As the authors point out, there have been a number of papers examining clustering of microswimmers due to gyrotaxis in homogeneous isotropic turbulence over the past one or two decades.

The main contribution of the present paper appears to be the addition of the inertial torque (second term on the RHS of Eq. 2) to an otherwise standard model of a microswimmer that has previously been studied. This inertial torque comes from the work of Candelier et al. (2022). Candelier et al. go to great lengths to describe the various orders of approximations in their work and the conditions under which their results are valid. However, the authors of the present paper do not clarify whether the situation they consider is consistent with including this extra term and whether they can include only this extra term without the need to include other extra terms for consistency. In a similar vein, the authors (between Eq 4 and Eq 5) state the microswimmer inertia is negligible and consider the limit where their St —> 0 without any explanation of whether the limit exits at finite Re (required for the inertial torque to be relevant).

Author's Reply:

Thank you for your constructive comments. One of your major comments is concerned with the validation of our model in the situation currently discussed. We apologize for the lack of clarification of model assumptions. We provide a clearer clarification as follows about how the assumptions of our model are met in the parameter range that is specified right after Eq. (9).

The first assumption is that we adopted Candelier's model for the fluid inertial torque. This requires the Reynolds number to be finite but much smaller than unity, which is satisfied for typical marine microorganisms. For instance, we assume the radius and the swimming speed of a typical plankton to be a = 0.1 mm and v_s = 0.5 mm/s, respectively. The obtained Reynolds number is $\mathrm{Re} = 2av_s/\gamma = 0.1$, where $\gamma = 1\,\mathrm{mm^2/s}$ is the kinematic viscosity of fluid. The Candelier's model is expected to describe the inertial torque quite precisely at such a low Reynolds number according to Candelier et al. (2022). A more detailed presentation of Reynolds number of typical plankton species can be referred to Table II in our earlier publication (Qiu et al., Physical Review Research 4, 023094 (2022)), which is shown below.

The second assumption is made when we derive Eqs. (5-8) from Eqs. (3) and (4). Here, we assume the Stokes number is negligibly small. The Stokes number, usually used to quantifying the inertia of a rigid particle, is defined as $\mathrm{St} = 2Da^2/(9\gamma\tau_f)$, where a, D, and τ_f are the radius of the plankton, the cell-to-fluid density ratio, and the time scale of fluid motion, respectively. For typical plankton parameters, we have a = 0.1 mm and D = 1.05. Considering the typical intensity of ocean turbulence, τ_f ranges from 1.0 to 31.6 s (Kiørboe & Enric 1995). Using all these parameters, we estimate that St ranges from 0.0001 to 0.0023. With such small St, our derivation is justified.

At the limit of zero St, the fluid inertial torque is not negligible as long as the swimming and settling speeds are not so small. This is shown in Eq. (4). When St ->0, the last term on the right-hand-side, which represents fluid inertial torque, is unaffected. Its magnitude is determined by a coefficient scaling with $\tau_f u_f^2/\gamma$ as well as the dimensionless swimming and settling speeds, Φ_s and Φ_g. In a turbulent flow, τ_f and u_f

are chosen as the Kolmogorov time and velocity scales, giving $\tau_f\,u_f^2\,/\,\gamma = 1$. This equation is given by the definition of Kolmogorov scales, where $\tau_f = (\gamma/\epsilon)^{1/2}$, $u_f = (\gamma\epsilon)^{1/4}$, and ϵ is the energy dissipation rate of turbulence. As a result, the fluid inertial term is comparable to other terms as long as the dimensionless swimming and settling speeds (Φ_s and Φ_g) are finite. According to the Table below, both Φ_s and Φ_g vary across a wide range depending on species and flow condition. For some zooplankton species, Φ_s and Φ_g are large enough so that the inertial torque is not negligible, while Re and St are still within the range of our model assumptions.

To address the validation of our model, we revised the text in section 2.1 (additional text is clearly marked), and we show typical values of St and Re using physical properties of marine plankton.

Around line 55:

"The motion of plankton in fluid flows is usually described by a micro-swimmer model (citations, ⋯), which assumes a plankton to be a point-like micro-swimmer carried by a fluid flow. This assumption is justified when the Reynolds number, $Re = a|v-u|/\gamma$, is much smaller than unity. Here, the Reynolds number is defined based on the radius of a swimmer, a, the differences between the velocities of a swimmer v and its ambient undisturbed flow u, and the kinematic viscosity of the fluid γ. For typical plankton species, this assumption is justified because of their tiny size and limited motility, as summarized in our recent publication (Qiu et al., 2022a). For instance, the typical size and swimming speed of zooplankton are $a = 0.1\,mm$, $|v-u| = 1.0\,mm$, respectively. Accordingly, we obtain $Re = 0.1$ using the viscosity of water $\gamma = 10^{-6} m^2/s$"

Around line 70:

"The model of fluid inertial torque is derived in the limit of Re → 0, but it has been shown to be justified when Re < 0.3 (Candelier et al., 2022), within the typical range of plankton physical properties (Qiu et al., 2022a)"

Around Line 80:

"Typically, St of planktonic microswimmers are usually negligibly small as summarized in Qiu et al. (2022a). For instance, using $a = 0.1 mm$, $\rho_p/\rho_f = 1.05$, and using typical turbulence Kolmogorov timescale $\tau_f = 31.6\,to\,1.0\,s$, one obtains $St = 1.0 \times 10^{-4}\,to\,2.3 \times 10^{-3}$."

Species		Φ_{swim}	Φ_{settle}	M	Re$_p$	St ($\times 10^{-5}$)	Ψ_I
Cochlodinium polykrikoides [39]	Single cell	2.17~0.39	0.14~0.03	−0.078	0.015	0.17~5.52	20.7~655.2
	2-cells	3.32~0.59	0.1~0.03	−0.101	0.029	0.22~6.95	9.1~287.2
	4-cells	4.44~0.79	0.23~0.04	−0.136	0.077	0.45~14.11	3.6~112.5
	8-cells	4.75~0.84	0.36~0.06	−0.137	0.147	0.90~28.51	2.1~67.4
Centropages typicus [28,41]	early nauplius	1.83~0.33	0.28~0.05	−0.114	0.041	1.29~40.78	8.7~274.2
	late nauplius	3.99~0.71	0.78~0.14	−0.114	0.153	3.75~118.48	1.4~44.9
Euterpina acutifrons [28]	late nauplius[1]	5.99~1.06	1.44~0.26	−0.114	0.204	2.96~93.61	0.5~16.1
Eurytemora affinis [28]	late nauplius[1]	9.09~1.62	1.01~0.18	−0.114	0.313	3.02~95.49	0.5~15.2
Temora longicornis [28,40]	late nauplius[1]	3.16~0.56	1.33~0.24	−0.114	0.166	7.02~222.01	1.0~33.1
	copepod[1]	4.55~0.81	0.94~0.17	−0.114	0.231	6.57~207.83	1.0~32.5
Ceratium tripos[2] [27]		0.93~0.16	0.91~0.16	−0.101	0.012	0.46~14.60	5.9~186.1
Ceratium furca[2] [27]		4.32~0.77	0.34~0.06	−0.101	0.033	0.17~5.50	3.3~105.9
Akashiwo sanguinea[2] [27]		1.66~0.30	0.30~0.05	−0.101	0.012	0.15~4.81	9.9~314.3
Dinophysis acuminata[2] [27]		1.84~0.33	0.18~0.03	−0.101	0.010	0.09~2.84	15.2~481.8
Alexandrium minutum[2] [27]		1.54~0.27	0.05~0.01	−0.101	0.005	0.03~0.89	58.3~1843.6
Prorocentrum minimum[2] [27]		1.14~0.20	0.03~0.00	−0.101	0.002	0.01~0.44	159.8~5053.4

Table II in Qiu et al., Physical Review Research 4, 023094 (2022). In this table, Φ_{swim} and Φ_{settle} means Φ_s and Φ_g in the present paper. We also note that Ψ_I in this table does not refers to the reorientation time scale defined in the present paper.

Referee's Comment #2

The final microswimmer equations used in the simulations (Eq 5 — Eq 8) are dubious for the reasons outlined above. However, taken them as a given, the results are not particularly novel because these microswimmer equations simplify to those investigated before (spherical gyrotactic settling microswimmers) and thus the results are not particularly novel.

Author's Reply:

You pointed out that our model simplifies to the classic Kessler's gyrotaxis model. However, despite the similarities in mathematical forms, the mechanism responsible for the orientation effect is totally different from that in Kessler's model. The fluid inertial torque makes a swimmer to swim in upward direction because the symmetry of flow ambient field is broken by the swimming behavior. In Kessler's model, a swimmer swim upwards under a gravity torque due to the offset between the centers of mass and the hydrodynamic force. The difference in the mechanisms can be seen in the definition of the reorientation time scale Ψ. In our model, Ψ_I depends on both swimming and settling speeds, while in Kessler's model, Ψ depends on the size of swimmer and the distance of the offset mentioned above.

We believe this difference is important and deserves investigation. First, it allows a plankton to modify its swimming speed and then control its reorientation behavior. Second, our results suggest that the settling speed also matters when we consider a motile plankton, because it generates a reorientation effect under fluid inertial. To address both points, we studied how clustering and preferential sampling are altered when micro-swimmers have different swimming and settling speeds.

This paper aims to understand the influence of fluid inertial torque in the reorientation phenomenon of micro-swimmers. Therefore, we isolate the term for fluid inertial torque in Eq. (4), which simplifies the discussion. To model the motion of an actual

plankton precisely, one must also consider other relevant mechanisms such as bottom-heaviness or phototaxis. However, they are not in the scope of this paper.

We have revised section 2.1 to address the difference between our model and Kessler's model.

Around line 90:

"This is analogous to the gyrotactic effect induced by bottom-heaviness, which is typically expressed as $2\Psi^{-1}(e_g \times n)$ (Kessler, 1986). We note that, however, they are two different mechanisms. The torque generated by bottom-heaviness depends on the distance of the offset between the center of gravity and hydrodynamic forces on a cell, which is usually determined by morphology. On the contrary, fluid inertial torque depends on the swimming and settling speeds and, determined by motility"

Referee's Comment #3
Additionally, the authors apply their model to extremely high swimming speeds and high settling speeds (L91) without any comment on whether this range of values are consistent with their assumptions. (I suspect they are not).

Author's Reply:
As shown in the Table above, large Φ_s and Φ_g can be reached by zooplankton species in weak turbulence, while Re and St are still within the range of our model assumptions. For instance, the nauplius of *Eurytemora affinis* has Φ_s = 9.1, Φ_g = 1.0, Re = 0.3 and St = $3*10^{-5}$.

To address our assumption about the range of dimensionless parameters, we added some sentences to discuss the typical parameters of marine planktons.

Around Line 105:

"Large Φ_s and Φ_g are reached by swimmers with strong motility in weak turbulence which u_η is small. In such case, the assumptions of our model are still justified. First, Re can be still small even for plankton that swim fast as long as their size is sufficiently small. Second, St is independent of plankton's motility, which has been shown to be negligibly small for typical turbulence conditions in the ocean (Qiu et al., 2022a)"

Referee's Comment #3
Minor comments:
L16, 'accumulates' — check grammar
L17, 'clustering' — check grammar
L22, 'the inverse of a timescale B' — unclear writing. It needs to be explained here what B is (timescale for reorientation against gravity in an otherwise quiescent environment).
Eq. 4 — v_s\prime should be \Phi_s

Author's Reply:
We thank you for your careful review. We have addressed these points in the revised manuscript.

Referee's Comment #4

L146, 'variance of Voronoi volumes' — I'm not sure whether the authors mean the location of the peak of the distribution or indeed the variance.

Author's Reply:

The term 'variance of Voronoi volumes' means the actual variance of the Voronoi volumes of each swimmer. To avoid ambiguity, we explain the term when it appears in the text first time.

Around line 160:

"We calculate the Voronoi volume of each swimmer, and obtain the variance of volume distribution normalized by the mean volume of each swimmer, $\sigma_{\hat{V}}^2 = E\left(\frac{V}{<V>} - 1\right)^2$.