers who are not familiar with deep learning models. The pedagogical aspect of our work has the potential to contribute to the hydrology community by providing a deeper understanding of the application of deep learning models and multifractal techniques in short-term rainfall prediction that remains a fundamental problem of hydrology starting with one-step-ahead prediction. This work specifically addresses the need in the HESS community for accessible methodological advances that maintain strong connections to hydrological theory and practice, demonstrating how modern deep learning techniques can enhance traditional approaches to precipitation forecasting while preserving the physical understanding of rainfall processes essential for water resource management.

2. *The Results section is still very meager (though I do appreciate the inclusion of the dataset details now), and while figures 11-13 give some measure of improvement over LSTM and a benchmark linear regression (the authors also don't mention the LR parameters), there is still some very large errors especially at the lowest rainfall values (<4 mm/hr) where there are either False Zeroes or High Influx related errors. This is not discussed anywhere, whereas that results discussion is crucial for any form of applicability: does the model perform poorly at low intensity so is it only useful for applications near the high-end of rainfall distributions, are there ways this can be alleviated, etc.*

We agree with your assessment that the Results section required a more detailed discussion, particularly concerning the model's performance at different rainfall intensities. We have substantially revised the Results section to provide a more in-depth analysis of the model's performance for both daily and hourly rainfall, including a discussion of the errors at low rainfall values.

4.1 Daily rainfall series

Figure 8 shows the predicted daily time series in the testing set. It compares the predicted results of the VMD-RNN hybrid model, the pure LSTM model and the linear regression method with the actual data. It can be clearly observed that the hybrid model has a better fit for most of the points, particularly during periods of high-intensity rainfall events that are critical for flood forecasting applications. The VMD-RNN model demonstrates enhanced capability to capture rainfall variability patterns, including the temporal clustering of precipitation events that characterizes real rainfall processes.

The comparison of prediction performance with and without VMD for daily time series in the testing set can be seen in Figure 9. The scatter plot demonstrates that the VMD-RNN model has superior performance in predicting both high and low values for daily time series, whereas the baseline models LSTM and linear regression exhibit systematic biases. Notably, the VMD-RNN model shows improved performance in predicting extreme rainfall events, which are crucial for urban flood warning systems. The predicted values obtained by the baseline models exhibit considerable deviation from the best linear fitting line (blue dotted line), with a tendency to underestimate high-intensity events - a critical limitation for hydrological applications where accurate prediction of extreme events directly impacts flood risk assessment and emergency response effectiveness.

All the parameter values estimated using the TM and DTM methods are listed in Table 6. The values of α and C_1 obtained using the DTM technique show slight differences from those estimated by TM, but remain within acceptable ranges for multifractal analysis. Importantly, the VMD-RNN predicted time series preserves multifractal properties more effectively than LSTM without decomposition, as evidenced by UM parameters that are closer to those of actual rainfall. This preservation of scaling properties is crucial for hydrological applications where the multifractal structure of rainfall directly influences runoff generation, infiltration processes, and the temporal distribution of streamflow in urban catchments.

4.2 Hourly rainfall series

Figure 11 displays the hourly time series in the testing set with 1024 data points. The qualitative analysis reveals that the predictive performance differences between VMD-RNN, pure LSTM, and linear regression are less pronounced for hourly rainfall time series compared to daily predictions. This reduced benefit of decomposition for hourly data can be attributed to the inherently higher noise level and lower signal-to-noise ratio characteristic of high-frequency precipitation measurements, which limits the effectiveness of decomposition techniques in extracting meaningful frequency components.

Figure 12 depicts the comparison between predicted and actual hourly rainfall values. The scatter plot reveals that the predicted values from VMD-RNN basically agree with the corresponding actual values, but the values predicted from the baseline LSTM model do not yield the same level of alignment. While the VMD-RNN model shows reasonable agreement with actual values for moderate to high rainfall intensities, significant challenges become apparent for low-intensity precipitation events.

The UM analysis results for hourly time series (Figure 13) and estimated parameters (Table 7) indicate that the predictive performance of VMD-RNN is comparable to pure LSTM for hourly data, without demonstrating the substantial benefits observed for daily predictions. The UM parameters α and C_1 show similar values between VMD-RNN and LSTM predictions, suggesting that both approaches preserve multifractal properties to a similar degree at hourly resolution. This finding reflects the scale-dependent effectiveness of decomposition techniques, where the benefits become more apparent at longer timescales where signal-to-noise ratios are higher and frequency separation is more pronounced.

3. *Additionally, the figure captions seem to be missing or are in any case in complete; figure quality is also quite poor, and in figure 12 there are no axis labels so indicate what is being displayed.*

   We apologize for the issues with the figures. We have improved the quality of all figures, added the missing axis labels, and ensured all captions are complete and descriptive.

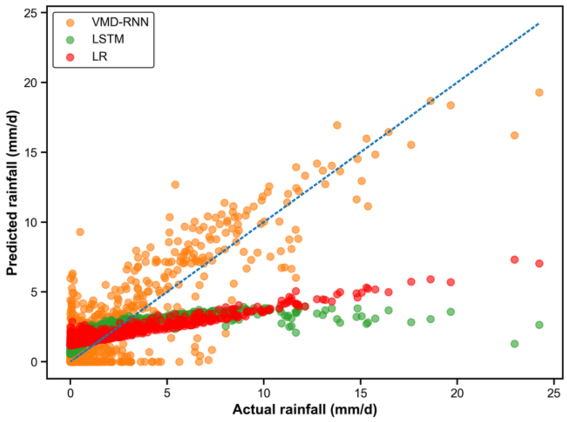

Figure 9. The comparison between predicted and actual daily rainfall values

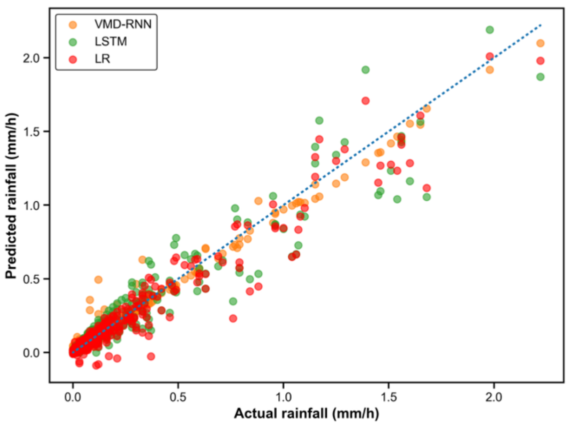

Figure 12. The comparison between predicted and actual hourly rainfall values

Response to editor:

1. *The latter mainly indicates that the paper's results are meager (I agree but on the other hand, the paper makes one point and does so with clear focus. So that is fine for me.*

   Thank you for your comment. We are pleased that you found the focus of the paper to be clear. At the same time, we hope that the current modifications make more obvious the results and their importance.

2. *The second point however, is that the paper is not that accessible to the Hess community (being hydrologists and hydrometeorologists). If I read the discussion and conclusion sections, I do see (and agree with) that point. The discussion, for example, has no references, no broader context. It is more like three paragraphs technical summary of the work. This broader (hydrological application) discussion of your results would be of added value. This then also should tickle down in the conclusions section.*

   We agree that the Discussion section needed to be expanded to provide broader context and better connect our findings with the hydrology community. We have completely rewritten the Discussion section (now Section 4.3) to address this, including a comparison with existing approaches, a detailed explanation of the hydrological significance, and a discussion of the model's limitations.

   4.3 Discussion

   4.3.1 Comparison with existing approaches

   The hybrid VMD-RNN approach addresses several limitations of traditional hydrological forecasting methods. Compared to physically based numerical weather prediction models, data-driven approaches like VMD-RNN can provide more computationally efficient solutions for short-term rainfall prediction, particularly for local-scale applications where high-resolution atmospheric models may not be practical or cost-effective (Wilks, 2011). However, it is important to note that these models complement rather than replace physical understanding of hydrological processes.

   Nowcasting systems, which typically rely on radar observations and numerical weather prediction models, face challenges in accurately predicting the timing and intensity of precipitation events (Hess and Boers, 2022). The VMD-RNN approach could potentially be integrated with existing nowcasting frameworks to improve short-term precipitation forecasts, particularly when combined with radar-based observations and ensemble forecasting techniques.

   The multifractal analysis using Universal Multifractals provides additional insights into the scaling properties of rainfall that are not captured by traditional error metrics. The closer agreement of VMD-RNN predictions with observed multifractal parameters suggests that the model better preserves the natural variability structure of rainfall processes across scales (Schertzeret al., 1997). This is particularly important for hydrological applications where the temporal distribution and intensity patterns of rainfall can significantly affect runoff generation and flood risk.

   4.3.2 Hydrological significance and applications

   The results of this study demonstrate that the VMD-RNN hybrid approach offers significant advantages for rainfall prediction in hydrological contexts, particularly at daily time scales.

This improvement has important implications for operational hydrology and water resource management.

The enhanced performance of VMD-RNN in capturing extreme rainfall events is particularly valuable for flood early warning systems. Extreme precipitation events are the primary drivers of flash floods and urban flooding, and their accurate prediction can provide crucial lead time for emergency response and flood mitigation measures (Berne et al., 2004; Beven, 2012). The improved prediction of high-intensity events could enhance the reliability of flood forecasting systems and reduce false alarm rates, which are critical factors in maintaining public trust and ensuring effective emergency response (Demeritt et al., 2007).

4.3.3 Model limitations and uncertainties

Despite the promising results, several limitations must be acknowledged. The currennt systematic overestimation of low-intensity rainfall represents a significant challenge for practical applications. This bias could lead to overestimation of cumulative precipitation over extended periods, affecting water balance calculations and long-term hydrological planning (Gardiya Weligamageet al., 2023). The issue of false positives in low-intensity predictions is a common challenge in precipitation forecasting and requires careful consideration in operational applications.

The temporal scale dependency observed in our results, where VMD decomposition shows greater benefits for daily compared to hourly predictions, suggests that the approach may be most suitable for applications requiring daily to weekly rainfall forecasts. This scale dependence may be related to the frequency content of rainfall signals, where longer time scales contain more distinct frequency components that can be effectively separated by VMD.

The current study focuses on a single location with a temperate climate. The performance of VMD-RNN may vary significantly across different climatic regions, particularly in areas with distinct wet and dry seasons, monsoon climates, or arid regions where rainfall patterns differ markedly from those observed in our study area. Further validation across diverse climatic conditions is essential for establishing the general applicability of the approach.

5. Conclusions and future work

In this study, the hybrid VMD-RNN model was used as a methodology for forecasting rainfall with a one-step lead time. The integration of variational mode decomposition with recurrent neural networks demonstrates significant potential for improving rainfall time series prediction accuracy, particularly for extreme events that are critical for flood risk assessment.

VMD was first used to extract hidden information to understand the complex original time series. Then variants of RNN were applied to handle problems involving sequential prediction. By combining the dominant characteristics of VMD in decomposing nonlinear time series and the favourable performance of variant RNN models in predicting complex sequential problems, the hybrid model based on VMD and RNN was employed to predict rainfall time series with daily and hourly resolution. The framework of UM was subsequently introduced to evaluate the performance of predicting rainfall time series. This adds a non negligeable originality to our current model and moreover points a direction for further improvements.

According to the above study, the following conclusions could be drawn:
(1) The VMD-RNN hybrid approach successfully addresses the challenge of predicting highly variable rainfall time series by decomposing the signal into frequency-specific components. The determination of optimal decomposition levels through power spectral density analysis provides a systematic approach for model configuration.
(2) For daily rainfall prediction, the VMD-RNN model significantly outperforms pure LSTM models, particularly in capturing extreme rainfall events that are crucial for flood forecasting applications. The improvement in prediction accuracy has direct implications for early warning systems and flood risk management.
(3) The closer agreement of VMD-RNN predictions with observed universal multifractal parameters demonstrates that the model better preserves the natural scaling variability of rainfall processes. This validation using α and C_1 parameters provide additional confidence in the model's ability to represent the complex intermittent nature of precipitation.
(4) The benefits of VMD decomposition are more pronounced at daily compared to hourly time scales, suggesting that the approach may be most effective for applications requiring daily to weekly rainfall forecasts rather than sub-daily nowcasting.

However, there are still some limits to this study, and corresponding improvements will be implemented in future work. First, extending the approach to multi-step-ahead predictions would significantly enhance its practical utility for hydrological applications. Second, incorporating spatial information through the development of spatially distributed VMD-RNN models could improve rainfall prediction for catchment-scale applications. Third, the integration of physics-informed constraints into the VMD-RNN framework could help address some of the observed limitations, particularly the overestimation of low-intensity rainfall. Finally, the development of ensemble forecasting capabilities would provide valuable uncertainty information for decision-making.

3. *Please also provide higher quality figures.*

We have replaced all figures with high-resolution versions as requested, particularly Figures 9 and 12.