#### **General comments**

In their study, Sedaghatkish et al. model ice melt in ice cleft and compare melting rates with or without free convection. They show that free convection is a key process to account for because it increase melt rates by an order of magnitude. The modeling works is based on a commercial software, validated against to modeling experiments from the literature as well as observations. When used in a real world case, their setup shows a good ability to produce a melt rate that matches the water flow observed in a monitored cave.

My overall impression is that the study tackles an interesting topic with a relevant angle and adequate methodology. The model validation is convincing and the results on the real world case are good. The article is well structured and reads well. Some illustrations could be prettier but do the job (see suggestions below). Overall I think it is a good study that maybe won't catch the attention of everyone because it tackles a very specific question but that TC should be interested in publishing because it is a relevant piece of work on the cryosphere. I don't see much to change, so I suggest minor revision to the editor.

I made a detailed lists of small comments, I think some significant progresses can be made on the clarity of the explanations. The biggest of my small comments are the following.

- I think the study can be more clear and pedagogic regarding the physics and the equations.
- I think the author should discuss to what extent, fast water flow through the macroporosity of this kind of massif could compete with free convection by disturbing the small scale density contrast that it requires. Who wins? Would it be nice to have a model that do both... (see below)
- I think the discussion should also try to discuss larger scale implications of the results in terms of consequence for permafrost disappearance at the scale of the massifs/mountains and regarding catchments water balance.

We thank the reviewer for the constructive comments, particularly related to the governing equations and discussion section. We are delighted to hear that our study is acceptable for The Cryosphere journal with minor revisions. We tried our best to modify or justify better our statements and claims in order to boost the quality of the manuscript. We have updated the methodology section by adding more details and made a better connection between our study and other fields by considering the effect of free convection also at larger scales. A list of references used in our answers is available at the end of the document.

Abstract: I have the feeling that it could be nice to remind in a few words what is free convection to the readers in the abstract as it took me a few minutes to realize what we are talking about. E.g. "free convection (convection driven by density contrasts within the water phase)". Instinctively I did not think that the water bodies in karstic environments are big enough for free convection to be important, like it would be in a lake for example.

The abstract will be modified as follows:

**"Abstract**. This research develops a conceptual model of a karst system subject to mountain permafrost. The transient thermal response of a frozen rock-cleft after the rise of the atmosphere temperature above the melting temperature of water is investigated by numerical simulations. Free convection in liquid water (i.e., buoyancy-driven flow) is considered. The density increase of water from 0 to 4°C causes warmer meltwater to flow downwards and colder upwards, resulting in significant enhancement of the heat transferred from the ground surface to the melting front. Free convection increases the melting rate by approximately an order of magnitude compared to a model based on thermal conduction in stagnant water. The model outcomes are compared qualitatively with field data from Monlesi ice cave (Switzerland) and confirm the agreement between real-world observations and the proposed model when free convection is considered. »

## L15: "the anomalous behavior of water between 0 and 4°C"Do you refer to the density increase of water from 0 to 4°C? If so state it in a less mysterious way.

The density increase of water between 0 and 4°C is better explained in the new version. The following paragraph has been inserted along with a new figure showing the water density as a function of temperature (see below).

"Figure 1-a displays the bottom of a frozen cleft hosted in an Alpine karst. Such clefts and fissures are characterized by distinct geometries accommodating contrasted volumes of ice. Our aim is to study the effect of free convection of water on the melting rate in frozen rock clefts and/or karst conduits at daily scale. Atmospheric warming at the upper boundary melts the ice from top of the fractures, and increases the meltwater temperature. While most fluids expand as temperature increases, water shrinks when warmed from 0°C to 4°C. Above this temperature, the common behavior is recovered (see the maximum density at 4°C in Fig.2). Therefore, the progressive warming of the meltwater at the top of the cleft results in an unstable situation (heavier fluid above lighter) that triggers free convection (buoyancy-driven flow). »



Figure 2: density of liquid water as a function of temperature.

L22: The first sentence talks about the impact of global climate change on permafrost and the ref is only about the French Alps, maybe add a more large scale ref as well.

We added some other references (Walvoord and Kurylyk, 2016; Jin et al., 2021) relating to larger scales. The first one is a review paper about climate warming and its consequences on soil thermal change which are expected to modify the distribution of permafrost, leading to changing hydrologic conditions, including alterations in soil moisture, connectivity of inland waters, streamflow seasonality. The second one is another review paper which is about permafrost degradation effect on arctic and alpine ecology and vegetation.

"With global climate change and rising temperatures, permafrost degradation has become a significant concern (Duvillard et al., 2021; Walvoord and Kurylyk, 2016; Jin et al., 2021)"

L33-34: "have shown that heat advected by water and air fluxes may significantly disturb the geothermal field, challenging classical models of heat propagation based on conductive fluxes"

Here you want to talk about the general case, not specifically karstic environment, so it would be good to come up with a reference that demonstrates that in non-karstic environments. Since you then talk about well-developed conduits right after, it would be also nice to add a line to explain the difference between convection in a porous media and convection in a conduit.

The references to the boreholes do not specifically refer to karstic environments. Nonetheless, we added. In contrast to porous media, karst systems concentrate water fluxes through well-defined conduits. It is rare to have similar fluxes in porous media. We do not think this statement needs to be underlined further.

## L36: "this permafrost" which permafrost? Maybe better "permafrost in karstic environments" or something similar. But the "this" refers to something undefined I believe.

Corrected.

"The discontinuous nature of permafrost in karst environments may lead to the formation of massive cave ice at depth (Bartolomé et al., 2022) but also explain unexpected speleothem formations during glacial times (Luetscher et al., 2015; Fohlmeister et al., 2023; Fohlmeister et al., 2023)"

#### L42: If rock-glaciers are relevant to this list, add it.

Added.

#### L43: lower than what? Than 2 or 3D models?

Corrected.

"they offer several advantages including lower computational costs than 2or 3D models and easy implementation"

#### L47: "This distributed model is efficient for large domains (catchment scale)."

#### In what regard? What does that mean?

We now specify "for calculation of temperature in large domains".

#### L49-50: Tubini et al., (2021)With which processes? Conduction only or also convection?

We now specify "based on conduction and latent heat flux".

"Tubini et al., (2021) proposed a numerical approach for modeling heat transfer of permafrost thawing based on conduction and latent heat flux in 1D domain which is capable to deal with high time steps and maintains conservation of energy in long-term simulations"

L53-54: "The effect of water flow inside the clefts is noticeable because of creating thermal shortcut between atmosphere and subsurface." This is a general statement that does not really fit the list of modelling studies you are going through. It probably fits better earlier when you talk about medias with conduits.

The statement refers to the model of Hasler et al., (2011) mentioned in the previous sentence. This has been clarified as follows:

"Hasler et al., (2011) investigated the effect of advective heat transport in frozen rock clefts and fractures at small scales. They built a conceptual model combining numerical modeling and laboratory experiments. These authors find a significant effect of water flow inside the clefts, due to the formation of thermal shortcut between atmosphere and subsurface."

L59-60: Maybe one more sentence to be more specific. You imply that atmospheric warming could warm melt water located close to the upper boundary close to 4°C, which would later sink right?

See our answer to L15 above.

Figure 1. As I am still at the point where I try to understand what you did, I am surprised that you show on the picture a volume which order of magnitude is tens of meters and your modeled domain is in the order of 80 cm. I missed earlier on some explanations on why 80 cm is a relevant size and how something that small can have a larger scale relevance (i.e. small features but very frequent I suppose).

Thank you for your feedback. Indeed, we realize that the description of the problem was confused. Section 2 will be completely rewritten. It will begin by a section 2.1 describing the computational domain with the main physical assumptions:

#### "2.1 Physical model and simplifying assumptions

We consider the upper part of a single vertical cleft of size aperture  $A_p$  filled with pure water whose melting point is  $T_m = 0^{\circ}C$ . This cleft is surrounded by a rock mass of width W (see Fig.1b). In karst massifs, water flow concentrates in well-defined conduits (Ford and Williams, 2007). The microporosity of the rock is thus disregarded, and impermeable rock mass is assumed.

The cleft is located at the center of the 2D domain of height  $H_{dom}$ . In the initial state, the system (water and surrounding rock) is at the uniform initial temperature  $T_i$ =-1°C, and all the water is frozen. At time t=0, the temperature of the ground surface  $T_s$  increases at the constant rate 1.77 °C/hour to reach 15°C after 9 hours. This temperature increase is similar to the daily warming between the early morning and the afternoon.

The effect of the cleft aperture size was investigated by varying  $A_p$  from 2 cm to 50 cm. We imposed  $H_{dom}$ =0.8 m and W =1 m in all simulations. These values are large enough so that the thermal perturbation induced by the presence of the cleft does not reach the domain boundaries at the end of

the simulated time (9 hours). The vertical and bottom boundaries of the domain can therefore be considered as adiabatic (see Fig.1b). It is important to note that the domain height  $H_{dom}$  contains only the upper part of the cleft, whose actual depth commonly ranges from 1 to 10 m. The value of  $H_{dom}$ used in this study is convenient for the daily time scale considered in the numerical simulations. Simulating larger time scales would require larger values of  $H_{dom}$ ."

Also Figure 1b can be improved. Remind what are the x and y axis (I am surprised you did not use z for the vertical axe by the way, I have the feeling it is what the general reader would expect to understand your work more smoothly). As it is I am still unsure which one is the vertical one. If you put dT/dx on the side I am tempted to believe it is a boundary condition for the side, but with the proximity of the other dt/dx to the H, I would be tempted to think x is the vertical axis. Add arrows as well maybe. The red arrows look like they were made with MS Paint. I think in general you can give more love in Fig 1B.

A new figure 1-b has been inserted (see below). The vertical axis will be called z instead of y throughout the manuscript. x and z will be explicitly defined in the governing equations section.



Figure 1: b) Computational domain with external boundary conditions; 2 cm  $\leq A_p \leq$  50 cm,  $H_{dom}$ =0.8 m and  $W_{dom}$ =1m (the sketch is not at scale).

#### L86: The sentence misses a point at the end.

#### Corrected.

L93: I feel there is maybe a lack of pedagogy regarding the A x... term that is introduced. I don't know what is a Darcy like pressure drop (a bit of physics "with the hand" to help intuition maybe) and I don't see why theta=0 will nullify the velocity. What I see is that you will add the term A v /

epsilon to the Navier Stokes equation (or A u / epsilon horizontally). How does that nullifies the velocity?

Similarly, I think you should explicit what is the Boussinesq approximation. I don't suspect many readers of TC will know that and it can be disorienting to see rho0 in your equations whereas you intend to work with density contrasts in your study (and your actual rho variable is hidden in beta, so at first glance, rho seems to be fixed).

We used a method that allows to define a single set of governing equations in both solid and liquid phases of water. We realize that the presentation of this method was confused. It will be rewritten in the future version, focusing on the main principles, and referring the reader to the literature for more detailed explanations.

The Boussinesq approximation consists in assuming constant density in the Navier-Stokes equation except in the buoyancy term. This is a standard approximation valid most of the time in liquids. In the new paper version, it will be defined and justified.

L109: "domain", to me at this stage, it is not clear if your domain is just the water/ice or also the surrounding rock. I see a mesh on the rock on Figure 1 but the rock is impermeable. So I think you should find a way to be clear on this, talking about the "water and ice domain only" or the "whole domain" or anything that would reach the same goal. So for Temperature, your domain is water/ice + rock? Because the cp you describe looks like it is for water only, it is not a freeze curve that would account for suction effect in the rock, that would spread the phase change below 0°C.

See the new section 2.1 above and new figure 1b. We hope that the new text of section 2 will be more clear.

L131: "no-slip" same, explicit quickly. Also what about heat fluxes between the water and the wall? Is it just conduction or also convection?

The following clarification will be inserted in the section dedicated to the governing equations:

"The boundary conditions are as follows. At the interface between an impermeable solid and a viscous fluid, the fluid velocity is equal to that of the solid (see Guyon et al (2015)). This is the so-called no-slip and impermeability conditions, resulting in u = v = 0 at the rock-water interface. The temperature continuity and the heat flux conservation through this interface are also considered (since the water velocity vanishes at the rock-water interface, the heat flux through the interface reduces to conduction). As already mentioned in section 2.1, the bottom and vertical external boundaries are adiabatic, and the temperature evolution of the top boundary is imposed (see Fig.1b)."

The following reference will be added: « Physical hydrodynamics, E. Guyon, J.P. Hulin, L. Petit, C.D. Mitescu, Oxford University Press, 2015 »

#### Section 2.1. Finite elements? Finite volumes?

The following sentence will be inserted in the new version:

"The system of partial differential equations (Eqs. 1-5) was solved by finite elements using the software Comsol Multiphysics version 6.0 (Galerkin method with quadratic Lagrangian elements, time discretization by implicit backward differentiation formula)."

# L151-152 : Here you just mention comparison with simulations even though Virag also has observation if I understood correctly. Reproducing observations is an even better validation, so make it more clear.

I think you are referring to Kahraman et al. We are comparing our result with the modeling results of Kahraman et al when free convection is absent. In fact, in their study, they investigated also the effect of free convection both experimentally and numerically. However, we prefer as a first step to consider the purely conduction scenario as the simplest possible test case before considering a more intricate case including free convection. The introduction to section 3 was clarified as follows:

"The validity of our model is tested by comparison with two studies from the literature. A simple test case assuming stagnant liquid water (no free convection) was selected as a first step (numerical simulation of ice freezing by Kahraman et al., (1998). In a second step, our model was tested against experimental results including free convection (ice melting experiment by Virag et al., 2006). "

#### L159: « with (Kahraman et al., 1998). »Fix parenthesis

corrected.

L172-174 « The isothermal line corresponding to T=4 °C is plotted inside the temperature contours implicating well the interface of the two counter rotating convection cells with two temperature ranges. »

#### I do not understand this sentence. Please reformulate.

In fact, the isothermal line shown in figure 3-a is the interface of two convection cells as shown in figure 3-b. The velocity direction of these two convection cells are opposite. We added some details for better understanding:

"The 4°C isotherm is plotted together with the temperature contours underlining the interface of the two convection cells rotating in opposite directions and showing two distinct temperature ranges (Fig. 3-b)."

#### Sect 3-2

#### Here be more precise whether you replicate the simulations from Virag or their observations.

We modified the text:

"Our numerical model replicates the observations of Virag et al (2006) depicted in Fig. 4 by displaying the ice-water interface at different times"

L165: "The density of water increases between 0 and 4°C, and decreases above 4°C by increasing temperature." If not already the case, this should appear earlier. Not in the experiment explanation.

This sentence has been removed, and more explanation given in the introduction about the density maximum of water.

#### L196-197 "thorough" Through

Corrected.

#### L200 "for two scenarios"

#### They are the same 2 scenarios as before right?

Yes. It was modified for better clarification:

*"Figure 6 depicts the melting rate for both previous scenarios considering stagnant water of free convection in the water phase"* 

#### L202: "and the extending the meltwater depth" Problem with the sentence.

The sentence was modified as follows: "In both cases, the melting rate and the meltwater depth increase with time in response to the temperature increase at the top surface"

#### Figure 8. Make a more explicit legend than W/WO.

In the new version, we will use the abbreviations SLW for stagnant liquid water and FC for free convection. They will be defined in the introduction and recalled in figure captions.

#### L221: "Irregular water circulations" What is that?

Deleted "Irregular".

#### L238: "completely are removed."

corrected into: "the effect of surrounding rock can be disregarded".

#### L240: You left an exclamation mark.

Deleted.

## L242-L245: not completely clear to me, can you reformulate? What is a aperture size threshold? So you back calculate Ra based on the empirical relationship and check for threshold values? With the last sentence do you mean that the empirical relationship is not valid?

This section has been rewritten as follows:

"In the present work, we simulated 9 hours of atmosphere temperature increase. When the aperture size  $A_p$  was varied from 2 to 50 cm, the liquid height H at the end of the simulation approximately ranged from 30 to 40 cm, and the convection cell occupied the entire liquid domain. However, the liquid height reached after 9 hours is only a small part of the actual height of the cleft (commonly up to 10 m). H is expected to increase if longer times are considered. The question arises whether the

free convection cells always fill the entire liquid domain at longer times, despite the increase of friction due to lower aspect ratio  $A_p/H$ . If the convection cell occupies only a part of the cavity, the efficiency of heat transfer between the ground surface and the melting front will be reduced. The significance of free convection can be assessed from the value of the dimensionless Rayleigh number

$$Ra = \frac{g\beta(T_c - T_H)H^3}{\alpha_l v_l}$$
(10)

where  $(T_{c}-T_{H})$  is the temperature difference between bottom and top surfaces,  $\alpha_{l}$  and  $v_{l}$  are the liquid water diffusivity and kinematic viscosity, respectively. Ra represents the ratio of the diffusion time over the free convection time  $(Ra \sim 10^{8} \text{ in the numerical experiments presented in this article})$ . In a cavity with infinite lateral dimensions, free convection is triggered when  $Ra \gtrsim 10^{3}$  (otherwise, the conductive state is stable, see Bergman et al (2017) for more information about the Rayleigh-Bénard instability). However, in the confined geometry considered in this work, the presence of the vertical walls must be considered. Rohsenow et al (1998) provide the following condition for convection onset, which takes into account the stabilizing effect of the vertical walls for  $A_{p} \ll H$ , in the limiting case of perfectly conducting walls:

$$Ra \gtrsim 10^2 \times \left(\frac{H}{A_p}\right)^4$$
 (11)

*Injecting Eq.(10) in Eq.(11) yields the maximum value of the liquid height H for which the free convection cell extends from the ground surface to the melting front:* 

$$H \lesssim 10^{-2} \times \frac{g\beta(T_c - T_H)A_p^4}{\alpha_l \nu_l}$$
(12)

Considering that the liquid region at temperature T>4°C is stable and that the isotherm 4°C is close to the top of the cleft when the free convection cell fills the entire cavity (see Fig. 7b), we get  $(T_c-T_H)$ =-4°C. Using the physical properties from section 2.3 and  $A_p = 2$  cm (the minimum aperture size considered in this study) yields  $H \leq 10$  m, which is also the order of magnitude of the maximum cleft height. Therefore, free convection cells should always extend throughout the melted region for  $A_p \gtrsim$ 2 cm. Note that the assumption of perfectly conducting walls used in Eq.(11) is less favorable to convection than finite conductivity (Rohsenow et al, 1998). Eq.(12) is thus expected to slightly underestimate the higher bound of H corresponding to fully developed free convection cells."

## L257: What is the soil temperature? Do you have a real pedological soil? To make things clear maybe you can talk about the ground surface (I guess it's the ground surface outside right, not in the cave?).

The soil temperature is given in Figure 11 and was measured at 10 cm depth in a true pedological soil (calcisol – B horizon).

L260-261: "The daily temperature variation induces a water..."Temperature of what? A cave is a complex setup compared to an homogenous media, be more specific to help the reader get a clear picture of what you describe.

Corrected as follows:

"The atmosphere daily temperature variation induces a water flow rate (0-12 l/min) with a trend similar to the surface temperature, supporting an origin associated with melting process of ice-filled clefts surrounding the cave."

## L264: "we allow" you allow but you are talking about the natural case no? Because you said "In contrast to our model" just before?

Thank you for your attention. The text was corrected: "In contrast to our model, meltwater drains deeper into the subsurface".

L273: "(red dash-dotted in Fig. 11-b)" If I understand correctly, should be "(red solid line and red dash-dotted line..." because the good news is that the dash-dotted line follows the moving average of the red solid line right? I feel you should make the 3 red lines thicker or anyhow more visible. They convey the key message of your paper right?

You are right. We changed them to thicker lines to make them more visible:



## Discussion : the 2 first paragraphs feel redundant with things you already said in the introduction and methods. I would start at line 285.

We feel it is important to remind the context for the discussion but agree it can be simplified to avoid to many overlaps. Accordingly, we deleted the first paragraph and now start with "A quantification of the melting rate ..."

L293-295: "Whether this water results from the melting of ice in the cleft or recharges from the surface (storm events or snowmelt) does not matter."I don't understand what you mean with this sentence. Additionally, if these are the sources of water, it is unlikely to be warm, so I miss the connection with the previous sentence.

The key element controlling the melting rate is the presence of a liquid phase which may result from the melting ice column or from hydrological recharge. Accordingly, permafrost thawing is also enhanced by rainfalls (and not only temperature). We agree that the formulation of this paragraph was unclear. It was clarified and simplified in the next version of the article.

# L302-306: I don't see where this paragraph goes. In soils things work differently than in karstic cavities right? The freezing will spread below 0 because of the suction in the soil. I don't see what perspective that gives on your work.

Indeed, the effect of suction in soils is not relevant for the freezing of a cleft. This paragraph will be removed. Instead, we will add in the conclusion that the mineralization due to karst dissolution should be considered in future works (expected effects are a shift of the melting temperature and also a contribution of salt concentration gradient to buoyancy).

# L312 :"... is in the same order of magnitude as the measured water flow rate (Fig. 11) in Monlesi cave." Here would be a nice place to discuss why the water flow is oscillating and your melting rate is smooth. If the average values are similar, what create the contrast in timing?

The measured water flow rate has variable amplitudes in its daily fluctuations which can be related to uncertainties in the epikarst flow paths and the meltwater saturation within the clefts. Here, our goal is to compare two different scenarios (with and without free convection) under the same thermal configuration and demonstrate that purely conduction case (without free convection) is too far from the measured water flow rate even though the "with free convection" scenario may not be too close to our observations.

We imposed a linearized soil temperature indicated by the black dash-dotted line in figure-11 at the top boundary in our model. Accordingly, the corresponding melting rate is linear too as expected but in the same order of magnitude with water flow.

L313: "The effect of free convection is not limited to hourly or daily oscillations and can be studied over much longer timescales, including centennial to millennial fluctuations. "When melting very big ice clefts? If the weather warms up from one year to another, once you start melting a cleft of one or 2 meters long, isn't it going to melt in a few years? Or are you discussing the melt at the scale of a massif?

Some ice-cleft can reach about 50 m length with a few meters aperture size. The effect of climate warming can be seen on the long-term. It is of interest to characterize the melting rate and/or the long-term changes in the meltwater depth due to freeze/thaw cycles. Further investigations are required in order to developing a model with feasible computational time.

L326-328: "In karst systems and fractured aquifers, where secondary porosity is exceptionally well developed, frozen conduits/fractures may all of a sudden drain water into depth and change the local hydrological regime leading a thermal anomaly within the surrounding permafrost (Phillips et al., 2016)."That's something you could discuss further to think against yourself. How much fast flow in karstic system is likely to actually advect temperature quickly over long distances and disrupt the peace of free convection? It gives an opportunity to discuss the representativity of the process you pinpoint in the perspective of the general functioning of a karstic/fractured massif exposed to seasonal/long term cold weather. It is also an opportunity to compare your work with Hasler et al. (2011). Is there a process more important than the other? Would we gain something trying to represent both at the same time?...

The model proposed by Hasler et al. (2011) relies on experimental data, which makes difficult a direct comparison with our model. However, water drainage can clearly enter in competition with free convection. Determining what mechanism dominates is an interesting question that will be mentioned in conclusion as an outlook for future works.

L333-335 "But also at shallower depth, acknowledging the potential role of convective heat fluxes in ice-rich permafrost degradation may help predicting the rate of greenhouse gas releases, mainly carbon dioxide and methane, due to the decomposition of formerly frozen organic matter (Schaefer et al., 2014; (Schuur et al., 2015)."This sounds a bit far-fetched. What carbon pools are you talking about? There is not much carbon in the fractures/karstic cavities of a rock massif right? And you do not expect much free convection in an organic peat soil right? I have the feeling that free convection is relatively low in the list of missing processes to accurately represent permafrost thaw where you find a lot of organic carbon, but I am happy to be proved wrong. You can check Kane et al. (2001, GPC, 10.1016/S0921-8181(01)00095-9).

Thank you for your pointing this out. We believe any kind of ice-rich media (not only fractures/karst cavities) can be subject to the effect of free convection. Global warming increases the top boundary temperature of any kind of ice-rich media in the long term. The intensity of free convection in porous media depends strongly on the Rayleigh number which in turns depends on the permeability and soil layer dimensions. So, further investigation is needed to address this effect. Of course, according to your cited research, free convection is negligible for some kind of soils close to surface with specific permeability but it may still be important for other soil types with different thermal conditions like the study published more recently by Najafian Jazi et al (Jazi et al., 2024). We tried to better express this in the manuscript.

"The intensity of free convection in soil depends strongly on the Rayleigh number which in turns depends on the permeability and dimensions of soil layer making it negligible, e.g. (Kane et al., 2001) or significant, e.g. (Jazi et al., 2024) with respect to the total heat transfer."

L335-338: Free convection is everywhere so it is beautiful, ok but not super relevant for your study. Funny that you did not explain the TC reader what is Boussinesq approximation but you do explain what is an iceberg ④.

The explanations about Boussinesq approximations were added.

In this discussion, since your main conclusion is that we need to be careful about not underestimating the melt rates in rock massifs with ice cleft, I missed a bit of large scale discussion on the implication for:

- catchments water balance. If you try to upscale your results, how can this impact runoff in mountain catchments, river flow, lake levels, at catchment scale and a global scale? Where should we start worrying more about this question?
- Permafrost disappearance at the scale of the massif. Does it change what we forecast for the Alps, by much?

#### So that we can grasp how significant these results could be at broader scales.

Thank you for your suggestions. This study is only a first step to underline the significance of convection on melting rates. Analyzing its impact at a regional/global scale is out of scope of this paper. Underlining that permafrost degradation along rock clefts could be enhanced by an order of magnitude as compared to classical models based on conduction, however, paves the way for investigating specific case studies.

#### L345: "only impact on the temperature" I suspect the "on" should be removed.

corrected:

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