Revision of the manuscript:

A hybrid ice-mélange model based on particle and continuum methods

September 20, 2024,

Dear Reviewer 1,

we thank you for your valuable remarks on our paper, which we address in the revised version of our manuscript. According to the main concerns which deal with the ice-mélange formulation in summer conditions (in the absence of sea ice between icebergs), we modified the parameterization of the tensile strength in the revised version. Now, we account for a weighted dependency on the sea-ice concentration in the tensile strength parameterization. This leads in particular to zero tensile strength between icebergs in the absence of sea ice. This new tensile strength mechanism is analysed in a test case which has been added to the manuscript. To address the concern of Reviewer 2 that sea ice is modelled on top of icebergs, we slightly modified the derivation such that this assumption could be dropped. Now, the effective sea-ice concentration and thickness is calculated by $\tilde{A}_{\text{sea-ice}} := \min\left(A_{\text{sea-ice}}/(1 - A_{\text{iceberg}}), 1\right)$ and $\tilde{H}_{\text{sea-ice}} := H_{\text{sea-ice}}/(1 - A_{\text{iceberg}})$. Here $A_{\text{iceberg}} < 1$, due to the assumption that icebergs are modelled by a finite number of small disks. We hope these changes address your suggestions.

Yours sincerely,

S. Kahl, C. Mehlmann and D. Notz

Answer to the Referee's comments

In the following, we respond to the reviewer's comments and explain the changes in the manuscript. All modifications in response to your comments are marked in magenta in the paper.

Major comments

1. *This review is for a revised manuscript now titled "A hybrid ice-mélange model based on particle and continuum methods" which describes a new method for modelling icebergs within sea ice (collectively called "ice melange") in existing viscous-plastic sea ice formulations within climate models. This is almost an entirely new and different model from the first version of this manuscript, and I appreciate the authors taking very seriously the reviewer suggestions and making substantial changes in response to those suggestions. Pretty much all of my major suggestions from the initial manuscript have been resolved, as I find this revised manuscript to make a much more compelling case for this model and its potential use within existing sea ice schemes in climate models. If I have one major comment about this version it is that I think the bounds of applicability of this model need to be defined more explicitly. Would this model be useful in a scenario where one has many icebergs and no sea ice (i.e. the way in which Robel 2017 and Burton et al. 2018 simulate iceberg melange within fjords)? My thought is not since then it would essentially just become a DEM model without many of the numerical advantages outlined here. This manuscript mainly makes the case for this model as a way to add icebergs into existing sea ice models, and I agree that is a compelling use of this model. However, there is also a community of scientists thinking about how to building melange models for use with ice sheet models, and I can see that taking this model to that extreme might break some assumptions inherent in this model. So, I would like to see more explicit mention of what assumptions are inherent in the model formulation and how that may limit its applicability.*
   Response: The model is developed from the sea-ice perspective with the aim to include the effect of small iceberg dynamics on the evolution of the sea-ice dynamics. So far, the prescribed iceberg dynamics are relatively simple. In the current version icebergs move either due to collision or with an averaged velocity calculated from the ice-mélange momentum equation. One perspective to allow for a more complex motion of icebergs is to use a particle model with higher fidelity, e.g the particle model used in Robel 2017. In this case one would need to derive the iceberg motion by taking into account both the particle dynamics and the ice-mélange velocity weighted by the sea-ice concentration per cell. We added a paragraph on the topic to the discussion.

Minor points

1. *Ln 2: ice melange has not been represented...realizations does not exist*
   Response: Done.

2. *Ln 11: property for computational efficiency and inclusion within large scale models. In idealized...*
   Response: Done.

3. *Ln 24: and melting at glacier termini*
   Response: Done.

4. *Ln 26: measurements in dense ice conditions*
   Response: Done.

5. *Ln 28: dynamics (Robel 2017, Burton et al. 2018)*
   Response: Done.

6. *Ln 41: It is challenging to efficiently solve the...(2017) with existing solvers.*
   Response: Done.

7. *Ln 45: to be explicitly resolved*
   Response: We reformulated the sentence.

8. *Ln 46: Thus, the typical grid size of several kilometres for a sea ice model can be used...*
   Response: Done.

9. *Ln 54: Amundson and Burton consider the case of iceberg melange without interstitial sea ice, where V-P might be used as a continuum approximation of granular flow. I'm not sure its a one-to-one comparison with the short of melange you are describing here (related to major comment above)*
   Response: In the revised version there is no tensile strength modelled if no sea ice is present between icebergs. In the absence of sea ice the motion of the icebergs are prescribed by the ice-mélange dynamics, which interpret the icebergs as a thick and compact piece of sea ice. The icebergs, if they do not collide, move with an averaged velocity which is a result of the viscous-plastic ice-mélange model. The resulting viscous-plastic motion can be viewed as an approximation of a granular flow, therefore we think the comparison can be made at this point.

10. *Ln 63: pieces of ice which cannot break*
    Response: We reformulated the sentence.

11. *Ln 158: would it make sense to describe this as a Hertzian contact model? This approach is obviously not entirely new, may be good to say explicitly what sort of contact model you are adopting here.*
    Response: We use a hard-disk model to prescribe floe–floe collisions, see Herman 2011. We added the information to the text.

12. *Ln 160: can you say if Gascoigne is open-source?*
    Response: We added a note that Gascoigne is open source code.

13. *Ln 172: We use a modified*
    Response: Done.

14. *Ln 190: Related to above comment on whether contact mechanics are Hertzian: What is the assumption inherent here? Particles are advected with melange velocity, and they can contact each other, but they don't have momentum? If sea ice concentration were zero, and you had*

a lot of particles densely packed together (like in many Greenlandic fjords in summer), would this model work well? My intuition is no...
Response: The assumption is that the ice mélange as a whole behaves like a viscous-plastic material and the icebergs in summer move according to the derived velocity from the viscous-plastic model without additional tensile strength. This viscous-plastic motion of the iceberg particles is slightly modified if two icebergs collide. The modification is based on a hard-disk model to prescribe the iceberg collision. We added a test case to show the behaviour in the absence of sea ice. We think that this model could work to some extent in summer conditions. It should be possible to capture jamming events in summer.

15. *Ln 203: Is this model approach more appropriate when you don't expect to have interacting particles? If so, then say that here.*
Response: No, we definitely need interacting particles. Especially for the description of the ice-mélange in the summer months, see our answer to your major comment. Furthermore, without particles a realization of ice mélange on coarse grid models is not possible.

16. *Ln 223: could you comment on the non-zero thickness along the terminus wall? I'm guessing this is sea ice thats been piled against the wall (not diffused iceberg thickness)?*
Response: Yes, this is accumulated sea ice. We added an information to the text.

17. *Figure 6: not sure what the colorbar is for here*
Response: The colorbar indicates the amount of the sea-ice concentration visible in the background of the image. The concentration is 0.7 in the initial state. We prefer to keep this information.

18. *Section 4.2.2: I'm still a bit confused about why the ungrounded icebergs don't move at all - my physical intuition is that this should be advected somewhat, but I suppose they are attached to the grounded icebergs. Is that physical?*
Response: The icebergs in the ungrounded setup move slightly. We added a closeup to show that and included a paragraph which describes this behaviour.

19. *Discussion: the discussion section seems quite short to me. There could be more comparison to Vankova and the advantages of this method (beyond simply that it uses a rheology consistent with sea ice models). How is this model more physical realistic than the model of Vankova - please explain. Also, this is a good place to further discuss exactly where this model would be useful - polynyas, where else? In ice sheet models? What biases in climate models would be improved by adding this to a sea ice model? I suggest this because you are essentially arguing to model developers to but this into their meticulously crafted sea ice models, and you could make a more vigorous case for others to put in the work to do so.*
Response: Thanks, we extended the discussion. Now we outline further potential applications.

Thank you for all your very helpful comments and suggestions for improvement. We feel that our paper has much improved by taking them into account!

Revision of the manuscript:

A hybrid ice-mélange model based on particle and continuum methods

September 20, 2024,

Dear Reviewer 2,

we thank you for your valuable remarks on our paper, which we address in the revised version of our manuscript. According to the main concerns which deal with the ice-mélange formulation in summer conditions (in the absence of sea ice between icebergs), we modified the parameterization of the tensile strength in the revised version. Now, we account for a weighted dependency on the sea-ice concentration in the tensile strength parameterization. This leads in particular to zero tensile strength between icebergs in the absence of sea ice. This new tensile strength mechanism is analysed in a test case which has been added to the manuscript. To address the concern that sea ice is modeled on top of icebergs, we slightly modified the derivation such that this assumption could be dropped. Now, the effective sea-ice concentration and thickness per cell is calculated by $\tilde{A}_{\text{sea-ice}} := \min\left(A_{\text{sea-ice}}/(1-A_{\text{iceberg}}), 1\right)$ and $\tilde{H}_{\text{sea-ice}} := H_{\text{sea-ice}}/(1-A_{\text{iceberg}})$. Here $A_{\text{iceberg}} < 1$, due to the assumption that icebergs are modeled by a finite number of small disks. We hope these changes address your suggestions.

Yours sincerely,

S. Kahl, C. Mehlmann and D. Notz

Answer to the Referee's comments

In the following, we respond to the reviewer's comments and explain the changes in the manuscript. All modifications in response to your comments are marked in blue in the paper.

Major comments

1. *My main concern is regarding the implementation of the tensile strength. From what I understand, a grid cell is assigned tensile strength, if sufficient fraction of its area is occupied by icebergs (icebergs being smaller than a grid cell). In reality, ice melange experiences large tensile strength over each individual iceberg (diverging winds will hardly break apart an iceberg) but the tensile strength between icebergs is zero, unless icebergs are glued/frozen together by sea ice (in absence of sea ice, diverging winds will easily blow apart two different icebergs). At the moment, it seems to me that in the proposed model this zero tensile strength between different icebergs is not present, and I anticipate this could lead to accumulation of icebergs, without the ability for them to disperse again. I propose the following example (or similar) to be included in the manuscript:*
-start with a grid cell full of ungrounded iceberg particles in the middle of the domain, away from boundaries, sea-ice concentration being 0 everywhere
-impose divergent wind (vectors pointing from the center of the domain towards boundaries Physically, these icebergs should fairly quickly disperse and move towards the boundaries in all directions. However, if I understand the proposed model well (and example 5 in Fig 8b is already showing that), I anticipate that the icebergs will stay all glued together in the center of the domain, held together by a large tensile strength, which is not what would happen in nature.
Response: Thank you for bringing up this point. We now account for sea-ice concentration when applying the tensile strength to the momentum equation. The inclusion of the sea-ice concentration ensures that there is no tensile strength added if there is no sea ice between the iceberg. The modified tensile strength reads as

$$T = \begin{cases} 0 & \text{if } A_{\text{iceberg}}|_K < \frac{\pi(0.5\sqrt{|K|})^2}{|K|}, \\ c_{\text{tensile}} P^\star H A_{sea-ice} & \text{else.} \end{cases} \quad (1)$$

To test this modified tensile strength we added a test case (Section 4.1.2) with a divergent wind field. The setup shows that icebergs disperse if there is no sea ice between the icebergs. If there is sea ice between the icebergs the tensile strength is active and the icebergs do not disperse in dense ice conditions.

2. *Related to the above, there is a bit of an inconsistency in the testing of the strength properties of the model. The compressive, shear, and tensile examples 1-3 are testing deformation properties on something that resembles and iceberg that is larger than a grid size. But if what is illustrated there as an iceberg was actually a collection of icebergs smaller than a grid cell (and clustered together in the middle of the domain), the desired outcome in terms of thickness distribution of these experiments would be very different.*
Response: Due to the modified tensile strength parameterization the icebergs now disperse also under shear and compression if there is no sea ice between the icebergs.

3. *It seems that the proposed model allows for sea ice and icebergs to overlap (Equation 1), physically that means that icebergs and sea ice would be piling on top of each other. In fact Fig 8a suggest that - does sea ice really pass through icebergs? This does not occur in reality. It is fine for a model to make approximations and assumptions, but it would be useful if this was*

stated explicitly as a limitation. It would also be nice to quantify how often this occurs, or what fraction of sea ice piles up on icebergs, to assess whether this assumption is significant.
Response: Thanks for bringing up this point. We now slightly modified the derivation (see Eq.(1)) such that sea ice and icebergs are not overlapping. Ice mélange is considered as a joint continuum of sea ice and icebergs, where icebergs are interpreted as thick and compact pieces of sea ice. The ice-mélange mass as well as the concentration and thickness of the ice mélange (considered in the momentum equation) includes both sea ice and icebergs. Therefore, the calculated ice-mélange velocity takes the joint mass of the ice mélange into account.

The concentration shown in Figure 9 is the joint concentration of sea ice and icebergs. The particles which are marked in red allow to identify how much of the joint continuum is covered by icebergs. In the presence of icebergs sea ice is more compressed and thicker than in areas without sea ice. The effective sea-ice thickness as well as the sea-ice concentration per cell can be calculated as $\tilde{A}_{\text{sea-ice}} := \min\Big(A_{\text{sea-ice}}/(1-A_{\text{iceberg}}), 1\Big)$ and $\tilde{H}_{\text{sea-ice}} := H_{\text{sea-ice}}/(1-A_{\text{iceberg}})$, respectively. In the context of ice mélange we assume that the icebergs are represented by a finite number of small particles, smaller than the grid cell size.

4. *Some clarifications about the sub-stepping procedure are needed. Are only the particle positions adjusted to account for inelastic collisions? Or is the velocity field modified, as suggested in equation 26? If the velocity fields is modified, is that also propagating back to the advection of the continuum model via some sort of iterative algorithm?*
Response: Only the particle positions are adjusted, the velocity field is not modified. We modified the paragraph and added a pseudo-algorithm for clarification. In test cases in which we consider iceberg particles the relative speed between ice mélange and iceberg particles is almost zero. Therefore, we neglected the feedback from the modified iceberg velocity to the sea ice velocity. You are right in cases where the relative velocity is larger, the ice-mélange velocity should be adjusted. One possibility is to derive an average velocity of the icebergs per cell and include a drag term in the ice-mélange momentum equation that accounts for the difference of iceberg velocity and ice-mélange velocity. We added this aspect to the discussion.

Minor points

1. *Figures - all figures need x y axis labels, dimensions and units*
Response: We added the axis descriptions to all figures.

2. *Ln 52: I think this should be the Hunke and Dukowicz 1997 reference*
Response: Yes, you are right. Thank you for pointing this out.

3. *Ln 62: "Ice mélange is considered as a joint continuum of sea ice and icebergs, where icebergs are prescribed as thick and compact pieces of ice" → Probably appropriate to put the Vankova citation for this.*
Response: We rephrased the sentence and highlight now the particle coupling into the continuum which is novel in this contribution.

4. *Eq 1: This is a bit strange, I would think that at any given point there is either iceberg, or sea ice, but not both of them (that is, iceberg and sea ice can't pile on top of each other). For this case I would expect $H = h_p$.*
Response: We modified the derivation (see Eq.(1)) such that there is either sea ice or icebergs at a given point in space.

5. *Ln 103: remove almost*
   Response: We removed the almost from the sentence.

6. *Eq 29: Is this density? If yes, how do the units of $H_{iceberg}$ work out?*
   Response: Sorry that's a typo. ρ should not appear in this equation.

7. *Eq 30: Is this overwriting equation 23? Or is this just its discretized form?*
   Response: It is the discretized form. We changed the paragraph to clarify this.

8. *Eq 30: I think it should be specified earlier whether all icebers are the same size, or if they can have variable sizes. In case they need to be all the same size, specify where this assumption is necessary.*
   Response: They can have variable sizes, we choose for simplicity a uniform size for our test cases. We added a sentence to clarify this.

9. *Eq 30: Why do you need this threshold all together, and why not allow some tensile strength for sea ice as well (as in Konig)? I guess you would still need some indicator as the tensile strength formulation would be different for sea ice (small multiple of compressive strength) and iceberg (large multiple of compressive strength)?*
   Response: In our paper we consider the viscous-plastic (VP) sea-ice model of Hibler. In Section 4.1 we have shown that a higher tensile strength is necessary to model icebergs in this model. There are also variants of the VP model with a higher tensile strength, see Hutter 2022. Qualitatively the proposed approach can be also used for modified versions of Hiblers rheology.

10. *Ln 187+189: should this be melange? aren't you advecting properties defined in equation 1 and 2?*
    Response: Eq.(1) and Eq.(2) is an analytic point-wise representation of the thickness and concentration of the ice mélange. Eq. (30) and (31) explain how this is numerically realized on a grid. The ice-mélange concentration and thickness only enters the momentum equation. Sea ice and iceberg properties are separately advected. We clarify this in Eq. (7) and Eq. (8)

11. *Ln 190: Is it really sub-stepping - it seems that you are doing these in parallel. Sub-stepping suggests that over the duration of one step of melange advection, you do many particle transport steps - is that what you do? Please clarify this.*
    Response: We transport sea ice and icebergs separately and build a joint continuum for the ice mélange. The thickness and concentration of the ice mélange only couple to the momentum equation. We modified the paragraph and added a pseudo-algorithm to clarify this.

12. *Ln 191: interpolation → what interpolation algorithm do you use? bilinear, or something like that?*
    Response: We use a piecewise linear finite element interpolation based on the corresponding basis function. We added the information to the text.

13. *Ln 191: advection-step → do you mean after the substepping?*
    Response: See answer to question 11.

14. *Ln 192: then replaced → do you mean the velocities are corrected?*
Response: We check the advected particles for collisions and if they collide we use the modified velocity (Eq. (26)) to calculate the new particle position due to the collision.

15. *Ln 193: sea-ice → melange tracers? or continuum tracers?*
Response: See answer to question 11.

16. *Paragraph ln 188-194: For this last paragraph, I would suggest making this bullet point list of the steps that are done and in which order, so that it is really clear and nothing is lost/ambiguous. For example, the way this is written now, I wouldn't be able to reimplement this algorithm myself without having to contact the authors for clarification.*
Response: We added a pseudo-algorithm to clarify the procedure.

17. *Ln 198: not sure what you mean by this or what it is specifically about the coupling you aim to test.*
Response: We reformulated the sentence. We qualitatively analyze the combination of particle and continuum methods to represent ice mélange.

18. *Ln 200: tensile → compressive, compressive → tensile*
Response: We corrected this.

19. *Ln 202: for → of*
Response: We corrected this.

20. *Figure 2: Is this shown on the native grid or interpolated?*
Response: It is shown on the native grid.

21. *Ln 220: move → maybe the word deform would be useful here?*
Response: Done.

22. *Figure 4: But if this was a cluster of sub grid scale sized iceberg particles, you would probably expect something more like (a). See for example iceberg distribution in Stern et al 2016*
Response: See our response to your major comment 1 and 2

23. *Figure 5: should the icebergs be circular? instead of octagonal?*
Response: The icebergs are circular. This is just due to the closeup of the visualization. We changed the visualization.

24. *Figure 6: Are we supposed to think of this as one grounded iceberg, or many grounded icebergs next to each other?*
Response: The particles are many grounded icebergs next to each other with sea ice between the icebergs.

25. *Figure 8: Icebergs that are ungrounded should flow away towards the right boundary if there is open water to the right of them and they are forced with wind from the left. There is no reason they should be held by some strength to the other ice bergs. There is no physical basis for this strength (apart from sea ice bonds = ice bergs frozen into sea ice - but I don't think that is the*

case you are simulating here?)
Response: This is a misunderstanding, the ungrounded icebergs are still moving, just very slow. We added another figure for clarification. There is densely packed sea ice between the icebergs. So icebergs are not in a free drift. The movement of the icebergs to the right could be further accelerated by lowering the constant c_{tensile} in Eq. (30). To properly choose the constant c_{tensile}, a comparison with observations is necessary. This is devoted to future work.

26. *Ln 273: through or around? Like iceberg, landfast sea ice (what you have here essentially) should still be able to form a polynya*
Response: It flows through the icebergs as the ice-mélange velocity in the cell is much larger than zero. The mass of the ice mélange in the cell with the grounded icebergs is higher than in the neighbouring right cell. This leads to a slow down of the velocity in the grid cell that contains icebergs compared to the velocity in the neighbouring right cell. Thus, a lower ice-mélange concentration (a "polynya") behind the grounded icebergs (in the neighbouring right cell) is modelled. This effect is also observed in the absence of tensile strength. The major point in Section 4.2.3 is that not a pile up of sea ice in the front of the icebergs is modelled without additional tensile strength. The missing pile up is unphysical.

27. *Ln 284: can you elaborate on why this is the case, and outline foreseeable difficulties involved with relaxing this assumption?*
Response: The use of geometric objects with other shapes can lead to a motion with different direction after the collision. But calculating the collision of more complex geometric objects such as polygons is numerically more expensive compared to the usage of disk shaped particles, Damsgaad et al 2021. We added a sentence to clarify this

28. *Ln 303: you keep using this, but 300 m is very small iceberg, definitely in the Antarctic context. And even in Greenland icebergs are often bigger than that. Rather than <300 m, I would suggest to say something like 'on the order of 100s of m in Greenland fjords'.*
Response: Thank you for pointing this out. We have rewritten the sentence.

Thank you for all your very helpful comments and suggestions for improvement. We feel that our paper has much improved by taking them into account!