Response to Anonymous Referee #2 (Round 3)

Dear Referee,

We would like to thank you for carefully reviewing our manuscript. Your input has greatly improved our model and manuscript. We appreciate the time you have invested in multiple reviews. Below we respond to your comments point-by-point.

Best Regards,
On behalf of the authors, Amy Jenson

This paper describes an R-channel model which takes into account the effects of saline water. Its main finding is that an R-channel conducting saline water at the local pressure and salinity melting point will open less fast that an R-channel conducting fresh water. This is because as the channel walls melt, salinity drops and with that the melting point increases. Thus some energy will be needed to heat up the water to the new melting point and thus is not available to melt the channel wall. Note that this is similar to the effect of the pressure dependence of the melting point of ice on R-channel dynamics for fresh water R-channels.

This is the third review for this paper I am doing and I am happy in how this progressed. I think the model is now correct, however, I am not 100% sure about the results. In particular the ones presented in Fig2d seem odd to me: a difference of 10^-5 C between brine and freshwater runs, as shown in Fig2d right axis, is a tiny difference compared to the difference in channel radius or discharge between those runs (Fig2a,c). To get some more context for the amount of melt and thus change in salinity consider the following back of the envelope calculation.

Thank you for noticing the error in Fig 2d. You are correct that the change in salinity (and consequently changes in temperature) should be higher. An error was found in the code that generates Fig. 2d (remnant from debugging). The first time steps were cut off and this is when the majority of the change in salinity occurs.

The total potential energy available is approximately 150m*rho_w*g (100m of ice thickness and about 50m from the 3-degree slope). Taking a typical discharge of 1m3/s the total melt along the channel is approximately (ignoring pressure and salinity effects, i.e. an upper estimate)

Q * H * rho_w * g / (rho_w L) ~ 1 * 150 * 1000 * 10 / (1000 * 3.3e5) ~ 0.004 m3/s thus the relative salinity change should be about 0.004/Q ~ 0.004. This is about 3 orders of magnitude larger than what is plotted in Fig2d. Maybe my above estimate is wrong or I do not understand what is plotted in Fig2d. Therefore this needs to be either explained better, in particular how such minuscule changes could have such a big effect, or the calculation needs to be revisited.

After correcting the error in the figure code, the changes in salinity from the end of the simulation from the initial salinity is ~0.004 psu which is close to the back of the envelope calculation you provide. Please see the corrected Fig. 2d.

We have updated the accompanying text describing these results (lines 210 - 212) to "The salt concentration decreases linearly along the channel in all scenarios, with the most significant changes occurring in cases with higher salinities (Fig. 2d). As the salinity decreases, the melting point increases along the channel (Fig. 2d)."

Additionally, what was previously lines 263-269 is now lines 274-283 and reads "The ratio of fresh meltwater from the channel walls to the saline discharge from the lake determines brine concentration. A channel with a smaller cross-sectional area has a higher surface-area-to-volume ratio compared to a larger channel, leading to a higher ratio of meltwater to saline discharge and larger changes in brine concentration along the channel. Channels are smaller for higher salinities (Fig. 2c). Brine concentration and the resulting changes in the melting point vary linearly along the channel, with larger gradients observed at higher salinities (Fig. 2d). Larger spatial gradients in brine concentration require more energy to raise the brine temperature to the new salinity and pressure-dependent melting point along the channel. This reduces the energy available for melting, which strongly inhibits rapid channel growth (Eq. 10). The effect of inhibited channel growth is largest at higher salinities, where the initial brine temperature is lowest (-7.63 ◦ C). However, the system is more sensitive at lower salinities, where even small increases in salinities lead to significant changes, as the initial brine temperature is close to 0◦C (−0.37 to − 0.67◦ C) (Fig.2d). Inhibited channel growth due to salinity results in more gradual increases in velocity over time (Fig. 2b)."

Similarly, the change in temperature along the channel is at most 10^-5C according to Fig2d. However, I would expect a change in temperature due to pressure melt point effects alone of c_n * 100m * rho_w * g ~ 0.07C as the water de-pressurizes from the ice overburden pressure at the lake to zero at the outlet. Again many orders of magnitude difference between this estimate and Fig2d.

As for the change in temperature due to the pressure melting point, there are a few differences between your calculation and our assumptions. We assume the effective pressure is zero at the lake (following Evatt et al., 2006) and we do not assume the effective pressure is zero at the end of the channel (a result of the setup and numerical method we choose). Instead we require that dN/ds = 0, however it does tend to zero at the end of the channel. Consequently, changes in effective pressure along the channel are small and do not substantially impact changes in the melting point.

Calculating only the change in temperature due to changes in salinity, $\Delta\theta$ = -c_b * $\Delta\beta$ ~ -0.06 *- 0.004 ~ 2e-4 C which is similar to what is shown in the updated version of Fig. 2d. Thanks again for your thorough attention to our results.

Line by Line comments

34: state the salinity of sea water for comparison for those of us who do not deal with brine regularly

Good point. It now reads, “The salinity of the englacial brine feeding Blood Falls is approximately 125 psu (compared to ~35 psu for seawater) but the precise geometry…”

43: not sure what is meant by "edge dynamics"

We have changed this to read, “Channel size and evolution are also expected to differ as the result of fluid chemistry.”

Table 1: for some variable their value is given (e.g. rho_i) but not for others (e.g. A). This should be consistent.

We state in the table caption that values of constants are specified, but A here is a function of temperature. Previously in line 93, we stated “We calculate A as a function of ice temperature using the Arrhenius relation and relevant calibrated values (Cuffey and Paterson, 2010, Eqns 3.35.” We realize the necessary parameter values to calculate A were not all listed in Eqn 3.35 so we have added Eqn 3.36 to this reference.

99-101: Likely the biggest uncertainty in Manning's n is due to the great variability in its value (e.g. Pohle et al., 2022 find order of magnitude difference for one R-channel as it evolves). Maybe state something in this regard.

We have added in line 101, “Regardless of salinity, large uncertainties exist in Manning’s formula even in fresh water systems due to the large variability in the value of Manning's friction factor (Pohle et al., 2022).”

Eq 7: I guess in this equation only the constant and linear term could be retained, the effects of the higher order terms are minimal for the salinities considered here.

Yes, true. We will delete the higher order terms.

143: use `\times` to typeset the "x" in scientific notation numbers

Good to know. Thanks.

Eq 9: it should be \sigma_w, the heat capacity of water (sorry, I miss-stated this in my last review); also in the summary of model equations

Thanks for pointing this out. We should have caught this after reviewing Eq 2.4 in Fowler (1999) upon your suggestion, but you are correct. It should be the heat capacity of the brine here, not ice.

The value of the specific heat capacity varies significantly as a function of salinity and temperature. We have now included this consideration in the model code and changed it in the manuscript. We have added the following to line 135 in the manuscript.

“Fluid properties such as the density, specific heat capacity, and the melting point of ice are functions of salinity (β in practical salinity units [psu]). The specific heat capacity of brine is calculated with the salinity(β in practical salinity units [psu]) and temperature of the brine [C] in the lake following,
σb = 4217.4 - 3.72 θ_b - 7.64 β. using the first order terms from Eqn. A3.11 in Gill (1982).”

After this change, we reran all simulations to include the specific heat capacity of fluid, given the temperature and salinity. The results are similar, although there are now larger differences between saline and fresh water due to the larger specific heat of fluid compared to ice. This larger value of sigma further limits energy available for channel growth (in Eq. 10). We have edited all figures to include updated results. Slight adjustments have been made to the text to capture these changes. See below.

Changed lines 239-241 because the specific heat capacity of brine is constant along the channel… “As the walls of the channel melt and the brine is diluted, the density of the brine is not constant along the channel. We have accounted for this in our simulations, but neglecting these changes does not have a substantial influence on the results because the changes are very small (data not shown).”

Rewriting lines 228-230 based on the new results and for clarity, “With a salinity of 10 psu and an initial channel radius of 0.2 m, the lake drains substantially slower (≈ 10 days) compared to an initial channel radius of 0.3 m. For each 0.1 m increase in initial channel radius, the peak discharge increases non-linearly for both saline fluid and freshwater, but the rate of increase is less pronounced for saline fluid.”

Replacing what was line 224, “For a freshwater lake, the peak discharge roughly triples when comparing Vi = 1 × 10^5 m^3 with Vi = 5 × 10^5 m^3 and Vi = 5 × 10^5 m^3 with Vi = 1 × 10^6 m^3, where as for a lake with a salinity of 10 psu the peak discharge approximately doubles.”

Changed percentage slightly in line 334. “For a lake with a salinity of 125 psu, which is approximately the measured value at Blood Falls, and an initial brine temperature of −7.63◦ C, the peak velocity reached is 40% lower and the lake drains 9% slower than for a freshwater lake.”

And in line 338 “22% higher than pure water…”.

Also changed subscript in Table 1, summary of model equations, and appendix equations.

190: repetition

Deleted “Higher salinities require smaller time steps.”

255: "evolving" -> "increasing"

Okay.

259: I'd rather use "reduce" than "limit" as the latter suggest a hard limit but it only reduces the effects. Also other occurrences, e.g. l295.

Agreed, changed this language everywhere.

287-289: state which assumption is more likely correct. Potentially cite, e.g. Röthlisberger here about his choice for the pressure melting point.

We have added "However, this assumption is likely unrealistic and differs from the convention in previous studies. In other models of subglacial channel flow that do not explicitly include temperature, the temperature is set to follow the pressure melting point of the ice walls within the channel (e.g., Röthlisberger, 1972; Werder et al., 2013)."

Fig 4: plot 0psu also.

We choose not to plot 0 psu here because 0 psu already has a density of 1000 kg/m^3 and so there would be no difference between them. Removing 0 psu also changes the scale which makes the slight differences easier to visualize. We edited the caption to remove beta = 0 psu from the list.

Eq 15: Pretty sure this is wrong. In both denominators should be rho_s. This then leads to, for instance, a density of 1056kg/m3 for a SSC of 91g/l. Probably easiest to just correct the vector of SSCs in line 315 to give the stated rho-vector.

We agree the equation is wrong. We will change it to
$\rho c= \rho w + (\rho s - \rho w) (SSC/\rho s)$

We believe this the equation you meant. We have changed the values of SSCs. We calculated the SSCs given the fluid densities we ran in the model. It now reads “We vary the suspended sediment concentrations from SSC = {0, 63, 125, 188, 251} g L^-1 to arrive at the combined fluid densities of rho_c = {1000, 1040, 1080, 1120, 1160} kg m^-3 to simulate different suspended sediment loading (for rho_s = 2760 kg m^-3).”

References

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