Modeling saline fluid flow through subglacial ~~ice-walled~~ channels~~and the impact of density on fluid flux~~

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Abstract. Subglacial hydrological systems have impacts on ice dynamics, as well as, nutrient and sediment transport. There has been extensive effort to understand the dynamics of subglacial drainage through numerical modeling. These models, however, have focused on ~~freshwater in warm ice and neglected the consideration of~~ fresh water in temperate ice and not considered variable fluid chemistry such as salts. Saline fluid can exist in ~~cold-based~~ cold glacier systems where ~~freshwater~~ fresh water cannot and understanding the routing of saline fluid is important for understanding geochemical and microbiological processes in these saline cryospheric habitats. A better characterization of such terrestrial environments may provide insight ~~to~~ into potentially analogous systems on other planetary bodies. We present a model of channelized drainage from a hypersaline subglacial lake and highlight the impact of salinity on melt rates in an ice-walled channel. The model results show that given a subglacial system at the salinity-dependent melting point, channel walls grow more ~~quickly~~ slowly when fluid contains higher salt concentrations which lead to ~~higher~~ lower discharge rates. We show this is due to a ~~higher density fluid moving through a gravitational potential which generates more energy for melting~~ feedback between melting of the channel walls and the energy needed to warm the fluid to the new melting point as the brine is diluted. This model provides a framework to assess the impact of fluid chemistry and properties on the spatial and temporal variation of fluid flux.

1 Introduction

Subglacial hydrology is of fundamental importance to the dynamics and evolution of ice masses (Flowers, 2018; Morlighem et al., 2014). The presence, distribution, and geometry of the subglacial water system have direct effects on rates of ice sliding, ice mass flux, erosion, and deposition (e.g. Russell et al., 2006; Bell et al., 2007; Stearns et al., 2008; Siegfried et al., 2016; Larsen and Lamb, 2016; Seroussi et al., 2017; Carrivick and Tweed, 2019; Keisling et al., 2020). The subglacial hydrological system affects the distribution and character of subglacial biological communities and influences water and nutrient flux into surrounding water bodies (e.g. Neal, 2007; Kjeldsen et al., 2014; Meerhoff et al., 2019; Mikucki et al., 2004; Vick-Majors et al., 2020). Whether the goal is to project future sea level rise, understand glacier bed forms, model ocean circulation, or to investigate potential extra-planetary ~~life~~ habitats through Earth analogs, we require an understanding of the distribution and dynamics of subglacial hydrological systems (e.g. Nienow et al., 2017; Forte et al., 2016).

Subglacial lakes have been observed to drain episodically through outburst floods and less catastrophically through longer-lived drainage events. There is a significant body of work on modeling the drainage of glacial lakes (e.g. Rothlisberger, 1972; Nye, 1976; Spring and Hutter, 1981; Fowler, 1999; Clarke, 2003; Evatt et al., 2006; Kingslake, 2015; Schoof, 2020; Jenson et al., 2022). Many subglacial hydrology models have assumed that drainage from a subglacial lake occurs through ice-walled channels at the ice-bed interface (e.g. Nye, 1976; Fowler, 1999; Clarke, 2003; Evatt et al., 2006). ~~This~~ Collectively, this work has focused on ~~freshwater~~ fresh water at the pressure melting point and neglected the consideration of water chemistry.

The chemistry of the subglacial water influences the character of the hydrological system. Depression of the pressure melting point through increased solute concentration is one potential mechanism to explain the presence of subglacial water in locations with a subglacial temperature below, sometimes significantly below, the pressure melting point (Mikucki et al., 2015). Locations with observable saline discharge occur in both Antarctic and Arctic settings such as Blood Falls, Taylor Glacier, East Antarctica and Borup Fiord Pass Glacier, Ellesmere Island Canada (Trivedi et al., 2018; Lyons et al., 2019). The salinity of the englacial brine feeding Blood Falls is approximately 125 psu but the precise geometry of the subglacial brine system beneath Taylor Glacier is not fully understood (Badgeley et al., 2017; Lyons et al., 2019). Hubbard et al. (2004) inferred that a zone 3-6 km upglacier from the terminus contained saturated sediments or ponded water, based on radar data, and widespread hypersaline groundwater has been detected as far as 5.7 km upglacier from the terminus using transient electromagnetic techniques (Mikucki et al., 2015). Hypersaline lakes have been inferred to exist beneath the Devon Ice Cap, Canadian high Arctic from airborne radio echo sounding data, with predicted salinity in the range of 140 to 160 psu ~~(Rutishauser et al., 2018)~~ (Rutishauser et al., 2018, 2022). However, the effects of increased salinity on the geometry and flow in subglacial hydrological systems remains unknown.

Subglacial water ~~chemistry~~ composition is expected to impact the geometry and dynamics of the subglacial hydrological system. For instance, the hydraulic potential field is modified through the density of the fluid; saline fluid can have a significantly different flow path than ~~freshwater~~ fresh water for the same glacier geometry (Badgeley et al., 2017; Rutishauser et al., 2022). The size and edge dynamics for a channel are also expected to differ as the result of fluid chemistry.

An understanding of the impact fluid chemistry has on subglacial systems is important for mapping and classifying subglacial hydrological features using radar. The size, continuity, and electrical conductivity of subglacial channels determines the detectability of subglacial features by radar remote sensing. Constraints provided by modelling inform radar system design decisions such as power requirements, center frequency, and antenna geometry ~~(?)~~(Scanlan et al., 2022). The ongoing development of multi-polarization radar system and radar processing algorithms increases the detectability of variations in subglacial hydrological organization (Scanlan et al., 2022, 2020). The expected geometry of subglacial features is an important specification for the design of radar ~~system~~systems (Scanlan et al., 2022). The response in the geometry of subglacial features to changes in the discharge, position along ~~glacier flow~~a flowline, and aqueous chemistry provides constraints for the technological and scientific development of new radar systems.

Basal thermal regimes have been shown to impact the solutes, nutrients, and microbes found in the subglacial systems (Dubnick et al., 2020) and subglacial fluid flow can transport these materials leading to a change in the geomicrobiology of local and nearby environments (Mikucki et al., 2004). We hypothesize that both the basal thermal regime and solute concentrations influence the subglacial hydrological system by altering the effective pressure and fluid flux (which in turn will influence geomicrobiology). A better understanding of the flow dynamics in cold ice is important for characterizing the distinct biogeochemistry in saline subglacial systems.

By modeling saline fluid flow through cold ice, we seek to address the following questions: how significant is the effect of salinity on channel wall melt rates?; how does the salt concentration change along the channel in response to the melting of the channel walls?; and in what systems is the consideration of fluid chemistry and fluid properties important for understanding subglacial hydrology~~.~~ ? To answer these questions, we mathematically investigate channel evolution ~~, effective pressure,~~ and discharge over time in response to variable fluid chemistry. The results show that the radius of an evolved channel is ~~larger~~ smaller for saline fluid than the fresh water equivalent when both glacier-lake systems are at their respective salinity and pressure-dependent melting points. The ~~larger~~ smaller channel cross-sections affect the temporal and spatial evolution of fluid flux for saline fluid. ~~This effect is of leading order for gravity driven fluid flow, due to the energy generated when a higher density fluid moves through a gravitational potential. We find that the same result occurs for high density freshwater such as water carrying high loads of suspended sediment.~~ The differences related to fluid chemistry are greatest for high discharge rates which are generated by high volume lakes and channels with steep bed slopes and circular geometry. Additionally, we find that fluid density has a substantial influence on discharge rates which has relevance to suspended sediments and outburst floods.

2 Model description

We construct a lake-drainage model in which the water flows from a subglacial lake through an R-channel (Rothlisberger, 1972; Nye, 1976). In contrast with previous approaches, we allow for varying salt concentrations in fluid flowing from a subglacial lake. We follow the implementation of Fowler (1999) and Kingslake and Ng (2013). In particular, the equations describing channel evolution and the conservation of mass, momentum, and energy are identical to those in Fowler (1999) with the differences being (i) the fluid density and the melting point of the fluid are functions of salinity, (ii) the assumptions around fluid temperature are related to salinity, and (iii) an additional equation is needed to solve for salinity along the channel and in time. The model equations from Fowler (1999) are described with our assumptions and in our notation in Sec. 2.1 and changes to the model equations are described in Sec 2.2. For a list of model variables and parameters along with the consistently used parameter values see Table 1.

2.1 Model equations

We ~~construct a lake-drainage model in which the water flows from a subglacial lake through an R-channel (Rothlisberger, 1972; Nye, 1976). We follow the implementation and notation of Fowler (1999) and Kingslake (2015). In our model, we~~ assume a subglacial conduit on an inclined bed slope beneath ice of constant thickness (Fig. 1). ~~In contrast with these previous approaches, we allow for varying salt concentrations in fluid flowing from a subglacial lake. The ice and fluid are assumed to be at the salinity-~~

Table 1. List of model parameters and variables. Values of constants are specified in brackets.

Variable	Description
ρ_i, ρ_w, ρ_b	densities of ice [917 kg m^{-3}], water [997 kg m^{-3}], and brine
g	gravitational acceleration [9.81 m s^{-2}]
$\mathcal{L}$, σ_i	latent heat of fusion [3.34 x 10^5 J kg^{-1}] and specific heat capacity for ice [2093 J kg^{-1} C^{-1}]
A, n	ice flow law parameter and exponent [3]
K	ice flow parameter for conduit closure
f, $\mathcal{R}$, n_i, n_b	friction factor, hydraulic roughness, and roughness of ice [0.6 m$^{-1/3}$ s] and bed material [0.16 m$^{-1/3}$ s]
x, s	horizontal and bed-parallel spatial coordinates
B, L	bed slope~~3° C~~, channel length
Q, m	discharge along the conduit and melt rate
S, r	cross-sectional area and channel radius
P_i, ψ	ice overburden pressure and basic hydraulic gradient
N	effective pressure
h, h_i	lake depth and initial lake depth
H	ice thickness from surface to bed adjacent lake
V, V_i	volume of lake and initial volume of lake
θ_i, θ_w, θ_b	temperatures of basal ice, water, and brine
$\hat{\theta}_w$, $\hat{\theta}_b$	~~freezing~~ melting point of water and brine at pressure
$\hat{\theta}$	salinity- and pressure-dependent melting point of ice
β, $\hat{\beta}$	salt concentration in psu and kg m^{-3}

~~and pressure-dependent melting point of the fluid. For a list of model variables and parameters along with the consistently used parameter values see Table 1.~~

The negative basic hydraulic gradient is the sum of the glacier geometry related terms,

$$\psi = \rho_b g \sin B - \frac{\partial P_i}{\partial s}, \tag{1}$$

where B is the conduit slope (assumed to be constant along the channel and the same as the bed slope), s is the along-flow coordinate parallel to the bed, and g is gravitational acceleration ~~(Fowler, 1999)~~(Kingslake and Ng, 2013, Eq. 5). The ice-overburden pressure P_i in [Pa] is given by $P_i = \rho_i g H$ where H is the glacier thickness. Since we assume the ice thickness is constant, the change in ice-overburden pressure along the channel is zero and $\psi = \rho_b g \sin B$. ~~The total negative potential gradient is~~

$$G = \psi + \frac{\partial N}{\partial s},$$

~~where N is the effective pressure which is the difference between the ice-overburden pressure and the water pressure in the channel.~~

We assume a channelalready exists and Assuming an existing channel, the channel walls open due to melt and close due to creep closure. Together these govern the rate of change of the conduit cross-sectional area S with respect to time t,

$$\frac{\partial S}{\partial t} = \frac{m}{\rho_i} - KSN^n, \tag{2}$$

where m is the melt rate in [kg m^{-1} s^{-1}] and (Fowler, 1999, Eq. 2.1). $K = 2A/(n^n)$ is a function of the Glen's flow law parameter A and exponent n (Evatt et al., 2006). The effective pressure N is the difference between the ice-overburden pressure and the water pressure in the channel. We calculate A as a function of ice temperature using the Arrhenius relation and relevant calibrated values (Cuffey and Paterson, 2010, Eqn 3.35).

Mass conservation relates the rate of change of conduit area to the spatial gradient in discharge Q, and the production of water due to melt , and additional water added to the system along the conduit, such that (Fowler, 1999, Eq. 2.2), such that

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s} = \frac{m}{\rho_b}\frac{m}{\rho_w}. \tag{3}$$

We assume turbulent flow and use Manning's formula to empirically relate the total potential hydraulic gradient (negative basic hydraulic gradient and the effective pressure gradient) to friction along the channel. Note that Manning's formula was derived for freshwaterfresh water, but due to a lack of empirical data with saline fluid in ice, we use the Manning's friction factor for freshwater in ice fresh water in ice as used in Fowler (1999). The conservation of momentum equation is then,

$$\psi + \frac{\partial N}{\partial s} = f \rho_b g \frac{Q|Q|}{S^{8/3}}\frac{Q^2}{S^{8/3}}, \tag{4}$$

where f is the friction factor (Fowler, 1999, Eq. 2.3). For a circular ice-walled channel, $f = (4\pi)^{2/3}\mathcal{R}^2$ and $\mathcal{R} = n_i = 0.06$ m$^{-1/3}$ s where $\mathcal{R}$ is the hydraulic roughness and n_i is the roughness of ice Clarke (2003)(Clarke, 2003). For a semi-circular channel at the bed, $f = (2(\pi+2)^2\pi)^{2/3}\mathcal{R}^2$ where $\mathcal{R} = \pi/(2+\pi)n_i + (1-(\pi/(2+\pi))n_b$ and $n_b = 0.16$ m$^{-1/3}$ s is the roughness of the bed material (Fowler, 1999, Eq. 2.24).

As brineflows through the channel, the viscous dissipation of heat causes melting along the channel walls and results in dilution of the brine. The salinity, or salt concentration, of the brine therefore varies The conservation of energy equation is

$$Q\left(\psi + \frac{\partial N}{\partial s}\right) = \rho_b \sigma_i \left(S\frac{\partial \theta_b}{\partial t} + Q\frac{\partial \theta_b}{\partial s}\right) + m\mathcal{L} + m(\theta_b - \hat{\theta}), \tag{5}$$

where θ_b is the temperature of the brine, $\hat{\theta}$ is the melting point of the ice, σ_i is the specific heat capacity of ice, and $\mathcal{L}$ is the latent heat of fusion for ice (Fowler, 1999, Eq. 2.4). Following Rothlisberger (1972) and Nye (1976), we neglect the heat transfer equation which is equivalent to assuming any heat generated from flow is instantaneously transferred to the channel walls. Consequently, we also need to make an assumption around fluid temperature and the melting point which are related to the salinity of the fluid.

2.2 Consideration of brine

As the brine flows from the subglacial lake, any melt occurring at the channel walls will add fresh water and dilute the brine along the channelin time. We derive an equation describing the evolution of the . We use a partial differential equation to describe the concentration of salt $\hat{\beta}$ [kg m^{-3}] at position s and time t in response to changes in channel cross-sectional area and discharge. The fluid is moving along the channel at velocity v which gives the flux of salts per square meter

$$\phi = v\hat{\beta} - D\frac{\partial \hat{\beta}}{\partial s}$$

where D is the diffusion coefficient. The mean velocity of the fluid is given by $v = Q/S$. We calculate a Péclet number of $(Pe) > 10^8$ which suggests advection dominates diffusion in fluid flow and assume diffusion is negligible. With this assumption, the flux of salt moving through a channel cross-section with area S is

$$\phi = \frac{Q\hat{\beta}}{S}.$$

Assuming there is no brine added along the channel and there is no accretion on the channel walls, the salt concentration equation is,

$$\frac{\partial}{\partial t}\left(\hat{\beta}S\right) = \frac{\partial}{\partial s}\left(-Q\hat{\beta}\right). \tag{6}$$

In the case of a semi-circular channel, contact with the ground could be a source for salts in the fluid flow. Although we do not include it here, such a mechanism could be accounted for in our model as a source term in Eq. 6.

The salt concentration of $\hat{\beta}$ discussed above is in [kg m^{-3}] in order to be compatible with the model. These values for salinity are converted to a standard unit for measuring salinity [psu] before calculating the density and melting point in Eqs. 7 and 8 respectively using the conversion $\hat{\beta}$ [kg m^{-3}] $= 1000\beta(\rho_b)^{-1}$ [psu]. The salinity in the lake is constant in time since no fluid is being added to the lake which gives the boundary condition $\hat{\beta}(0,t) = 1000\beta(0,t)(\rho_b)^{-1}$ where $\beta(0,t)$ in [psu] is prescribed at the beginning of the simulation. The dilution of the brine along the channel is minimal so at the beginning of the simulation we assume that the salt concentration in the channel is equal to the concentration in the lake, that is $\hat{\beta}(s,0) = \hat{\beta}(0,t)$.

As salt concentration changes, the salt (see details in Sec. 2.2). Due to changing salt concentrations, the density of brine and the melting point of ice also vary spatially and temporally. The density of the brine in [kg m^{-3}] as a function of salt concentration β in practical salinity units [psu] under 1 bar using the FREezing FREeZing CHEMistry (FREZCHEM) model from Wolfenbarger et al. (2022) results in

$$\rho_b = 9.98\text{x}10^{-7}1000 + 0.763\beta^{3} + 5.53\text{x}10^{-5}\beta^2 + 7.639.98\text{x}10^{-1-7}\beta + 1.00\text{x}10^{33}. \tag{7}$$

Note we do not account for changes in density due to pressure or temperature. Using the same FREZCHEM model, we calculate the melting point of ice due to salinity and adjust for the ice-overburden pressure(Chang et al., 2022)water pressure. The melting point in [°C] of ice in contact with saline fluid at pressure P_i is P_w is

$$\hat{\theta} = -5.81\text{x}10^{-7} - c_b\beta^3 + 1.24\text{x}10^{-6} - c_n P_w = -c_b\beta^2 - 6.05\text{x}10^{-2}\beta - 7.45\text{x}10^{-8} - c_n(P_i. - N) \tag{8}$$

where $c_b = 6.05\text{x}10^{-2}$ °C psu^{-1} and $c_n = 7.45\text{x}10^{-8}$ °C Pa^{-1}. We assume the lake and surrounding ice system is in thermal equilibrium which requires that at the lake, the ice and brine temperatures, θ_i and θ_b respectively, are equal and at the salinity and pressure-dependent melting point $\hat{\theta}$. For a given salt concentration in the lake, we calculate the melting point at the lake and set the ice and brine temperatures equal to that temperature at the lake. We assume the ice and brine temperatures remain temperature remains constant in time and along the channel; this is realistic for most freshwater systems (Clarke, 2003) . However, the melting point only remains constant at the lake and evolves .

We assume the brine temperature and the melting point are equal (Rothlisberger, 1972; Werder et al., 2013) and evolve in response to the changes in salinity along the channel and in time. With these assumptions, the conservation of energy equation is

$$Q\left(\psi + \frac{\partial N}{\partial s}\right) = m\mathcal{L} + m\sigma_i(\hat{\theta} - \theta_i),$$

Given that expected changes in the brine temperature are small and we are modeling slow drainage events, we make the assumption that the brine temperature is in quasi-steady state ($\frac{\partial \theta_b}{\partial t} = 0$). Using the assumption that fluid temperature is equal to the melting point ($\theta_b = \hat{\theta}$) along with Eq. (8) gives the simplified conservation of energy,

$$Q\left(\psi + \frac{\partial N}{\partial s}\right) = \rho_b \sigma_i Q\left(-c_b \frac{\partial \beta}{\partial s} + c_n \frac{\partial N}{\partial s}\right) + mL. \qquad (9)$$

where σ_i is the specific heat capacity of ice and L is the latent heat of fusion for ice. The term on the left hand side of Eq. 5 is the total work done which must be balanced by the sum of the energy lost to melting due to latent heat and lost to raising the ice (i) needed to raise the brine temperature to the changing melting point . Therefore the amount of energy available for melting the channel walls is a function of salinity. Following Rothlisberger (1972) and Nye (1976), we neglect the heat transfer equation which is equivalent to assuming any heat generated from flow is instantaneously transferred to the channel wallsnew salinity and pressure-dependent melting point and (ii) lost to melting due to latent heat.

We assume a circular channel for most simulations, but we do compare the effect of circular vs semi-circular channels in Sec. 3. The main differences between these assumptions are that in the semi-circular case (1) the fluid is flowing along the bed and therefore the roughness of the bed must be accounted for instead of the roughness of ice and (2) the substrate may contain some salts. We do not account for (2) in our model. We do account for (1) through the friction factor which appears in the conservation of energy equation Eq. 4 and changes depending on the channel geometry and roughness of the channel walls or the bed.

2.3 Consideration of brine

As the brine flows from the subglacial lake, it will be diluted as meltwater is added along the channel length. To account for this, we have developed a partial differential equation describing the concentration of salt $\hat{\beta}$ kg m^{-3}at position s and time t. The fluid is moving along the channel at velocity v which gives the flux of salts

$$\phi = v\hat{\beta} - D\frac{\partial \hat{\beta}}{\partial s}$$

where D is the diffusion coefficient. The mean velocity of the fluid is given by $v = Q/S$. We calculate a Péclet number of $(Pe) > 10^8$ which suggests advection dominates diffusion in fluid flow and assume diffusion is negligible. With this assumption, the flux equation becomes

$$\phi = \frac{Q\hat{\beta}}{S}.$$

Assuming there is no brine added along the channel and there is no accretion on the channel walls, the salt concentration equation is,

$$\frac{\partial}{\partial t}\left(\hat{\beta} S\right) = \frac{\partial}{\partial s}\left(-Q\hat{\beta}\right).$$

In the case of a semi-circular channel, contact with the ground could be a source for salts in the fluid flow. Although we do not include it here, such a mechanism could be accounted for in our model as a source term in Eq. 6.

The salt concentration $\hat{\beta}$ discussed above is in kg m^{-3}in order to be compatible with the model. These values for salinity are converted to a standard unit for measuring salinity psubefore calculating the density and melting point in Eqs. 7 and 8 respectively using the conversion $\hat{\beta}$ kg m^{-3}] $= 1000\beta(\rho_b)^{-1}$ psu. The salinity in the lake is constant in time since no fluid is being added to the lake which gives the boundary condition $\hat{\beta}(0,t) = 1000\beta(0,t)(\rho_b)^{-1}$ where $\beta(0,t)$ in psuis prescribed at the beginning of the simulation. When the cross-sectional area of the channel is small, there is less melting and the dilution of the brine is minimal so at the beginning of the simulation we assume that the salt concentration in channel is equal to the concentration in the lake, that is $\hat{\beta}(s,0) = \hat{\beta}(0,t)$.

2.3 Channel boundary conditions

The only fluid flux from the subglacial lake is the brine flowing out of the channel. Thus the rate of change of lake volume V is given by

$$\frac{dV}{dt} = -Q(0,t). \tag{10}$$

We assume a box-shaped lake which gives the lake hypsometry

$$\frac{h}{h_i} = \frac{V}{V_i} \tag{11}$$

where h is the depth of the lake, h_i is the initial lake depth, and V_i is the initial lake volume. For the treatment of more complicated lake geometries, see Kingslake (2013).

Implicitly differentiating gives

$$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} = \frac{V_i}{h_i}\frac{dh}{dt} \tag{12}$$

and by substitution, the lake depth evolves with time following

$$\frac{dh}{dt} = \frac{h_i}{V_i}(-Q(0,t)). \tag{13}$$

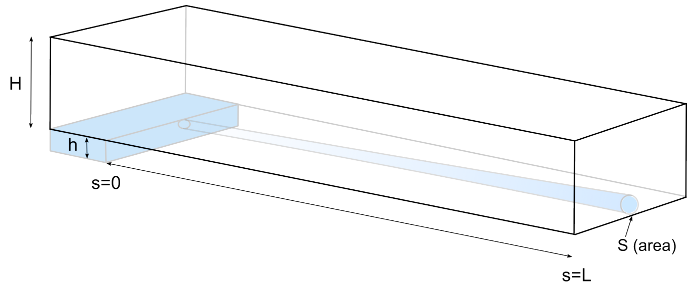

Figure 1. Schematic of simple glacier geometry and subglacial hydrological system with a R-channel draining a subglacial lake.

We assume the lake drains slowly enough that the ice roof drops with the lake depth following Evatt et al. (2006), so as the lake drops the effective pressure at the lake is still the effective pressure is the difference between the ice overburden pressure and the fluid pressure in the lake. The boundary condition where the conduit meets the lake is $N(0,t) = 0$ (Evatt et al., 2006). We impose a Neumann boundary condition at the end of the channel where

$$\frac{\partial}{\partial s} N(s,t)\bigg|_{s=L} = 0. \tag{14}$$

We choose this boundary condition (opposed to $N = 0$) in order to solve the system numerically in a more efficient way (see Appendix A and Sec. ??for details). Neumann boundary conditions on effective pressure at the end of the channel have been used to solve similar systems of equations without an influence on the qualitative results (Kingslake, 2015; Evatt et al., 2006).

2.4 Summary of model equations

The full model contains five unknowns (N, S, m, Q, and β) and five model equations (Eqs. 2, 3, 4, 5, and 6) which are solved simultaneously. The model equations contain the derived variables $\hat{\theta}, \rho_b$, and ψ which depend on salinity. The model equations written in terms of the salinity-dependent derived variables are listed below.

Summary of model equations

Channel evolution: $\frac{\partial S}{\partial t} = \frac{m}{\rho_i} - KSN^3$ Conservation of mass: $\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s} =$ Conservation of momentum: Conservation of ene

Conservation of mass: $\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s} = \frac{m}{\rho_b}$

Conservation of momentum: $\psi + \frac{\partial N}{\partial s} = f \rho_b g \frac{Q^2}{S^{8/3}}$

These five equations are non-dimensionalized and solved numerically as described in Appendix A. The system of equations are is solved using a constant time step of approximately 3 seconds and a constant grid spacing of 20 m . 50 m and a constant time step of less than 1 second for each simulation (smaller time steps are required for higher salinities). Higher salinities require smaller time steps. After the solution to the salt concentration equation is obtained at each time and space step, the melting point $\hat{\theta}$ ~~,~~ and the density ρ_b are updated along with the basic hydraulic gradient ψ, which is a function of density using Eqs. 8, 7, 1 respectively.

3 Results

Idealized model simulations were run to investigate the impact of brine on discharge rates, channel radius, effective pressure, and the duration of lake drainage. The model runs until open channel flow occurs, at which point the model run ends. Unless otherwise specified, the parameters used for the baseline simulations are as follows: ice thickness above channel $H = 100$ m, initial lake volume ~~$V_i = 1$~~ $V_i = 5$ x ~~10^6~~ 10^5 m^3, initial lake depth $h_i = 10$ m, channel length $s = 1000$ m, initial channel radius $r = 0.25$ m, bed and conduit slope $B = 3°$, and circular channel geometry. A range of different values were explored for each parameter listed in Table 2 while holding all other parameters equal to the baseline simulation values.

To investigate the effect of saline fluid, we ran ~~five scenarios with $\beta = \{0, 50, 100, 150, 200$ psu$\}$~~ six scenarios with $\beta = \{0, 5, 10, 25, 50, 1$ to explore the range of possible outcomes. Based on the salinity, we set the ice temperature and initial melting point of the ice and initial brine temperature to $\hat{\theta} = \{-0.06, -0.37, -0.67, -1.58, -3.09, -7.63°\text{C}\}$ respectively. The discharge rates are ~~greater~~ lower for fluid with higher salt concentrations (Fig. 2a). ~~Early in the simulations, the discharges at the lake are nearly equal for all salinities. Later on there is a greater difference in discharge rates.~~ Higher salt concentrations ~~increases~~ decreases the peak velocity reached and ~~decreases~~ increases the amount of time to reach peak velocity (Fig. 2b). The peak velocity and drainage duration change ~~nearly linearly~~ nonlinearly with increased salt concentrations. ~~The channel radius increases slightly along the channel for all scenarios~~ That is, a difference in salinity of 5 psu from fresh water has a substantial influence on fluid velocities, while a difference in salinity of the same amount at higher salinities results in much smaller velocity changes (Fig. 2b). After the lake has drained, the channel radius is larger than the initial channel radius for all salinities tested (Fig. 2c)~~and at~~. At the end of the simulations~~after the lake has drained~~, the channel radius is ~~greater for the higher salt concentrations~~greatest for fresh water, which is more than twice the channel radius of a lake with a salinity of 125 psu.

The ~~difference between the channel salt concentration after the lake has emptied and the initial salt concentrations is small~~ salt concentration decreases along the channel slightly for all scenarios, but largest changes occur for scenarios with lower salinities (Fig. 2d). ~~However greater dilution occurs in the channels with higher salinity and therefore greater fluid flux.~~ Changes in salinity and water pressure result in increases in the melting point along the channel (Fig. 2d). Changes in the melting point along the channel are higher for lower salinities.

We systematically vary parameters to explore the sensitivity of the model and the impact of channel geometry, lake volume, initial channel radius, and bed slope on discharge and the duration of drainage (Fig. 3). The channel geometry (circular vs. semi-circular) changes the time of lake drainage, as well as the peak discharge for ~~freshwater~~ fresh water and for a lake with salinity of ~~100~~10 psu (Fig. 3a). For a semi-circular channel, the difference in time until lake drainage is more significant than the difference in peak discharge between the freshwater and brine scenarios. For the circular channel, the difference in peak discharge is greater than the difference in the duration of time until lake drainage. In both scenarios, the circular channel drains the lake in less than half the time than for a semi-circular channel and the peak discharge is ~~about~~ over twice as high when

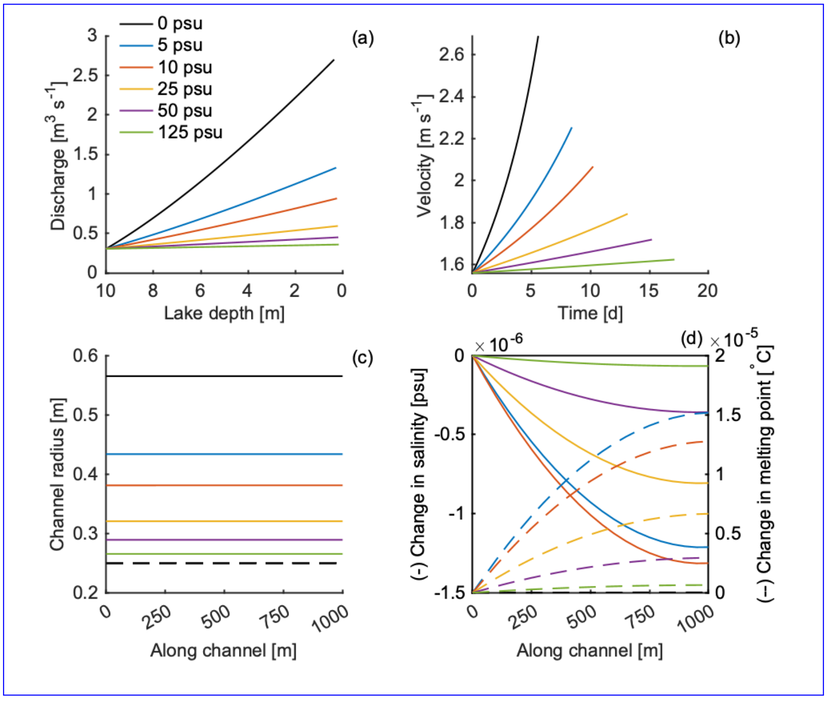

Figure 2. The impact of brine on discharge, velocity, channel radius, and changes in salt concentration ~~,~~ and ~~effective pressure~~ melting points for various initial salt concentrations, shown by the colors in (a). (a) Discharge at the lake outlet as the lake drains. (b) Velocity at the lake outlet over time in days. (c) Channel radius along the length of the channel at the time the lake has emptied. The dashed line is the initial channel radius of 0.25 m. (d) The solid lines ~~are~~ (associated with the left axis~~which is~~) indicate the difference between the ~~final salt concentration after the lake has emptied and the~~ initial salt concentrations shown in the legend of (a) along the channel and the final salt concentration after the lake has emptied. The ~~right axis~~ (dashed lines (right axis) refers to the difference between the initial melting point and the melting points at the end of the simulation along the channel~~and the initial melting point~~, where the change in the melting point is ~~only~~ due to the change in the salinity (shown in the left axis) and changes in water pressure.

the channels are circular ~~. This is because the initial cross sectional area is doubled~~ which is partially due to double initial cross-sectional area for those simulations.

The volume of the lake impacts the discharge and the timing of drainage by extending the amount of time the model is run (Fig. 3b). The discharge curves for all lake volumes follow the same curve until the smaller lakes drain. The lake continues to drain for greater lake volumes~~and~~. For both a freshwater lake and a lake with salinity of 10 psu, the peak discharge ~~increases by two orders of magnitude~~ roughly doubles when comparing $V_i = 1\times10^5$ m^3 ~~and $V_i = 1\times10^7$~~ with $V_i = 5\times10^5$ m^3 and $V_i = 5\times10^5$ m^3 ~~.~~ with $V_i = 1\times10^6$ m^3. Larger lake volumes extend the time until the lake drains which results in non-linear increases in the differences between fresh water and saline fluid. The differences in timing and peak discharge for saline and freshwater lakes are largest for greater lake volumes (Fig. 3b).

Table 2. Prescribed model parameter variables and descriptions with baseline simulation values and ranges explored.

Variable	Description	Baseline	Range
s	length of channel	1000 m	[500 – 5000 m]
B	bed (conduit) slope	3°	[2 – 4°]
r	initial channel radius	0.25 m	[0.1 – 0.5 m]
h	initial lake depth	10 m	[5 – 15 m]
H	ice thickness above bed and channel	100 m	[100 – 1000 m]
V_i	reference volume of lake	~~1~~5 x ~~10^6~~10^5 m^3	[1 x 10^5 – 1 x ~~10^7~~10^6 m^3]
β	salt concentration of brine in lake	~~0, 100~~0, 10 psu	[~~0 – 200~~0 – 125 psu]

~~For smaller channels ($r < 0.25$ m), the~~ The initial channel radius impacts the amount of time until the lake drains ~~but does not change~~ and the peak discharge ~~reached~~ (Fig. 3c). ~~The channels with smaller initial radius take longer to reach peak discharge. For larger~~ For a simulation with salinity of 10 psu and an initial channel radius of 0.2 m, the lake drains substantially slower ($\approx$ 8 days) compared to an initial channel radius ~~($r > 0.25$ m), both timing and peak discharge are impacted by an increase in radius, where larger channels tend to reach higher peak discharges in less time~~of 0.3 m. For increases of 0.1 m in initial channel radius, the peak discharge increases more linearly with saline fluid compared to fresh water. Increasing the bed and channel slope increases the peak discharge and decreases the time to reach that peak (Fig. 3d). We varied fresh water ($\beta = 0$ psu) and brine (~~$\beta = 100$~~$\beta = 10$ psu) along with the channel slope and found that for lower slopes there is a larger difference in timing between brine and ~~freshwater~~ fresh water and for greater slope there is a larger difference in peak discharge between brine and ~~freshwater~~fresh water. Higher bed slopes lead to higher discharge rates more quickly (Fig. 3d). For all choices of parameters, we find that brine decreases the discharge rates and increases the time until lake drainage.

We explored lake depths of 5 – 15 m, ice thicknesses of 100 – 1000 m, and channel lengths of ~~500 – 5000~~100 – 5000 m and found that these parameters do not substantially impact the results for the parameter combinations described ~~. For~~ (data not shown). For all initial conditions on effective pressure, the effective pressure along the channel tends towards zero over time. For a list of parameters and the range of values explored, see Table 2.

As the walls of the channel melt and the brine is diluted, the properties of the brine, such as specific heat capacity and density, ~~change~~are not constant along the channel. We have accounted for these changes in our simulations, but neglecting these changes in brine properties does not have a substantial influence on the results (data not shown).

4 Discussion

The results of this model suggest that the consideration of brine in relevant glacial systems is important for capturing the dynamics of drainage through ice-walled channels: a failure to ~~include brine~~ consider salt even for low salinities leads to substantially different estimates on channel formation and drainage timescales (Fig. 2). The consideration of ~~brine~~ salinity is more important when considering systems with high discharge rates (high lake volume and steep bed slopes~~(~~, see Fig. 3b,d).

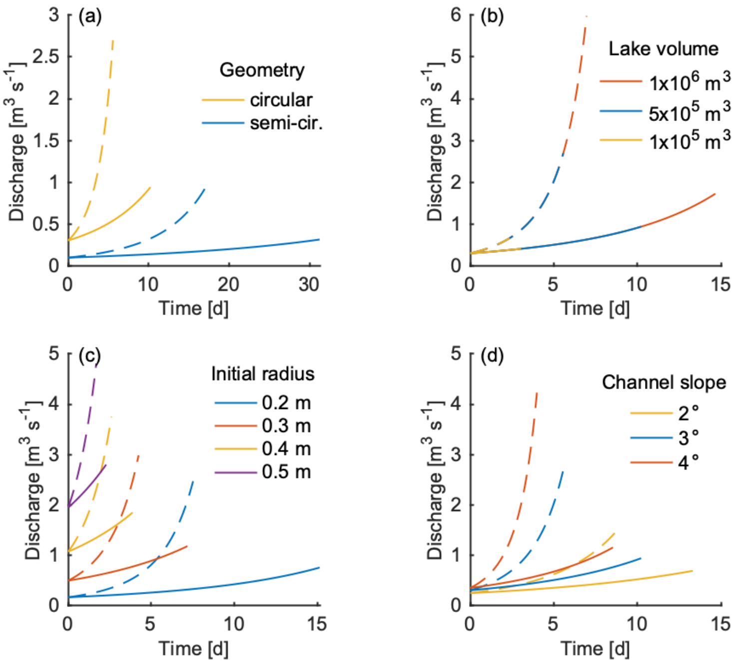

Figure 3. Discharge over time for different channel geometries, lake volumes, initial channel radius, and channel (bed) slope ~~. (a) A channel with semi-circular geometry~~ for ~~a lake~~ lakes with ~~salinity 0 psu~~ fresh water (dashed ~~line~~lines) and ~~100~~10 psu (solid ~~line~~lines). (a) Channels with semi-circular geometries are denoted in blue ~~. A~~ while circular ~~channel geometry for a lake with salinity 0 psu (dashed line) and 100 psu (solid line)~~ geometries are ~~denoted~~ in yellow. (b) Lake volumes for $V_i = 1$ x ~~10^7 m^3 (blue line), $V_i = 5$ x~~ 10^6 m^3 (red line), ~~$V_i = 1$~~$V_i = 5$ x ~~10^6~~10^5 m^3 (~~yellow~~ blue line), and $V_i = 1$ x 10^5 m^3 (~~purple~~ yellow line) are plotted over top of one another~~for a lake with salinity of 100 psu~~. (c) The initial channel radius ~~from a lake with salinity of 100 psu~~ is varied such that the blue line is ~~$r = 0.1$~~$r = 0.2$ m, the red line is ~~$r = 0.2$~~$r = 0.3$ m, the yellow line is ~~$r = 0.3$~~$r = 0.4$ m, ~~purple line is $r = 0.1$ m,~~ and the ~~green~~ purple line is $r = 0.5$ m. (d) Channel slope is varied ~~for~~ to 2° (in yellow), 3° (in blue), and 4° (in red)~~for a lake with salinity of 100 psu (solid) and 0 psu (dashed)~~.

This is particularly important for gravitationally generated flows where the high density significantly increases fluid flow rate. Large circular channels also tend to lead to more dramatic substantial differences in the fluid dynamics between high and low concentrations of salt (Fig. 3a,c).

4.1 Density, a first order effect Effects of salinity

In our model for the parameters we explored, density is the leading order effect of salinity on the fluiddynamics. A fluid with higher salt concentrations has a higher density. As the denser material moves through the gravitational potential, more energy is available for melting . The presence of salt in fluid decreases the melting point and increases the density of the fluid, both of which have implications for how fluid drains through an R-channel from a subglacial lake. An obvious impact is that saline fluid can remain liquid at subzero temperatures and a subglacial lake system can exist below the pressure melting point. The differences related to the salinity-dependent melting point also influence how the channel grows in time. The observed higher discharge for higher salinity systems results from the increased density. In Fig. 4a, fresh water is modeled with varying densities equivalent to the density of brine with salinities $\beta = 0, 50, 100, 150, 200$. These velocity curves are nearly identical to the curves in Fig. 2b.

The presence of salt in the system tends to increase decrease the amount of energy needed to melt available for melting the channel walls because the ice temperature must increase energy is needed to change the brine temperature to the evolving melting pointbefore melting. As the salinity along the channel changes,

For freshwater systems, a positive feedback allows for dynamic channel growth where higher discharge rates generate more energy for melting, opening the channels, and allowing for even higher discharge rates. This feedback is the mechanism responsible for developing an efficient drainage system with large channels and for glacier lake outburst floods. A system with saline fluid at a sub-zero temperature limits this positive feedback. The melting of the channel walls results in an increase in the melting point changes and subsequently the energy needed to melt the channel walls, although this change is minimal as seen in of ice due to the changes in salinity after the addition of fresh meltwater. While melting of the channel wall increases the cross-sectional area and therefore the discharge, more energy is now required to increase the brine temperature to the new melting point and less energy is available for melting.

The effect of inhibited channel growth is largest for highest salinities, where the initial brine temperature is lowest ($-7.63°$ C), but most sensitive for lower salinities, where the initial brine temperature is close to $0°$ C (-0.37 to $-0.67°$ C) (Fig. 2d. The latent heat of fusion is far greater than this term and thus any changes in the melting point has little effect on the total energy c). For lower salt concentrations, there are higher melt rates, which lead to larger changes in the melting point and the brine temperature. Larger changes in brine temperature limits the energy available for future melting, which strongly inhibits rapid channel growth. As salt concentrations increase, less melting occurs which reduces the changes in the melting point and brine temperature (Fig. 2d). Therefore higher salt concentrations lead to more linear increases in velocity over time with smaller rates of change for higher salinities (Fig. 2b).

Density impacts channel growth in the opposite way. A fluid with a higher salt concentration has a higher density. As a denser material moves through a gravitational potential, more energy is generated and available for meltingand therefore fluid velocities. . In Fig. 4a, a temperate freshwater system ($\hat{\theta} = -0.06$°C) is modeled with the density of fresh water and a fluid density of $\rho = 1098$ kg m^{-3}, equivalent to the density of brine with a salinity of 125 psu. The fluid with the higher density results in a higher peak discharge (Fig. 4a).

Modeling all other changes related to salinity (including the treatment of the melting point and fluid temperature) while holding the density of the brine constant and equal to that of fresh water ($\rho = 1000$ kg m^{-3}) results in the velocities discharge rates shown in Fig. 4b. Without the consideration of a change in densityaccurate brine densities, there is almost no change in the velocity curves. difference in the peak discharge even for the highest salinities with the highest densities (Fig. 4b). Higher fluid densities amplify the dynamic feedback leading to channel growth and higher discharge rates (Fig. 4a). However, when the channel growth is limited by the effects of salinity, sub-zero initial brine temperatures, and thus changes in the melting point, this dynamic feedback does not occur and the influence of density is limited. In part, this is due to the fact that density changes almost linearly with salinity (Eq. 7) while the impact of salinity on discharge is non-linear.

It is important to note that the increased density increases melt generates more energy for melting only when the flow itself is gravity driven (i.e., an downward sloped inclined channel). For example, if the flow of a saline fluid is driven uphill by glacial overburden pressure, the fluid velocity will be slower compared to the fluid velocity of fresh water.

Due to the substantial influence of changes in the melting point on energy available for melting, the considerations of thermodynamics and assumptions around the brine temperature have large impacts on the results when modeling saline fluid in channelized systems. Assuming constant brine temperature (opposed to assuming the brine temperature remains equal to the melting point as we do here), results in substantially different results, where the influences of density become the first order effect.

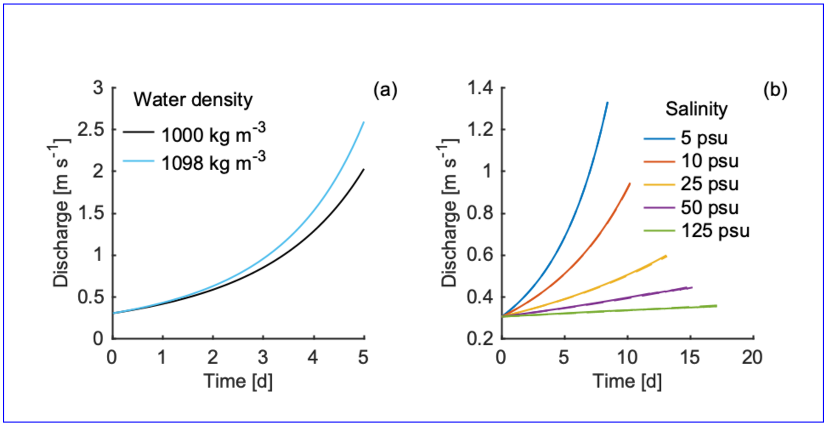

Figure 4. (a) Velocity Discharge curves for scenario drainage from a temperate freshwater system with fresh water at 0° C with densities equivalent to those that of brines fresh water and brine with salinities $\beta = 0, 50, 100, 150, 200$a salinity of 125 psu. (b) Velocity Discharge curves for brine with $\beta = 0, 50, 100, 150, 200$$\beta = 0, 5, 10, 25, 50, 125$ psu, but all with constant density equal to fresh water (1000 kg m^{-3}). Simulations from the full model, where both density and the effects of saline fluid at subzero temperatures are considered, are plotted in the dashed lines which are hardly visible.

4.2 ~~Outburst floods~~ Drainage types of saline systems

There are limited observations of saline outflows and subglacial lake drainage events which makes it difficult to understand the hydrological state of such systems. Each drainage system has a unique combination of (i) source and concentration of salt, (i) brine and ice temperature, (iii) bed topography and glacier geometry determining the hydraulic potential, and (iv) the mechanism initiating drainage which all play a role in the state of the subglacial system.

Our model results show that when the brine temperature remains equal to the melting point, the sub-zero saline fluid limits the positive feedback that leads to the rapid channel growth and increases in fluid velocity. Additionally, our results show that effective pressures along the entire channel tend towards zero over time, that is, the modeled channel is a high water pressure system. Consistent high water pressure and limited channel growth may suggest that saline systems do not tend toward channelization, and may exist as distributed systems. However, further modeling of a distributed system with sub-zero saline fluids would be required to characterize such behavior.

Alternatively, Badgeley et al. (2017) hypothesize that the mechanism of drainage at Blood Falls is not by means of opening and closing of R-channels, but by brine injection into basal crevasses. More observational and modeling work is needed to understand more how saline fluid behaves in a spatially connected subglacial hydrology system.

4.3 Implications for outburst floods

As shown in Sec. 4.1, density can have a significant effect on fluid flow regardless of salinity. Outburst floods often result in disproportionate amounts of suspended ~~sediments~~ sediment which increases the density of the water (Snorrason et al., 2002; Church, 1972). Discharge from outburst floods are typically on the order of $100-1000$ m^3 s^{-1} (Walder and Costa, 1996, Table 1) and can contain suspended sediment concentrations (SSC) of up to 70.7 g L^{-1} (Beecroft, 1983; Old et al., 2005) and in some extreme cases over 400 g L^{-1} (Maizels, 1997). The density of sediments ρ_s depends on the rock type and clast size, but typically range from $2350-2760$ kg m^{-3} (Frederick et al., 2016; Guan et al., 2015; Chikita, 2004). The combined fluid density ρ_c of water with suspended sediments is related to suspended sediment concentration by,

$$\rho_c = \frac{SSC}{\rho_w}\rho_s + \left(1 - \frac{SSC}{\rho_w}\right)\rho_w. \tag{15}$$

~~As shown in Sec. ??, density has a significant effect on fluid flow regardless of salinity.~~ We model an outburst flood from a subglacial lake with ~~freshwater~~ fresh water at 0° C and a volume of $V_i = 1\text{x}10^7$ m^3 with all other parameters equal to those in the baseline simulation (Table 2). We vary the suspended sediment concentrations from ~~$SCC = \{0, 23, 45, 68, 91\}$~~ $SSC = \{0, 23, 45, 68, 91\}$ g L to arrive at fluid densities of ~~$\rho = \{1000, 1040, 1080, 1120\}$~~ $\rho = \{1000, 1040, 1080, 1120, 1160\}$ kg m^{-3} to simulate different suspended sediment loading. ~~As can be seen in Fig. 5, there~~ There is a significant difference between the peak discharge of floods with a lower density fluid ($Q \approx 115$ m^3 s^{-1}) than with a higher density fluid ($Q \approx 140$ m^3 s^{-1}) as well as the timing ~~.~~ of the peak discharge (Fig. 5). Neglecting to account for sediment loading and accurate water densities could lead to inaccurate results when modeling outburst floods where ~~fluid density drives fluid flow.~~ gravity assists in driving fluid flow. The influence of density is particularly important for systems with high gravitational potential energy and high fluid density which together can significantly increases fluid flow rate.

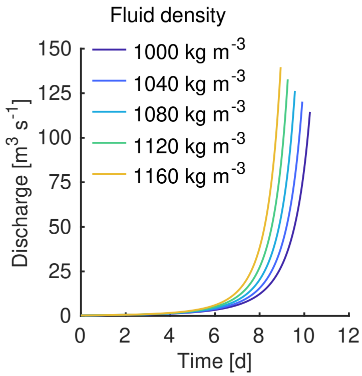

Figure 5. Outburst flood hydrographs for water with different suspended sediment concentrations and therefore varying fluid densities.

4.4 A different formulation of the energy equation

Realistically, not all energy generated from the viscous dissipation of heat will go into melting as is assumed in the model. Some energy will be used to increase the brine temperature. To address this and evaluate the influence of warming brine temperatures on fluid flow, we assume here that the temperature of the brine remains equal to the melting point of the ice ($\theta_b = \hat{\theta}$) which will increase over time as the brine is diluted. We add a term to the conservation of energy to account for the energy needed to warm the mass of fluid per unit length by the change in the brine temperature over time $\frac{\partial \theta_b}{\partial t} = \frac{\partial \hat{\theta}}{\partial t}$ (since $\theta_b = \hat{\theta}$). The updated conservation equation becomes,

$$m\mathcal{L} + m\sigma_i(\hat{\theta} - \theta_i) + \sigma_b \frac{\partial \hat{\theta}}{\partial t} \rho_b S = Q\left(\psi + \frac{\partial N}{\partial s}\right).$$

The dilution of the brine is minimal (less than 1% for all simulations, see Fig. 2e) and therefore the change in the melting point and brine temperature over time $\frac{\partial \hat{\theta}}{\partial t}$ is small. The energy needed to change the bulk fluid temperature is also minimal, especially in comparison to the latent heat of fusion $\mathcal{L}$. Considering this term in the energy balance does not significantly influence the results. The difference in discharge between the simulations with and without this additional energy term is on the order of 10^{-3} m^3 s^{-1} and therefore we claim neglecting this term is justified.

4.4 Effective pressures and channel closure

In solving the system of equations, we follow a similar method to that used in Kingslake (2015) and assume that the discharge is constant along the channel (Eq. A9). This is equivalent to assuming the water generated by melting of the channel walls is negligible compared to the flux of fluid from the lake. Additionally, we force the effective pressure gradient to be zero at the end of the channel (Eq. 14), and arrive at an expression for $Q(t)$ that is only a function of the cross-sectional area $S(1,t)$ and the basic hydraulic gradient $\psi(1,t)$ at the end of the channel over time. As a result, the dimensionless effective pressure gradient can be written as a function of the cross-sectional area and the basic hydraulic gradient both at the end of the channel and along the channel

$$\frac{\partial N}{\partial s} = \frac{1}{\delta}\left(\left(\frac{S(1,t)}{S(s,t)}\right)^{8/3}\psi(1,t) - \psi(s,t)\right).$$

The cross-sectional area is always smallest at the end of the channel because this is where the melting point is the highest and there is less energy available for melting. This implies that

$$\left(\frac{S(1,t)}{S(s,t)}\right)^{8/3} \leq 1$$

for all (s,t). Recall the basic hydraulic gradient is a function of fluid density which decreases (very slightly) along the channel as a result of the brine dilution. So the basic hydraulic gradient reaches a minimum at the end of the channel. This causes the effective pressure gradient to be negative (very slightly) for simulations with saline fluid . The effective pressure for all simulations are extremely close to zero ($-200 < N \leq 0$ Pa) and therefore we claim that the sign of the effective pressure is negligible and does not qualitatively affect our results. Physically, this suggests that there is high fluid pressure throughout the channel and there is no channel closure due to ice-overburden pressure. In systems where ice thicknesses are high, this may not be realistic.

5 Conclusions

We have presented a subglacial hydrology model which includes the consideration of saline fluid. Salt allows fluid to exist below the freezing point of freshwater fresh water and increases the density of the fluid. We show that if a channel exists, hypersaline fluid can flow through an ice-walled channel when the brine and ice are at the salinity and pressure-dependent melting point. Our results suggest that a higher salt concentration increases and commensurate lower brine temperature decreases the peak discharge and decreases the duration of increases the time for a fixed volume to drain from a subglacial lake.

The main driver of the increased discharge rates as a function of salinity is the higher fluid density associated with higher salt concentrations. More energy is generated and available for melting decreased discharge rates for higher salinities is the influence of changes in the salinity-dependent melting point. As the salinity along the channel changes due to melting of the channel walls, the melting point changes and consequently the energy needed to melt the channel walls. While more energy is generated when a higher density fluid moves through a gravitational potential. While some of this energy is used to warm the ice to , this increase in energy for higher salinities is minimal compared to the new melting point down the channel as the brine is diluted by meltwater, the melt is minimal compared to the discharge from the lake and therefore does not impact the discharge rates. Aside from the influence of salinity on the depression of the melting point, the greatest difference on fluid flux when considering saline fluids is related to the change in densityenergy needed to raise the brine temperature to the melting point.

This study shows that accounting for fluid ~~chemistry~~ properties is crucial for accurately modeling subglacial hydrology in relevant systems (i.e., ~~high density fluid, high lake volumes, steep bed slopes~~saline fluid, more or less dense fluid). For a lake with a salinity of ~~150~~125 psu, which is ~~close to the measured values~~ approximately the measured value at Blood Falls~~(125 psu) and the inferred values at the Devon Ice Cap (140 – 160 psu)~~, and an initial brine temperature of $-7.63°$ C, the peak ~~discharge reached is 14% greater~~ velocity reached is 41% lower and the lake drains ~~10% faster~~ 9% slower than for a ~~lake with freshwater~~ freshwater lake. The duration of drainage is most sensitive to initial channel radius and channel geometry while peak discharge is most sensitive to lake volume, channel slope, and channel geometry. We explored the influence of varying fluid density related to suspended sediment loads on outburst floods and found that peak discharge is significantly higher for a high density fluid (21% higher than pure water when the fluid density is 1160 $m^3\ s^{-1}$). ~~There may be implications for how fluid moves from high pressure, low fluid density systems to high density, low pressure systems and how these interactions change fluid dynamics~~In our model, subzero saline fluid causes high pressure channels with limited channel growth which may have implications for the type of subglacial drainage system (channelized vs distributed) that forms with such fluids.

We make a number of simplifying assumptions in the model and use arbitrary parameters for ice thickness, lake volume, channel length, bed slope, and initial channel radius due to a lack of available data on subglacial hypersaline systems. We also impose brine temperatures that remain equal to the ice melting point. In this light, the model presented here should be viewed as an initial exploration of the impact of brine on the dynamics of a subglacial hydrological system. ~~The model can be used to explore the impact of other solutes on drainage.~~ Additional modeling efforts are needed to provide a thorough sensitivity and stability analysis. Further research is required to understand the initiation of drainage in cold saline environments and the influence of fluid density on drainage networks and outburst floods.

Code availability. MATLAB script files for full model are available at https://doi.org/10.5281/zenodo.10775488 (Jenson et al., 2024).

Appendix A: Numerics

After non-dimensionalization, the model equations to be solved numerically are the following.

Dimensionless model equations

Channel Evolution: $\frac{\partial S}{\partial t}\frac{\partial S}{\partial t} = Q\left(\psi + \gamma_1 \frac{\partial N}{\partial s} + \gamma_2 \frac{\partial \beta}{\partial s}\right) - SN^3,$ (A1)

Conservation of Mass: $\frac{\partial Q}{\partial s} = \epsilon (r-1) + \epsilon S N^3 \left(rm - \frac{\partial S}{\partial t}\right),$ (A2)

Conservation of Momentum: $\frac{\partial N}{\partial s} = \frac{1}{\delta}\left(\frac{Q^2}{S^{8/3}} - \psi\right),$ (A3)

Salt Concentration: $\lambda \frac{\partial}{\partial t}\left(\hat{\beta} S\right) = \frac{\partial}{\partial s}\left(-\hat{\beta} Q\right),$ (A4)

Boundary Conditions: $N(0,t) = 0,$ (A5)

$$\frac{\partial}{\partial s} N(s,t)\bigg|_{s=1} = 0, \quad \text{(A6)}$$

$$\hat{\beta}(0,t) = \hat{\beta}(0,0) \quad \text{(A7)}$$

$$\frac{dh}{dt} = \zeta Q(0,t). \quad \text{(A8)}$$

Model and Scaling Parameters:

$$N_0 = (K t_0)^{-1/3}, \quad m_0 = \frac{Q_0 \psi_0}{\mathcal{L}}, \quad S_0 = \left(\frac{f \rho_b g Q_0^2}{\psi_0}\right)^{3/8}, \quad t_0 = \frac{\rho_i S_0}{m_0},$$

$$r = \frac{\rho_i}{\rho_b}, \quad \epsilon = \frac{s_0 m_0}{Q_0 \rho_i}, \quad \delta = \frac{N_0}{s_0 \psi_0}, \quad \gamma = \frac{\theta_0 \sigma_i}{\mathcal{L}}, \quad \lambda = \frac{S_0 s_0}{t_0 Q_0}, \quad \zeta = \frac{t_0 h_i Q_0}{p V_i h_0}$$

We use the subscripts $j = 1, 2, \ldots n$ to denote the grid points along the channel which are separated by Δs and the superscripts $i = 1, 2, \ldots m$ denote time steps separated by Δt. Note, the conservation of energy equation has been substituted into the channel evolution equation to give Eq. (A1).

To solve Eq. (A8), we follow Kingslake (2013) in using the Forward Euler Method to evolve the lake depth forward in time.

$$h^{i+1} = h^i + \Delta t \zeta Q_1^i.$$

Similarly, we solve Eq. (A1) using the same method. The channel cross-sectional area S is moved forward in time at all grid points by

$$S_j^{i+1} = S_j^i + \Delta t \frac{(Q_j^i)^3}{(S_j^i)^{8/3}} \frac{1}{1 + \gamma(\hat{\theta}_j^i - \theta_i)} \left(Q_j^i \left(\psi + \gamma_1 \frac{N_j^i - N_{j-1}^i}{\Delta s} + \gamma_2 \frac{\beta_j^i - \beta_{j-1}^i}{\Delta s} \right) - S_j^i (N_j^i)^3 \right)$$

for $j = 1, 2, \ldots n$.

To evolve these two equations forward in time, the discharge and effective pressure at time step i is needed. These variables can be found simultaneously solving the mass and momentum equations.

We follow Fowler (1999) and Kingslake (2013) in assuming ϵ is small enough to neglect the terms containing ϵ in Eq. (A2). With parameter values, m_0, $s_0 = 1000$ m, and $\rho_i = 917$ kg m^{-3}, ϵ is on the order of 10^{-3} and thus we neglect these terms which simplifies Eq. (A2) to

$$\frac{\partial Q}{\partial s} = 0. \tag{A9}$$

This is equivalent to assuming that any melt generated along the channel is small in comparison to the volume of fluid moving through the channel from the lake. Solving Eq. (A3) for the discharge Q and evaluating at the end of the channel gives

$$Q^2(1,t) = \left(\delta \frac{\partial N}{\partial s}(1,t) + \psi(1,t)\right) S(1,t)^{8/3}. \tag{A10}$$

From Eq. (A6) and the assumption that the discharge is always positive (flowing out of the lake),

$$Q(1,t) = \sqrt{S(1,t)^{8/3}\psi(1,t)}. \tag{A11}$$

From Eqs. (A11) and (A9), $Q(s,t) = Q(1,t)$ and we arrive at the following equation for discharge as a function of time

$$Q(t) = \sqrt{S(1,t)^{8/3}\psi(1,t))}.$$

We solve this equation by calculating the discharge profile at each grid point by

$$Q_j^i = \sqrt{(S_n^i)^{8/3}\psi_n^i}. \tag{A12}$$

To calculate the effective pressure along the channel, we start with the boundary condition at the lake given in Eq. (A5) and use Eq. (A3) to iterate

$$N_{j+1}^i = N_j^i + \frac{\Delta s}{\delta}\left(\frac{(Q_j^i)^2}{(S_j^i)^{8/3}} - \psi_j^i\right) \tag{A13}$$

from $j = 1, 2, ...n$.

To solve the dimensionless brine equation Eq. (A4), we use an upwind difference scheme such that

$$\hat{\beta}_j^{i+1} = \hat{\beta}_j^i - \Delta t\left(\frac{\beta_j^i}{S_j^i}\left(\frac{S_j^i - S_j^{i-1}}{\Delta t}\right) + \frac{Q_j^i}{\lambda S_j^i}\left(\frac{\hat{\beta}_j^i - \hat{\beta}_{j-1}^i}{\Delta s}\right) + \frac{\beta_j^i M}{\lambda S_j^i}\right).$$

After calculating the non-dimensional salt concentration, we re-dimensionalize the salt concentration to be in units of [kg m^{-3}] and convert this to [psu] using,

$$\beta_j^{i+1} = \frac{\hat{\beta}_j^{i+1}\hat{\beta}_0 1000}{\rho_{b}{}_j^i}. \tag{A14}$$

~~Using the dimensional salt concentration in psuat time $i+1$ and location j along the channel, the updated salinity-dependent melting point of the ice can be calculated using~~

$$\hat{\theta}_j^{i+1} = -5.81\text{x}10^{-7}(\beta_j^{i+1})^3 + 1.24\text{x}10^{-6}(\beta_j^{i+1})^2 - 6.05\text{x}10^{-2}(\beta_j^{i+1}) - 7.45\text{x}10^{-8}P_i,$$

for $i = 1, 2, ...m.$

The density of brine along the channel can be updated similarly with the new salt concentration using Eq. 7. The Similarly the basic hydraulic gradient is a function of brine density and can be updated using the new brine density. In order to solve the system, initial conditions are needed for cross-sectional area, effective pressure, discharge, salinity along the channel. At the beginning of the simulation, we assume a constant cross-sectional area and salt concentration along the channel. We use the initial cross-sectional area at the end of the channel to calculate the initial discharge curve along the channel using Eq. A12. We assume the initial effective pressure profile linearly increases from $N = 0$ at the lake to $N = P_i$ at the end of the channel.

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Competing interests. The authors declare that they have no conflict of interest.

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