Revision of the manuscript:

Modelling ice mélange based on the viscous-plastic sea-ice rheology

November 15, 2023,

Dear Reviewer 1,

we thank you for your valuable remarks on our paper. The goal of our paper is to realize ice mélange in the context of Hibler's viscous-plastic sea-ice model, which is currently the most used rheology for sea ice in climate models. So far, there is no description for ice mélange within the viscous-plastic model of Hibler. We demonstrate in our work that the composite of sea ice and icebergs (the ice mélange) is represented in a physical meaningful way. A similar approach was realized by Vankova and Holland using Flato's rheology. However, in a sea-ice model intercomparison project, it was shown that Hibler's rheology leads to more realistic results (Kreyscher 2000). Therefore, and due to its widespread use, we consider Hibler's model in this paper. Based on a suggestion by reviewer 2, we now replace the tracking of icebergs in the ice-mélange model by a particle tracking of icebergs. This allows us to describe ice mélange also on a sub-grid level. Thus, we do not need meshes that resolve icebergs, which clearly distinguishes our approach from existing ones. Due to the particle tracking the resistance to shearing forces and the separation of icebergs is straight forward, as well as the handling of multiple icebergs within a grid cell. Finally, we address the need of efficient numerical methods to solve ice-mélange models, which has been formulated as an open research question. We do so by analyzing a modified Newton method. We highlighted this aspects in the abstract and introduction to clearly state the novelty of the work and rewrote the corresponding parts of the paper. We hope these changes address your suggestions.

Yours sincerely,

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Answer to Referee's comments

In the following, we respond to the reviewer's comments and explain the changes in the manuscript.

Major comments

1. *As far as I can tell the only new thing in this paper is the implementation of spatially variable strength within the Hibler viscous-plastic rheology.*
   Response: The aspects which are novel in the paper are highlighted above.

2. *The representation of icebergs as thick and compact pieces of sea-ice and the concept of spatial variable strength depending on sea-ice or iceberg label is identical to that of Vankova and Holland. The only differences are the advective scheme and the shape of the yield curve in the implementation.*
   Response: Both approaches use an indicator type function to distinguish icebergs and sea ice via the ice strength. In our approach this modification is justified by a reformulation of the yield curve in the viscous-plastic model. The realization of the iceberg tracking and the resulting consequences differ significantly between the two approaches. We now use a particle tracking that allows us to describe ice mélange also as a sub-grid process. Therefore, we do not require grids which have at least the same size as the icebergs. In the approach of Vaňková and Holland the mesh size needs to be high enough so that icebergs are resolved.

3. *The yield curve the authors use is identical to Konig and Holland.*
   Response: So far, this modification of the yield curve has been used in different contexts like landfast ice (König and Holland) or as modification for the sea-ice rheology of Hibler (Ringeisen et al.). In our contribution we show that the modification is also useful for modelling ice mélange in the viscous-plastic sea-ice model.

4. *The authors emphasise throughout the paper the implementation of the strength modifications within viscous-plastic rheology, but it is important to realise that the Flato model, where the same was implemented is also viscous-plastic in its 2-D form.*
   Response: The goal of our paper is to realize ice mélange in the context of Hibler's viscous-plastic sea-ice model, which is currently the most used rheology for sea ice in climate models. We consider the full viscous-plastic dynamics of Hibler and present a numerical efficient way to solve it. The model of Hibler differs from the model of Flato. Furthermore, an efficient numerical treatment has not been presented for the model of Flato in 2D.

5. *In terms of the advection scheme and labelling icebergs, the authors do not show whether their scheme is able to handle separation of two icebergs that had been joined by convergent winds. I think this is a crucial feature to show. As of now there have been no iceberg-iceberg interactions tested and its unclear how the scheme handles the presence of multiple iceberg portions within a single grid cell. The authors also do not show (although it is claimed they do) that the scheme handles shearing - in test case 3 (which claims to test that) wind shear is applied away from the icebergs, the wind over icebergs is uniform and so the ability to rotate instead of deforming has not been shown.*
   Response: By the use of particles (rigid bodies) for the tracking of icebergs it is straight forward to handle separation of icebergs, multiple icebergs in a grid cell as wellas the resistance of icebergs to shearing forces. We added corresponding numerical test cases.

6. *Finally, I think the second selling point, after Hibler rheology, is the improved computational efficiency. The conclusion of the authors is that modified Newton leads to 40% improvement over Picard. However, there have been much more efficient solvers developed for the same system of*

equations. See for example Lemieux et al 2012, they show that Jacobian-Free Newton–Krylov (JFNK) solver is 3–7 times faster than the Picard solver for the same system of equations. The thing that makes the equations less likely to converge to a solution is whenever there is large stress buildup, which is what happens over the thick and compact bits of mélange as material strength scales with thickness per the parameterization (that is typically an issue for thick and compact sea-ice cover in continuum sea-ice only models as well, but the presence of thicker icebergs this is made worse). The test case on which the authors calculate the convergence properties is not one that experiences large stresses - it is icebergs moving through very thin and not very compact sea-ice, so they are essentially freely drifting. In summary, I don't think there has been made any improvement towards computationally efficient tractability of mélange as a continuum.
Response: We apologise the presentation is misleading. The main point is that the Picard solver failed to reach the required residual in 19 steps. The solver has been stopped after 100 iterations, with a much higher residual than the modified Newton solver. The fact whether a Newton solver converges or not depends on whether the initial guess is in the area of Newton convergence. There are situations in which the Newton method (also JFNK) does not reduce the nonlinear residual but the Picard solver reduces the nonlinear residual. However, in such cases the reduction by the Picard solver is very slow e.g. Mehlmann and Richter (2017). In the paper "*Robust and efficient primal-dual Newton-Krylov solvers for viscous-plastic sea-ice models*" as well as in the PhD thesis of Mehlmann it has been shown that the modified Newton solver is much more robust and efficient than the JFNK solver. In this paper we demonstrate that the modified Newton solver is also more robust in the ice mélange setting, which is in line with previous results for the standard viscous-plastic application. We now address this point and added corresponding references in the text.

Minor points

1. *Good chunk of the abstract sounds like copy pasted abstract from Vankova and Holland 2017. It would be good to separate out clearly what it is that has been done before and what is the new development and lay that out unambiguously. Arguably the new thing is that the existing sea-ice/iceberg prototype model (continuum model with spatially variable rheology and icebergs represented as thick and compact pieces of sea-ice held together by large shear and tensile strength) was implemented within the much more frequently used Hibler rheology instead of the original Flato cavitating fluid one.*
   Response: We reformulated the abstract and highlighted the novelty of our approach.

2. *The references are bit out of date - there have been some more new observational and modeling papers out that should probably be mentioned in this paper (I made a few notes in the pdf, but there is more out there). Some of the provided references are inaccurate (e.g. observational reference for a modeling claim). I didn't get to check all, but a few random ones were wrong so please check and fix all references.*
   Response: Done.

3. *There is an imbalance between the description of the equations which is quite comprehensive although already present in the literature, and the numerics section which is really thin and barely touched on. We don't even learn what grid the authors use.*
   Response: Thanks for the hint. We now describe the numerical setup more in detail.

4. *figs 2-4: arrows are not clear - it was those figures that made me thing the grid might be unstructured, but then I realized it is just arrows overlapping which makes it difficult to compare their relative size.*
   Response: We changed the representation.

5. *Fig6: why is 48th time step interesting? Show all time steps or some statistics from them so that we can see the spread. Here Picard actually does better than Newton in this case, it seems, which is not typical.*
Response: See our response to major comments point 6.

6. *Conservation of shape rectangle vs cylinder - this is a big misconception. The preservation of the shape of a moving object has to do with the advection scheme properties, not with the shape of choice.*
Response: We reformulated the sentence.

7. *Line 17: "Observations show that the seasonal pattern of glaciers and ice mélange overlap" → There are also observations pointing to no relation between the two, e.g. Amundson et al 2020*
Response: We rephrased the sentence.

8. *Line 19: suggested*
Response: Done.

9. *Line 20 - 21: I don't think this is a correct definition of buttressing. Buttressing and calving are not necessarilly related. See e.g. Reese et al 2018 for buttressing.*
Response: We removed the sentence.

10. *Line 23 - 24: That is not what is driving sea-level rise, the effects described here are minor and only come down to the difference of densities between freshwater and seawater. Sea level rise is mainly driven by acceleration of grounded ice towards the margins and enhanced ice flux across the grounding line.*
Response: We reformulated the paragraph.

11. *Line 28: "satellite images" → There are other methods for observing the ice mélange, e.g. terrestrial radar interferometry. See e.g. Xie et al 2019, Cassotto et al 2021*
Response: We added the information.

12. *Line 30: what do you mean by explicit, there are still parameterizations within mélange models*
Response: We mean explicitly modelling the icebergs in h and A. We reformulated the sentence.

13. *Line 37: Peters et. al., 2015 → not a modeling paper*
Response: We reformulated the paragraph.

14. *Line 40 - 41: Vankova and Holland, 2017 → this is not a particle model reference*
Response: We reformulated the paragraph.

15. *Line 41: "currently unclear" → Are there really no particle models coupled to continuum models in any fluid application?*
Response: We refer to sea-ice models.

16. *Line 41: "coupled" → coupling of what to what do you mean here?*
Response: We rephrased the sentence.

17. *Line 41 - 42: "used continuum sea-ice formulation" → there are also particle sea-ice models being developped/used, perhaps worth mentioning*
Response: We added literature to that point.

18. *Line 46:* cavitating
Response: Done.

19. *Line 48: "viscous-plastic material" → The 2D Flato model is also viscous-plastic*
Response: We rephrased it to the viscous-plastic sea-ice model of Hibler.

20. *Line 51: "productive" → what is the measure of productivity? provide reference*
Response: We rephrased the sentence.

21. *Line 52: "valid" → depends on the timescale of interest*
Response: We rephrased the paragraph.

22. *Line 53: "modification of the viscous-plastic material law" → you haven't introduced one so it is unclear what and why is being modified*
Response: We rephrased the paragraph.

23. *Line 55: Here we develop*
Response: Done.

24. *Line 55 - 56: "viscous-plastic rheology, introducing a spatially varying strength parameterization" → Probably should cite here Vankova and Holland as that is exactly what they did, and not only later on*
Response: We restructured the introduction.

25. *Line 58: "sea-ice" → and also use a spatially variable strength parameterization within a viscous-plastic continuum model*
Response: We rephrased the paragraph.

26. *Equation 1: You should first include all terms before dropping selected ones and you should provide justification for dropping terms - the advective term is missing at the moment*
Response: We added an explanation. The advection term in the momentum equation is standardly neclegted in the viscous-plastic sea-ice model.

27. *Line 151: prove*
Response: Done.

28. *Line 153: "Essential for this proof" → where is this proof?*
Response: The proof is given in the mentioned paper Mehlmann and Richter 2019. We reformulated the sentence.

29. *Line 162: "shear (Section 3.3) strength" → unlike announced here, the 3rd experiment does not test the ability of the iceberg part of mélange to withstand shear forcing without deformation, for that you would have to place the sheering wind on top of the iceberg*
Response: See our response to major comments point 5.

30. *Line 163: "extreme" → extreme in which sense?*
Response: We reformulated the sentence.

31. *Line 164: "play a major role in the center of the fjord. " → depends on the fjord shape*
Response: Done.

32. *Line 165: "neglected" → there is a difference between being neglected and being set to zero, presumably you are still keeping the ocean stress term that depends on mélange velocity as well?*
Response: We set the ocean velocity to zero. The sentence has been rephrased.

33. *Line 171: "A grid cells size of 110m is used" → You haven't said anything about the grid cells and stencills etc yet, are the regular? c grid?*
Response: We added more information to the numerical methods section. It is a regular A-grid.

34. *Line 176: "the iceberg and the sea-ice move" → you are modeling one material so it is just mélange....*
Response: We rephrased it.

35. *Line 176 - 177: "he glacier and accumulate in front of the glacier terminus" → isn't it really just the iceberg getting deformed what we see? sea-ice should indeed move and accumulate towards the terminus but the figure doesn't show that*
Response: We changed the figure.

36. *Line 187 - 188: "This test case shows in particular that the iceberg maintains its shape even under diverging wind conditions at the location of the iceberg." → That is by design, isn't that the whole point of the experiment? why the word even?*
Response: We removed even from the sentence.

37. *Line 189: "Two travelling icebergs" → this test properties of the advection scheme, not ability of iceberg to withstand shear*
Response: We changed the setting.

38. *Line 220: "The" → In line with prior findings, the*
Response: Done.

39. *Line 224 - 225: "Furthermore, the volume-in-fluid method cannot represent the corners of the rectangles exactly such that the corners are easily smoothed out by the advection scheme." → but that doesn't have to do so much with iceberg corners does it? if there is a cylinder peaking into a corner of a grid cell, its shape will too get diffused, right?*
Response: We change the tracking method to a particle tracking.

40. *Line 226: "whose shape is easier to conserve." → provide reference*
Response: We reformulated the paragraph.

41. *Line 228: "representation" → do you mean preservation of shape?*
Response: We reformulated the paragraph.

42. *Line 247 - 248: "Likewise, we demonstrate that icebergs can also interact while maintaining their shape" → no iceberg interaction has been shown*
Response: We added numerical test cases.

Thank you for all your suggestions!