Further Response to the Reviewer

General Remarks

The following response is presented in a self–contained manner, *i.e.*, the response can be read without referring to the original review comments. Yet the original comment texts are also inserted with blue. On the other hand, the Reviewer's comments are quoted by »...« when they are quoted in the response text.

Response

I would like to recommend this manuscript for publication after a minor revision. Following the reviewer's previous comments, the authors have corrected Fig 2 and added more examples, all of which provide valuable insights for the readers to understand when and how the proposed method would work efficiently. Below, I have raised a few additional points to further enhance the quality. Further review of the revised manuscript will not be necessary, but please consider incorporating them into the manuscript.

We are very glad to read that the present Reviewers now »would like to recommend this manuscript for publication«. In response, we have performed »a minor revision« by following the Reviewer's »few additional points to further enhance the quality.«

Major Comments

1) [question] P.46 ll.1182–1186 "In the case with the model (ii), the calculations with the exact distribution gets its own problem: ...":

The mean and variance cannot be defined for some fat-tailed distributions such as the Cauchy distribution. I speculate that this may be the cause of the slow numerical convergence. What do you think?

Yes, the difficulty with the model (ii) is due to the »fat-tailed distributions«. More precisely, the distribution increasingly presents a long tail to the positive direction with time, that leads to the difficulty. This tendency is reflected to singularities in the solutions (C.4a, b), that lead to a more direct cause of difficulties in the integrals. These points are elaborated in revision (ll.1196–1198).

2) [suggestions] Figs. 2 and 9:

If the exact solutions of the ODE do not blow up in finite time (e.g., m = 0 and 1 in Sec. 5.5, and (i) and (iii) in Appendix C), the moments predicted by the proposed method do not deviate significantly from the exact solutions. I think this is an interesting property worth highlighting somewhere in the manuscript.

The singular behavior of the solutions for the PDF parameters has been augmented in in Sec. 5.5 (ll.794–807), and explicitly noted in the Appendix C (ll.1204–1205) in revision.

As the Reviewer correctly points out, it is an important conclusion from the present study that the proposed prognostic assumed–PDF calculation overall works well, when the solutions for the assumed PDF parameters are not singular. We also note a similar issue in multiple–dimension cases in Yano (2024). These

points have been iterated in a newly–introduced paragraph in the conclusion section (ll.1037–1042) in revision.

Minor Comments

3) [typo] P.21 l.589: "reply" → "rely"

"reply" has been modified to "rely" as suggested.

4) [suggestion] P.28 l.803: "..., and in this case we find:" → "..., and in this case we find for all n:"

"..., and in this case we find:" has been modified to "..., and in this case we find for all n:" as suggested.

5) [typo] P.47 Fig.9: "(i) and (ii)" → "(i) and (iii)"

"(i) and (ii)" has been modified to "(i) and (iii)" as suggested.