Revision of the manuscript:

Modelling ice mélange based on the viscous-plastic sea-ice rheology

November 15, 2023,

Dear Reviewer 2,

we thank you for your valuable remarks on our paper. The goal of our paper is to realize ice mélange in the context of Hibler's viscous-plastic sea-ice model, which is currently the most used rheology for sea ice in climate models. So far, there is no description for ice mélange available in the context of the viscous-plastic sea-ice model of Hibler. We demonstrate in our work that the composite of sea ice and icebergs (the ice mélange) is represented in a physical meaningful way. A similar approach was realized by Vankova and Holland using Flato's rheology. However, in a sea-ice model intercomparison project, it was shown that Hibler's rheology leads to more realistic results (Kreyscher 2000). Therefore, and due to its widespread use, we consider Hibler's model in this paper. We thank you for asking how to handle our ice mélange approach on coarser grids. Now, the volume of fluid tracking of icebergs in the ice-mélange model is replaced by a particle tracking of icebergs. This allows us to describe ice mélange also on a sub-grid level. Thus, we do not need meshes that resolve icebergs, which clearly distinguishes our approach from existing ones. Due to the particle tracking the resistance to shearing forces and the separation of icebergs is straight forward, as well as the handling of multiple icebergs within a grid cell. Finally, we address the need of efficient numerical methods to solve ice-mélange models, which has been formulated as an open research question. We do so by analyzing a modified Newton method. We highlighted this aspect in the abstract and introduction to clearly state the novelty of the work and rewrote the corresponding parts of the paper.

We hope these changes address your suggestions.

Yours sincerely,

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Answer to Referee's comments

In the following, we respond to the reviewer's comments and explain the changes in the manuscript.

Major comments

1. *In its current form, the manuscript is a straightforward model description paper, which looks at how to get from existing sea ice models to this new model, and certain improvements in accuracy and computational performance by adding tensile stress and using a modified Newton Solver. At that level, this paper is more appropriate for a journal like Geoscientific Model Development than The Cryosphere. To be of interest to the broader readership of The Cryosphere, the authors could take this wonderful new tool and use it to answer certain outstanding questions about ice melange that current tools cannot answer. One is how ice melange responds differently to forcing than just sea ice. Why does it matter that you have bits of thick ice which can resist tensile stress embedded within sea ice? The answer would likely be from a ocean/atmosphere perspective, as the manuscript does not currently argue that this tool would necessarily be used in conjunction with ice sheet models..*
   Response: We show that with the ice-mélange model a correct physical description of the composite of icebergs and sea ice is obtained. Our approach is realized in Hibler's viscous-plastic sea-ice model, which is currently the most used material law for sea ice in climate models. Therefore, we think it is a topic that is of interest for a large community. Furthermore, the Cyropshere invites for contributions in numerical modelling and the ice mélange fits into over arching topics of the journal such as sea ice and ice sheets. This is why we think the Cyrosphere is an appropriate choice.

2. *The motivation for this work, as described in this manuscript, it to be able to efficiently include ice melange into climate models. However, the test cases here exclusively look at cases where the resolution is much, much finer (0.1 km) than it is in any global climate model (10-100 km). What do the author's anticipate being the specific use case for such a formulation? Should it be used in global simulations? Should it only be used for regional simulations at very high resolution? Is the indicator function Phi (Equation 20) strictly necessary to have a discrete division between grid boxes with sea ice and icebergs, or could one have a continuous function (phi) between 0 and 1 that can capture the scenario where you have a large climate model grid box with some icebergs in it (but not completely filled with them)?*
   Response: Thanks for bringing this point up. We now changed the iceberg tracking and apply a particle tracking of icebergs. This allows us to describe ice mélange also on a sub-grid level. Thus, we do not need meshes with a high resolution that resolve icebergs.

3. *It remains unclear how the results shown here with the new method specifically improve upon the method of Vankova. The comparisons made in section 3 are between a standard VP rheology versus a VP rheology with tensile strength, whereas Vankova uses an entirely different rheology with tensile strength. Section 3.1 makes the most direct comparison by looking at a similar setup, but there needs to be a specific argument made about why the inaccuracies in Vankova's method are problems (i.e. related to error in representing aspects of the physical climate system that we care about). In general, this comparison with Vankova is critical to arguing why this new method is necessary, and it should be explored in more detail throughout the results and discusison section.*
   Response: First, Hibler's model is more realistic for the simulation of sea ice. Second, the idea of our method is that one can still model sea ice with the viscous-plastic rheology of Hibler. Thus, one has a simple coupling and integration into existing sea-ice modules in climate models, which to a large extent still use Hibler's viscous-plastic material law. In the approach of Vaňková

and Holland sea ice is represented by a different material law, which makes an integration into sea-ice modules more difficult. Third, the method of Vaňková and Holland requires that the resolution of the grid is at least of the size of the icebergs. By using a particle tracking, we can also represent ice mélange on a sub-grid level.

Minor points

1. *Line 1: influence on sea-ice-ocean interactions. Ice melange currently is not represented in climate model as numerically efficienty realizations do not exist. This motivates our development*
   Response: We rephrased the sentence.

2. *Line 11: Newton's method*
   Response: Done.

3. *Line 12: melange into climate models*
   Response: Done.

4. *Line 14: Fjord with marine-terminating glaciers can be filled with sea ice into...*
   Response: Done.

5. *Line 17: the ice melange at some glaciers disintegrates [melange is year-round at some glaicers]*
   Response: We rephrased the sentence.

6. *Line 17: ...the seasonal cycle of glacier flow is in phase with the presence of ice melange, leading to the...*
   Response: We rephrased the sentence.

7. *Line 22: First sentence of this paragraph can be deleted as its redundant*
   Response: Done.

8. *Line 24: through releasing their freshwater storage from icebergs.*
   Response: Done.

9. *Line 25: delete "also"*
   Response: Done.

10. *Line 28: Delete "This is the more the case as"*
    Response: Done.

11. *Line 29: and the challenges of taking in-situ measurements within densely packed melange or near claving fronts*
    Response: We rephrased the sentence.

12. *Line 39: several studies have implemented a particle-based approach for representing icebergs into large-scale ocean models, including those by Alon Stern and Anders Damsgaard.*
    Response: We rephrased the sentence.

13. *Line 41: to commonly used continuum sea-ice formulations.*
    Response: Done.

14. *Line 43: model using a continuum approach.*
    Response: Done.

15. *Line 45: Using such an approach*
    Response: Done.

16. *Line 46: cavitating*
Response: Done.

17. *Line 50-53: Amundson and Burton (2018) don't do a direct comparison between observations and a VP rheology. They do conclude that continuum granular rheologies based on non-local or friction dependent on inertial number work well in describing granular flow through channels, but they don't directly compare to observations. My suggestion is to rewrite this sentence to make it a more accurate reflection of that paper. Even if there isn't observational support for using a VP rheology for ice melange, there are good modeling reasons, and for now, that is sufficient.*
Response: Thanks for the hint we included the sentence.

18. *Equation (in general): typically div is written as "del dot" ($\nabla\cdot$ in LaTeX) since the term "divergence" is not the same in all languages, whereas "del dot" is more "universal" notation*
Response: Done.

19. *Line 75: describes the divergence of the two-dimensional...*
Response: Done.

20. *Equation 2: I'm a bit confused by how forces and stresses are being mixed here since they do not have the same units. Either some notation is mixed up, or some constants are missing.*
Response: We carefully went trough the notation and fixed it.

21. *Equations 3-4: why does the melange velocity enter into equation 4, but not 3? (In the most general sense)*
Response: The wind velocity is much larger than the sea-ice velocity. Therefore, we follow the traditional approach in sea-ice modelling and neglect the sea-ice velocity in the atmospheric drag term.

22. *Line 88-89: this sentence is a bit odd. Ice melange is melted from below by the ocean, this is an important sink term. I think the point is that you are just considering idealized cases where melting is ignored. If so, then just state that.*
Response: Thanks, we included a sentence.

23. *Equation (6): It would help the exposition to explain what e and P are right after this equation*
Response: We added an explanation right after the sentence.

24. *Line 92: expressed in terms of the principal components...*
Response: Done.

25. *Line 105: It would help if equation 14 came before equations 12 and 13 as a way to explain that there are two regimes with a smooth transition between them before giving equations for the two regimes.*
Response: We changed the order of the equations.

26. *Line 107: what is meant by the "viscous closure"?*
Response: We rephrased it to the viscous regime.

27. *Line 118: strength lead to a...*
Response: Done.

28. *Line 126: what is meant by "normal flow rule"?*
Response: The normal flow rule models the relationship between the stresses and the strain rates. In the viscous-plastic model it is assumed that a change in strain is only allowed in

normal direction to the stress on the yield curve. The model assumption goes back to Truckers postulate. We added a citation.

29. *Equation 19: Why is the indicator function Phi needed if you already have phi? Also, what is the meaning of phi? It seems like a concentration of icebergs. The reason for this question relates to major point #2 above. If you are allowed to have a continuous concentration of icebergs between o and 1, then the model could make sense at grid resolution compared to global climate model, where a grid box might have some icebergs in it (while not being fully filled with icebergs).*
Response: We changed the iceberg tracking.

30. *Equation 19: What is c and how is it set?*
Response: We changed the iceberg tracking.

31. *Line 141: Is this the same thing as operator splitting? If so, that seems like the more common terminology to me.*
Response: Splitting in time means that some terms are treated implicitly in time and others explicitly, such that the coupled system of equations is split in the momentum equation and the transport equation which can be solved after each other.

32. *Line 151: prove not proof*
Response: Done.

33. *Figure 2-4: it is somewhat confusing to me why P-T is the correct contour to plot here (as opposed to concentration), and also that there are maybe three different quantities plotted? (P-T, arrows and a black contour for the iceberg, but not clear what quantity that represents). There should probably be a legend for these plots and a better explanation of what is plotted in the caption. It is also unclear why the quiver arrows indicating melange velocity have such variable velocity.*
Response: We changed the visualization.

34. *Line 200: Vankova and Holland (2017) stated the need for efficient*
Response: Done.

35. *Line 204: what about the third setup makes it "difficult"*
Response: Done.

36. *Line 213: residual threshold for convergence*
Response: Done.

37. *Line 218: It would be useful to have some kind of explanation as to why the modified Newton solver appears to work so much better than the others.*
Response: The method makes use of the structure of the equation. We added an explanation to the numerical setup.

38. *Line 222: icebergs*
Response: Done.

39. *Line 231: Most climate models represents the*
Response: Done.

40. *Line 232: Delete "An"*
Response: Done.

41. *Line 233: we recommend implementation*
    Response: Done.

Thank you for all your suggestions!