GENERAL COMMENTS

The manuscript presents numerical analysis of the thawing of ice-saturated cleft in rocks, demonstrating the relative importance of gravity-driven convection of liquid water due to the subtle increase in density as the temperature rises from 0 to 4 degrees. While it is not difficult to imagine the significance of free convection based on observations in other environments, it is important to advance the quantitative understanding of the effects of free convection in ice-filled clefts. Therefore, this study has the potential to make a significant new contribution to cryospheric sciences. The manuscript is reasonably well organized and written in a clear language. However, it is missing some essential information on the methodology and as such, it is difficult for the reviewer to evaluate the rigor of numerical and experimental methods in some places. Theoretical interpretation of numerical modelling results are sound except for some missing details (see above), but the comparison between numerical results and field observation is much weaker. To strengthen the comparison, I suggest that the authors consider the following: (1) enhance the description of field methods, (2) acknowledge the magnitude of uncertainty in flow measurements more specifically, and (3) use independent evidence to support the match between model results and field observation. I will elaborate more on these in my specific comments below.

We thank the reviewer for his constructive comments which will improve the quality of the manuscript. Since many questions of the reviewer deals with the model definition, we copy and paste the new version of the governing equation section that we tried to clarify. Our detailed answers to the specific questions of the reviewer follow. A list of reference cited in our answers is provided at the end of this document.

##

NEW SECTION 2.2

"2.2 Governing equations

The temperature T is the only dependent variable in the impermeable rock domain. It satisfies the standard heat equation

$$\rho_r c_{p,r} \frac{\partial T}{\partial t} = k_r \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (1)$$

where ρ_r, $c_{p,r}$ and k_r are the rock density, specific heat and thermal conductivity, respectively.

The water domain is more intricate. The velocity field must be calculated in the liquid part of the domain, but reduces to zero in the frozen region. To avoid the difficult task consisting in tracking the moving boundary between ice and liquid water, we adopted a strategy that allows to define the same set of dependent variables and governing equations in the entire water domain. To this end, we do the approximation of smooth phase transition between solid and liquid phases. We assume that ice melting begins à temperature $T_{m1} = T_m - \Delta T$ and ends at $T_{m2} = T_m + \Delta T$ (water is in solid state for $T < T_{m1}$, in liquid state for $T > T_{m2}$, and both phases coexist for $T_{m1} \leq T \leq T_{m2}$). It is important to note that in this study, ΔT is a numerical parameter with no physical meaning. Ideally, the behavior of a pure substance melting at temperature T_m would be recovered for $\Delta T \rightarrow 0$. Decreasing ΔT thus improves model accuracy, but requires more computational resources (see Nazzi Ehms et al (2019) and Caggiano et al (2018) for more details).

The dependent variables defined in the water domain are the temperature T, the horizontal and vertical components of the velocity vector u and v, the pressure p and the liquid volume fraction θ. Because of the assumption of smooth phase transition, θ varies continuously from 0 (solid phase) to 1 (liquid phase) throughout the water domain (see Fig.2b). The corresponding governing equations read:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}=0 \tag{2}$$

$$\rho_0\frac{\partial u}{\partial t}+\rho_0\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)+A\frac{(1-\theta)^2}{\theta^3+\varepsilon}u \tag{3}$$

$$\rho_0\frac{\partial v}{\partial t}+\rho_0\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right)-\rho_0 g\beta(T-T_0)+\rho_0 g+A\frac{(1-\theta)^2}{\theta^3+\varepsilon}v \tag{4}$$

$$\rho c_p\frac{\partial T}{\partial t}+\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial z}\right)-k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)=0 \tag{5}$$

$$\theta=\begin{cases}0 & T<T_{m1} \quad \text{(Solid)}\\ \dfrac{T-T_{m1}}{T_{m2}-T_{m1}} & T_{m1}\le T\le T_{m2} \quad \text{(Diphasic)}\\ 1 & T>T_{m2} \quad \text{(Liquid)}\end{cases} \tag{6}$$

This set of equations includes the mass balance Eq.(2), the momentum balance Eqs.(3-4), the energy balance Eq.(5) and the relation between the liquid volume fraction θ and the temperature field Eq.(6). g is the gravity acceleration, ρ_0 is the density of liquid water at the reference temperature T_0, μ and β are the dynamic viscosity and thermal expansion coefficient of liquid water, ρ, c_p and k are the density, specific heat and thermal conductivity of water.

The last term in Eqs.(3-4) is used to impose the velocity transition between the liquid and solid phases. A and ε are numerical parameters imposed by the user. A must be as large as possible provided that solver stability is insured. In contrast, ε is a small parameter used to prevent divisions by zero in numerical calculations (Mousavi Ajarostaghi et al., 2019). It can be demonstrated that for $\theta=0$ (i.e., in the solid phase), the solution of Eqs.(2-4) turns to $u\simeq v\simeq 0$, which is the solution expected in the solid phase (see Nazzi Ehms et al (2019) and Caggiano et al (2018) for more details). Conversely, the last term in Eqs.(3-4) vanishes for $\theta=0$ (i.e., in the liquid phase). In this case, Eqs.(3-4) turns to the standard Navier-Stokes equations (momentum balance in an incompressible Newtonian fluid), required to calculate the velocity field in the liquid phase.

The boundary conditions are as follows. At the interface between an impermeable solid and a viscous fluid, the fluid velocity is equal to that of the solid (Guyon et al., 2015)*. This is the so-called no-slip and impermeability conditions, resulting in $u=v=0$ at the rock-water interface. The temperature continuity and the heat flux conservation through this interface are also considered (since the water velocity vanishes at the rock-water interface, the heat flux through the interface reduces to conduction). As already mentioned in section 2.1, the bottom and vertical external boundaries are adiabatic, and the temperature evolution of the top boundary is imposed (see Fig.2b). "*

##

ANSWERS TO SPECIFIC QUESTIONS

Line 18-19. The agreement the model and real-world observations is qualitative at best (see my comment on Line 273). 'The close agreement' is an overstatement. Please rephrase the sentence.

corrected:

« The model outcomes are compared qualitatively with field data from Monlesi ice cave (Switzerland) and confirm the agreement between real-world observations and the proposed model when free convection is considered. »

Line 91. Eq. 1 assumes no volume change in water, implying that the change in volume associated with ice-to-liquid transition is neglected in the analysis. If this is the case, then the model domain will have void space, presumably at the top. How does the model take this into account? Please present a clear explanation.

It is true that the contraction due to ice melting is neglected in our model. This is equivalent to assume that an external water flow replenishes the top layer domain with water at the same temperature as the atmosphere. This would result in the additional vertical velocity in the liquid phase (Heitz and Westwater, 1970):

$$v_l = \frac{(\rho_l - \rho_s)}{\rho_l}\frac{dH}{dt}$$

This velocity would be that of the liquid in the absence of free convection, or would be added to the free convection velocity field in the other case. This additional flow can be safely neglected if the heat advected in that way is negligible compared to the heat absorbed by the motion of the melting front:

$$\rho_l\, v_l\, c_{pl}(T_{atm} - T_0) \ll L_m \rho_s \frac{dH}{dt}$$

Both equations above yield the condition of validity:

$$\frac{(\rho_l - \rho_s)}{\rho_s}\frac{c_{pl}(T_{atm} - T_0)}{L_m} \ll 1$$

With $(\rho_l - \rho_s)/\rho_s \simeq 0.09$, $c_{pl} \simeq 4200$ J/(kg.K), $T_{atm} - T_0$ =15°C and $L_m \simeq 334$ kJ/kg, the LHS of the above equation is approximately equal to 0.02, much lower than 1. The flow due to contraction can thus be safely neglected. This analysis will be inserted in the new version of the paper.

Heitz and Westwater (1970) display an interesting comparison of mathematical solutions with equal and unequal phase densities for the freezing of water initially at 13.8°C, with top boundary suddenly decreased at -46.5°C. The melting front velocities computed from both models are in very good agreement.

Line 92. Reduced pressure. This term is not familiar to most readers of the journal. Please define it.

The momentum equations have been rewritten to avoid the use of the reduced pressure. In the new version of the manuscript, p is the fluid pressure (this modification is purely formal, the model is unchanged).

Line 94. A kind of Darcy-like pressure drop. I do not understand what this means. Please explain it more clearly.

The Darcy law is $u = -\frac{k}{\mu}\nabla p$, k is permeability and μ is dynamic viscosity. If we rewrite this equation like $\nabla p = -\frac{\mu}{k}u$, there is a similarity between this pressure gradient and the term $A\frac{(1-\theta)^2}{\theta^3+\varepsilon}u$ in the momentum equations. In fact, the term $A\frac{(1-\theta)^2}{\theta^3+\varepsilon}$ and $\frac{\mu}{k}$ has the same unit and that's why it is called often Darcy-like pressure drop (or gradient) in the literature (specially in numerical modeling of melting and solidification)(Nazzi Ehms et al., 2019). However, this is a technical (and quite complicated) aspect of the numerical method not necessary to understand the paper. In the new version, the presentation has been simplified. We now focus on the main principles of the numerical method, and we refer the reader to the literature for more details (see the new version of section 2.2 above).

Line 96. What does the variable 'A' physically represent? What is the unit?

The variable *A* is a numerical parameter with no physical sense. In the new version, we tried to clarify the distinction between physical and numerical parameters (see the new version of section "*2.2 Governing equations" above).*

Line 109. The heat transfer in the rock phase is limited to conduction, implying negligible rock porosity. This is contrary to numerous field-based studies of karst hydrogeology, where the fracture network in karstified rocks can play a major role in water transfer. What is the expected porosity of the system the authors are intending to model? Please add a sentence or two on porosity and fractures.

Thank you for your feedback. We assumed an impermeable (solid) rock massif containing one ice-clefts with an aperture size ranging between 2 to 50 cm (10 cm as a reference value in figure-1). These figures are consistent with field observations and correspond to a karst porosity of between 0.2 and 5 %, depending on the density of fractures. Our study is not relevant for smaller fractures with lower aperture sizes, where the effect of free convection is expected to be less significant. The thermal impact of water flow on the surrounding rock temperature will be addressed elsewhere.

Line 122. The authors assume -0.5 to +0.5 C as the temperature rage of ice-liquid transition. While ice and liquid water can co-exist under negative temperature due to freezing-point depression, there is no known mechanism to sustain the ice-liquid mixture under positive temperature. Please justify the choice of temperature range. If it is not justifiable, please re-run the simulations using a physically feasible temperature range.

This melting temperature range is a numerical parameter, with no physical sense. See the second paragraph of the new version of section "*2.2 Governing equations*" above. We added a paragraph to explain how its numerical value was chosen:

"Regarding the numerical parameters required to model melting, we imposed ΔT=0.5 ℃, A =1000 kg.m^{-3}.s^{-1} and $\varepsilon = 10^{-3}$. We checked that imposing ΔT = 0.3 ℃ or 0.7 ℃ did not significantly change the results (see Appendix A). The selected values of A and ε produced vanishingly small velocity fields in the ice with no deterioration of the solver stability."

Line 127. Impermeable solid rock. Please see my comment on Line 109.

See our answer to the question about line 109.

Line 132. How is the depth of the cleft defined with respect to the actual clefts in the field. In natural systems, water will drain from the bottom of the cleft as shown in Figure 1a.

The following paragraph has been added in the introduction:

"We consider the upper part of a single vertical cleft of size aperture A_p filled with pure water whose melting point is $T_m = 0°C$. This cleft is surrounded by a rock mass of width W (see Fig.1b). In karst massifs, water flow concentrates in well-defined conduits (Ford and Williams, 2007). The micro-porosity of the rock is thus disregarded, and impermeable rock mass is assumed.

The cleft is located at the center of the 2D domain of height H_{dom}. In the initial state, the system (water and surrounding rock) is at the uniform initial temperature T_i=-1°C, and all the water is frozen. At time t=0, the temperature of the ground surface T_s increases at the constant rate 1.77 ℃/hour to reach 15°C after 9 hours. This temperature increase is similar to the daily warming between the early morning and the afternoon.

The effect of the cleft aperture size was investigated by varying A_p from 2 cm to 50 cm. We imposed H_{dom}=0.8 m and W =1 m in all simulations. These values are large enough so that the thermal perturbation induced by the presence of the cleft does not reach the domain boundaries at the end of the simulated time (9 hours). The vertical and bottom boundaries of the domain can therefore be considered as adiabatic (see Fig.1b). It is important to note that the domain height H_{dom} contains only the upper part of the cleft, whose actual depth commonly ranges from 1 to 10 m. The value of H_{dom} used in this study is convenient for the daily time scale considered in the numerical simulations. Simulating larger time scales would require larger values of H_{dom}.

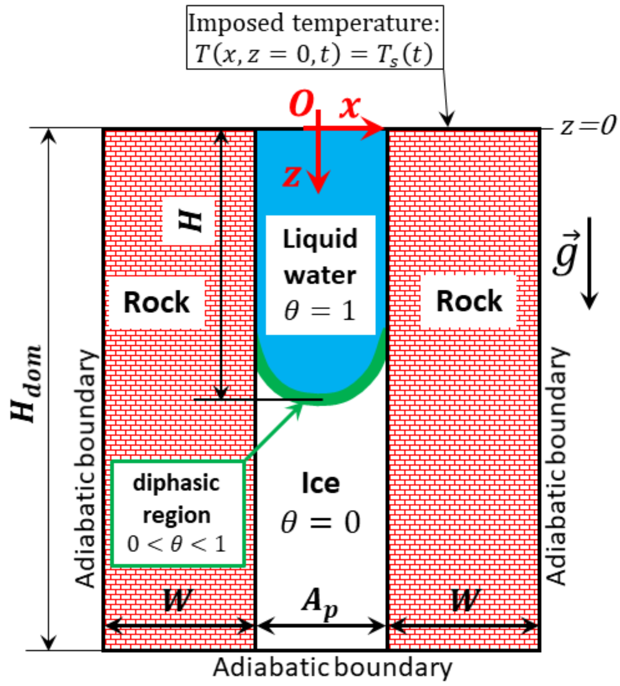

Fig. 1-b

Line 132. 80 and 10 cm. How are these values chosen? Please provide an explanation.

Please see above our previous reply for Line 132.

Table 1. Density 2320 kg/m3. This is much smaller than a typical density of solid carbonate rocks, and implies substantial porosity (14%?). Is this consistent with the model assumption? Please explain.

Table 1. Thermal conductivity 1.656 W/m/K. This is much smaller than that of solid carbonate rocks, implying substantial porosity. Is this consistent with the model assumption?

The assumed rock density and heat capacity were taken from (Covington et al., 2011). The thermal conductivity is from (Guerrier et al., 2019). The table was updated.

The literature reports a broad range of thermal properties. Wenk and Wenk, (1969) reported a density range between 2510 and 2840 [kg/m^3] and a thermal conductivity range between 0.97 and 1.99 [W/m/K] for carbonate alpine rocks and Zappone and Kissling, (2021) obtained a density range between 2150 and 2823 [kg/m^3] for limestone. To test the model sensitivity to rock properties, we run a simulation with a density value of 2700 [kg/m^3] and thermal conductivity of 2.2 [W/m/K] (Luetscher et al., 2008). The fig. R2-1 displays the melting rate as a function of time for modified (new values of rock density and thermal conductivity) and unmodified (initial) properties. These results show that after 9 hr the melting rates only differs by 10%.

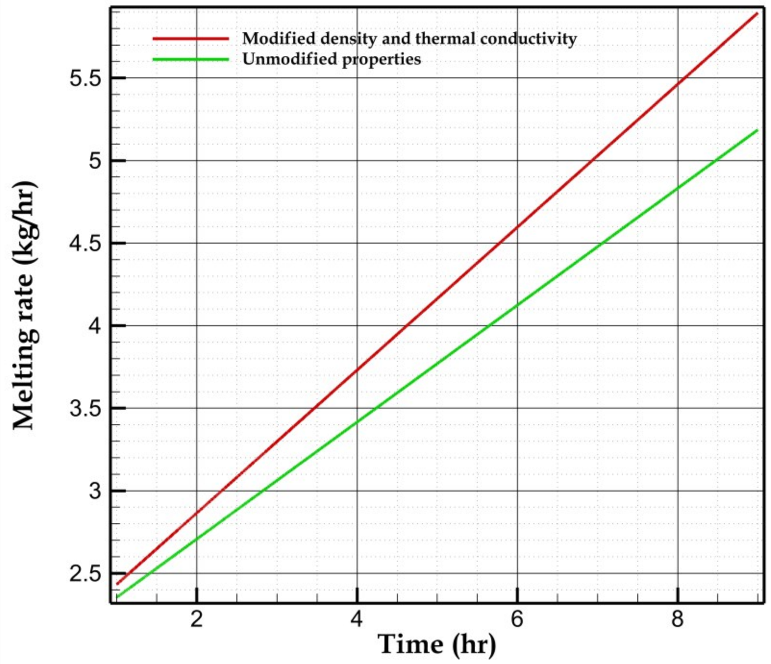

Fig. R2-1 Melting rate for modified and unmodified rock thermal properties

Table 1. Thermal properties and numerical parameters. The liquid water properties are temperature dependent. The properties of ice and rock are assumed constant.

Thermal properties	values	Reference
ρ_s (kg/m^3)	916.2	-
k_s (W/m/K)	2.22	-
$c_{p,s}$ (J/kg/K)	2050	-
ρ_l (kg/m^3)	see Fig.2	(Comsol, 2018)
k_l (W/m/K)	0.556 at 0°C 0.585 at 15°C	(Comsol, 2018)
$c_{p,l}$ (J/kg/K)	4216 at 0°C 4192 at 15°C	(Comsol, 2018)
μ (mPa.s)	1.79 at 0°C 1.43 at 7.5°C 1.15 at 15°C	(Comsol, 2018)
ρ_r (kg/m^3)	2320	(Covington et al., 2011)
k_r (W/m/K)	1.656	(Guerrier et al., 2019)
$c_{p,r}$ (J/kg/K)	810	(Covington et al., 2011)
L_m (J/kg)	334000	-

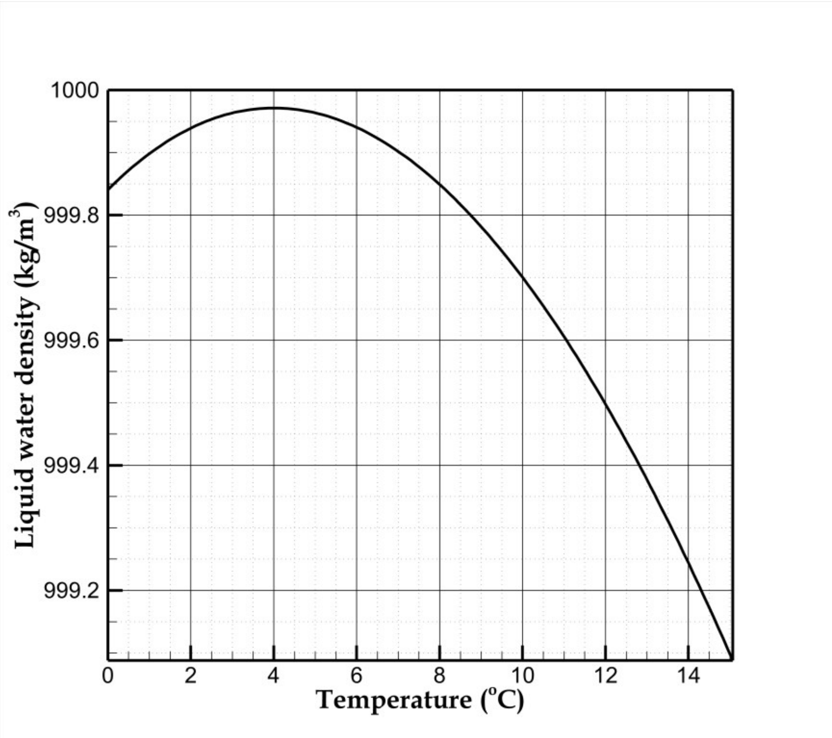

Figure 2: density of liquid water as a function of temperature.

Line 141. Please spell out 14k, 24k, etc.

It was corrected: *"The total number of elements in each of the four cases was almost 14000, 24000, 32000, and 47000, respectively".*

Line 179. At the bottom of the cavity. The model also under-simulates the advance of thawing front in the upper part by 2300 sec. Please point this out.

It is difficult to point out what specific assumptions cause this discrepancy. The following sentence has been added in the manuscript:

"Although some discrepancies exist between the experiments and the numerical model, especially at the bottom of the cavity at the start of the simulation and in the upper part at later times, the overall performance of our model is suitable to represent the free convection cells and their effect on ice melting despite the simplifying assumptions made in the model (including 2D geometry and negligible volume contraction upon melting)."

Line 170. The overall performance of our model is sufficient. This is a subjective statement. Please explain the basis for this statement.

The sentence was modified (given in previous comment Line 179).

Line 188-189. The authors state that the model conceptualization (Figure 1b) is similar to the physical setting depicted in Figurer 1a. However, I do not see a clear similarity. Please improve the description. A schematic diagram depicting typical clefts observed in the field will be useful to bridge the gap between Figures 1a and 1b.

This comment is related to Line 132. Figure-1-b was updated and better descriptions about the differences of computational domain and the real ice-clefts were added to the manuscript.

Line 193. Total volume of liquid water. The total volume of liquid water should be much smaller than the volume of ice before melting. Therefore, there should be some void spaces if the model obeys the mass-conservation law. Violation of mass conservation is considered a major deficiency of any mass and water transfer models. Please explain how the water mass is conserved in the model.

See our answer to the question related to line 91.

Line 223. Rayleigh number. Please report the Rayleigh numbers computed for the numerical experiments presented in the manuscript.

The depth of meltwater after 9 hr is about 0.3 m and the resulting Rayleigh number is in the order of 10^8. This information has been included in the new version.

Line 230. Melting rate. Does it refer to the melting rate (kg/hr/m2) or cumulate amount of melt (kg/m2)? Linear plots in Figure 10 seem to suggest the latter. Please clarify.

The legend of Fig.10 is correct. The melting rate actually increases with time. This clarification has been included in the text.

Line 244. 8 m meltwater depth. Is this 8 m or 0.8 m? The model has 0.8 m, not 8 m. Please clarify.

See our answer to the question related to line 132.

Line 253. Seasonal freezing seals them periodically. This implies that the clefts (or chimneys) are not always saturated. How can this be adequately represented in the saturated model? Please explain. By the way, are clefts and chimneys the same thing? If so, please use a consistent term.

Chimneys and clefts refer to different geometries. For the purpose of this paper, we can indeed focus the description only on clefts. The sentence was corrected accordingly.

The monitored site freezes seasonally from the bottom, rising the hydraulic head within the cleft. With further freezing the water within the cleft will completely solidify until the next thawing season. We believe our model is the best possible analogue for approaching the complexity of this natural system.

"The c. 600 m2 cave chamber is partly filled with perennial congelation ice fed by a number of vertical clefts of between 10^1 and 10^3 mm width"

Line 254. The distance. Does this refer to vertical distance? Does the external surface indicate the ground surface? Please clarify. A schematic diagram will be useful (see my comment on Line 188-189).

Yes. It was modified as follows: *"The vertical distance between the external ground surface and the cave ceiling reaches c. 20 m …"*

Line 255. Clefts of different sizes. Please indicate a rage of sizes observed at the site.

The sentence was modified as follows: *"[…] fed by a number of vertical clefts of between* 10^1 *and* 10^3 *mm width."*

Line 256. A few centimeters. Please report the actual depth, even if it is approximate.

Modified : *" […] soil temperature at 10 cm depth "*

Line 257. 4.5 days. Please report the actual dates.

The date of measurement was added in new version of manuscript: *"from April 13 to 17"*

Line 259. Cave temperature. Where in the cave (in relation to clefts) and how was it measured? Please explain.

Line 260. How was the water flow monitored? This information is critical. Please explain it carefully with sufficient details.

We added two paragraphs for measuring cave temperature and water flow rate:

"The main water inlet at Monlesi (subcutaneous flow) was instrumented to measure discharge rates at 30 min intervals using a pressure probe set at the bottom of a 1 m long perforated PVC tube capturing the inlet. The water height measured in the tube is converted to discharge (Q, in l/min) with an empirically calibrated rating curve checked by nine manual "bucket gauging" between 0 and 13 l/min with an uncertainty of ±10% (Luetscher et al., 2008).

Cave air temperatures were measured using negative temperature coefficient thermistors with a resistance of c.29.5 kOhm at 0°C and a temperature coefficient of about 5% °C-1 (YSI 44006). The thermistors were calibrated in a bath of melting ice to an accuracy of ±0.1°C. The thermistors were spaced at 2 m intervals in two chains comprising 19 sensors dispatched in the main cave chamber (Luetscher et al., 2008). Air temperatures were recorded at 1 h intervals and logged externally on a Campbell CR10X data logger with two multiplexer logging units."

Line 264-265. I do not exactly understand this sentence. 'We allow for the meltwater to drain deeper' implies that a new model is set up to simulate drainage, but 'in contrast to our model' implies that the same model without drainage is still used. Please clarify. Also, if drainage is allowed, please explain how it is done in the model.

This sentence has been deleted in the course of the revision.

Line 271. 3 meters depth. Is this based on the measured depth in the field? If not, how is it selected? The same applies to 10 cm aperture.

The details about the selection of the total depth of the domain were explained above (Line 132). The total depth of meltwater considered in the model depends on the duration of the warming which is about 4.5 days here (in section 4.3). This figure compares to the previous cases which considered a duration of 9 hours in the two predefined scenarios shown in section 4.1 and 4.2. The depth of the entire domain should be sufficiently high to ensure that its basis remains continuously obstructed by ice. In the field, this is confirmed by the negative temperature measured in the cave. In other words, the meltwater should exit the domain from somewhere above the ice interface although this outlet is not considered in our conceptual model. We changed a little the corresponding sentence for more transparency:

"A rough estimate of the cavity geometry (computational domain) assumes an initial ice-filled cleft with 3 meters depth (H_{dom}=3 m) and 10 cm aperture size (A_p=10 cm) subject to a linear temperature rise at the top boundary (black dash-dotted in Fig. 11-b)."

Line 271. In general, how long are the clefts observed at this site? Please indicate it in the sentence. I am referring to the third dimension in addition to the depth and the aperture.

The typical length of these clefts reaches an order of 1 m. The sentence was amended accordingly.

Line 273. The same order of magnitude. This is not a meaningful comparison because the measured flow rate may have a large degree of uncertainty depending on how it was measured (see my comment on Line 260). It is not uncommon for this kind of measurements to have uncertainties greater than an order of magnitude. This is a major weakness of the manuscript. Independent evidence demonstrating the qualitative match between the model and field observation will be useful (see my comment on Line 311-312).

The uncertainty on the measured flow rate is ±10 % (Luetscher et al., 2008). This figure is now mentioned in the revised manuscript (cf also response to comment on Line 260)

Line 290. Sufficiently high. Please indicate the number and compare it with the critical Rayleigh numbers reported in previous studies of free convection (not necessarily in water-ice systems).

This section has been rewritten as follows:

"In the present work, we simulated 9 hours of atmosphere temperature increase. When the aperture size A_p was varied from 2 to 50 cm, the liquid height H at the end of the simulation approximately ranged from 30 to 40 cm, and the convection cell occupied the entire liquid domain. However, the liquid height reached after 9 hours is only a small part of the actual height of the cleft (commonly up to 10 m). H is expected to increase if longer times are considered. The question arises whether the free convection cells always fill the entire liquid domain at longer times, despite the increase of friction due to lower aspect ratio A_p/H. If the convection cell occupies only a part of the cavity, the efficiency of heat transfer between the ground surface and the melting front will be reduced. The significance of free convection can be assessed from the value of the dimensionless Rayleigh number

$$Ra = \frac{g\beta(T_c - T_H)H^3}{\alpha_l\, \nu_l} \quad (10)$$

where (T_c-T_H) is the temperature difference between bottom and top surfaces, α_l and ν_l are the liquid water diffusivity and kinematic viscosity, respectively. Ra represents the ratio of the diffusion time over the free convection time ($Ra{\sim}10^8$ in the numerical experiments presented in this article). In a cavity with infinite lateral dimensions, free convection is triggered when $Ra \gtrsim 10^3$ (otherwise, the conductive state is stable, see Bergman et al (2017) for more information about the Rayleigh-Bénard instability). However, in the confined geometry considered in this work, the presence of the vertical walls must be considered. Rohsenow et al (1998) provide the following condition for convection onset, which takes into account the stabilizing effect of the vertical walls for $A_p \ll H$, in the limiting case of perfectly conducting walls:

$$Ra \gtrsim 10^2 \times \left(\frac{H}{A_p}\right)^4 \quad (11)$$

Injecting Eq.(10) in Eq.(11) yields the maximum value of the liquid height H for which the free convection cell extends from the ground surface to the melting front:

$$H \lesssim 10^{-2} \times \frac{g\beta(T_c - T_H){A_p}^4}{\alpha_l\, \nu_l} \quad (12)$$

Considering that the liquid region at temperature T>4°C is stable and that the isotherm 4°C is close to the top of the cleft when the free convection cell fills the entire cavity (see Fig. 7b), we get (T_c-T_H)=-4°C. Using the physical properties from section 2.3 and $A_p = 2$ cm (the minimum aperture size considered in this study) yields $H \lesssim 10$ m, which is also the order of magnitude of the maximum cleft height. Therefore, free convection cells should always extend throughout the melted region for $A_p \gtrsim 2$ cm. Note that the assumption of perfectly conducting walls used in Eq.(11) is less favorable to convection than finite conductivity (Rohsenow et al, 1998). Eq.(12) is thus expected to slightly underestimate the higher bound of H corresponding to fully developed free convection cells."

Line 311-312. The modeled melting rate alone is not sufficient to support the model performance due to the large degree of uncertainty in flow measurements. Independent evidence will be useful. For example, how long does it usually take to thaw a cleft completely from the top to the bottom? Will it be possible to estimate the thawing rate and compare it with the modelled thawing rates? Please explore this and other approaches further to provide independent evidence.

The uncertainty on the flow measurements is low compared to the uncertainties on the cleft geometry and we do not consider this as an issue. Providing a more robust estimate on the thawing rate is impossible in absence of an exact estimate of the ice volume hosted in the cleft. Nonetheless, one can consider that the cleft is completely thawed if the water flow responds to rainfall after a period of drought. This case is observed on April 30, 2003, 13 days after the end of our monitoring period. Assuming the cleft (3x0.1x1 m) was completely saturated with ice this yields a maximum thawing rate of 0.3m3/350h = 0.9 l/h to be compared with a measured flow rate averaging 0.5 l/h.

Line 447. At 8 m meltwater depth. Where does 8 m come from? The numerical model was 0.8 m deep, not 8 m.

See our answer to the question related to line 132.

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