

Response to Anonymous Referee #2

Thank you for your thorough feedback that will help us improve our manuscript.

This paper describes a jökulhlaup model, based on R-channel drainage, taking into account effects of saline water on the dynamics. It finds that the impact of salinity on melt-rates is limited and the biggest impact is due to the increased density of the fluid on hydraulics.

This short paper has interesting findings and is novel as no-one has looked into impacts of salinity on R-channel dynamics. It has however a few shortcomings which need to be rectified before publication.

We have responded to each of your comments below.

Major comments

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The somewhat surprising take-home (that the main difference compared to freshwater discharge is due to the higher density and not due to higher melt rates) should be elaborated a bit more. I would state the following, possibly in the Abstract & Conclusion:

- salinity makes liquid water possible at sub-zero temperatures: fresh water would be frozen. So this is really the biggest difference: liquid water versus no liquid water.

- once the equations are solved at the sub-zero temperature given by the melting point of the saline solution, it is actually pretty obvious that impact on melt is minimal: dilution of the water as it traverses the R-channel is minimal as melt is really small compared to discharge. Even for a setting where there is much more potential energy available, this statement holds. Thus it makes sense that density has the biggest impact.

A few comments in response:

1. We agree that the main impact of salinity is the depression of the melting point and we will explicitly make that point in the manuscript, as suggested (see below, especially l. 63)
2. Once we assume that the melting point is depressed and the ice and brine are at that temperature, the main difference compared to freshwater is due to higher density. The greater density of saline fluid does **result in higher melt rates** due

to the additional energy generated. We will make this more clear in the Abstract and Conclusion.

3. The dilution of the brine would have the opposite impact than what we see in our results. If the brine was diluted more, then the melting point would increase (as a result of lower salinity) and less melt would occur which would result in lower discharge rates. You are correct that the melt is minimal compared to the discharge and this impacts the results. We will clarify this in the conclusion.

In response to these concerns (and similar concerns from Referee #1), we will make the changes shown below.

Line 8:

“The model results show that given a subglacial system at the salinity-dependent melting point, channel walls grow more quickly when fluid contains higher salt concentrations which lead to higher discharge rates. We show this is due to a higher density fluid moving through a gravitational potential which generates more energy for melting.”

Line 63:

“The results show that the fluid flux is greater with saline fluid than the fresh water equivalent when both glacier-lake systems are at their respective salinity and pressure-dependent melting points. The larger channel cross-sections affect the temporal and spatial evolution of fluid flux for saline fluid.”

Line 233:

“Modeling all other changes related to salinity (including the treatment of the melting point and fluid temperature) while holding the density of the brine constant and equal to that of fresh water results in the discharge rates shown in Fig. 3b.”

Line 293:

“We show that if a channel exists, hypersaline fluid can flow through an ice-walled channel when the brine and ice are at the salinity and pressure-dependent melting point.”

Line 295:

“The main driver of the increased discharge rates as a function of salinity is the higher fluid density associated with higher salt concentrations. More energy is generated and available for melting the channel walls when a higher density fluid moves through a gravitational potential. While some of this energy is used to warm the ice to the new pressure melting point down the channel as the brine is diluted by meltwater, the melt is

minimal compared to the discharge from the lake and therefore does not impact the discharge rates. Aside from the influence of salinity on the depression of the melting point, the greatest difference on fluid flux when considering saline fluids is related to the change in density."

Provide a sketch (as fig. 1) of the setup for an overview, that way also the coordinate system and orientation are defined.

Okay, thank you. We will include the following schematic as Figure 1.

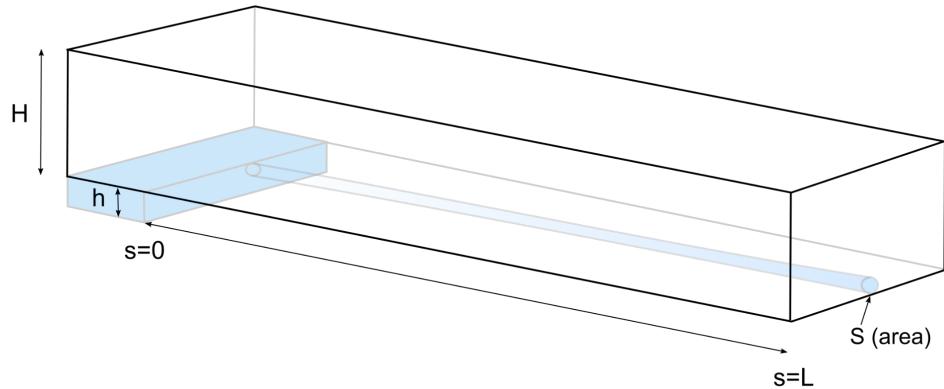


Figure 1. Schematic of simple glacier geometry and subglacial hydrological system with a R-channel draining a subglacial lake.

With many of the equations in section 2 I struggle:

- Eq6 and the following three unnumbered eqs have some issues, see line-by-line comments
- my understanding is that the authors assume that water-temp equal to the local melting point as stated on l72 (i.e. temperature is dictated by pressure and salinity). Thus θ , θ_i and θ_b are equal and hence the last term in Eq7 is zero. If temperature was treated as a free variable, as in Fowler 1999, then this term would be needed.

You are correct that θ_{hat} , θ_i and θ_b are all equal at the beginning of the simulations. However, because the brine is diluted as freshwater is added, the melting point θ_{hat} decreases along the channel. Therefore, the last term in Eq. 7 is non-zero except (i) along the channel at time zero and (ii) where the channel meets the lake throughout the simulation. The terms on the right hand side in Eq. 7 are identical to those presented in Fowler (1999), but the variables differ in that the brine temperature is

assumed constant and the melting point is allowed to vary in space and time as a result of the changing salt concentration along the channel. We will clarify this by deleting the sentence in line 115 and adding the following discussion of the assumptions. "We assume the lake and surrounding ice system is in thermal equilibrium which requires that at the lake, the ice and brine temperatures, θ_i and θ_b respectively, are equal and at the salinity and pressure-dependent melting point $\hat{\theta}$. For a given salt concentration in the lake, we calculate the melting point at the lake and set the ice and brine temperatures equal to that temperature. We assume the ice and brine temperatures remain constant in time and along the channel; this is realistic for most freshwater systems (Clarke, 2003). However, the melting point only remains constant at the lake and evolves in response to the changes in salinity along the channel and in time."

--> this whole temperature treatment is confusing. I had a quick look at the code and I think that temperature is indeed not a free variable.

You are correct that ice and brine temperatures are parameters and not free variables. They are constant in space and time (except in Section 4.3). However, the melting point $\hat{\theta}$ is calculated along the channel after each time step given the updated salt concentration, β . So $\hat{\theta}$ is a derived variable that depends on the free variable β . It was pointed out by Referee 1 as well that it is not clear what are free variables. In order to address this, we will change line 168 to read, "The full model contains five unknowns (N , S , m , Q , β) and five model equations (Eq. 3, 4, 5, 7, and 8) which are solved simultaneously. The model equations contain the derived variables $\hat{\theta}$, p_b , and ψ which depend on salinity. The model equations written in terms of the salinity-dependent derived variables are listed below."

- the melt rate m can be solved for in Eq7. However, I do not understand what is going on here. As stated above, the last term should ==0. Also there should be a pressure dependent term to capture the pressure melting point effects. At least when a linear relation between temperature and pressure is assumed then it takes the form $\rho_w c_t c_t dP_w/ds * Q$ (e.g. Röthlisberger 1972, Eq 17); maybe the authors try to capture this with their last term of Eq7, but I don't think that can be done like this. A term in similar spirit would need to be added for the salinity dependent effects, presumably featuring a $d\beta/ds$. In fact, this is the quintessential equation to be stated as that is the novel one, all others are known.

The term you are referring to ($\rho_w c_t c_t dP_w/ds * Q$) is described in Eq. 3 in Röthlisberger 1972 as the energy needed for the change in water temperature to the new pressure melting point. This is because in Röthlisberger's model, (i) the water temperature always remains at the pressure melting point and (ii) there is a change in

ice overburden pressure which results in a change in the melting point. However, in our model we assume (i) the fluid temperature remains constant and (ii) the ice thickness is constant. Because the melting point increases as a result of the brine dilution, some energy is needed to raise the ice temperature to the new pressure melting point before melting, which is accounted for in the last term of Eq. 7.

The brine temperatures in our simulations are considered constant, except in Section 4.3 where we explore a small change in temperature equal to the change in the melting point. In this section in Eq. 15, we include a term to account for the energy needed to the change in brine temperature. In this term ($\sigma \partial \theta^*/\partial t \rho S$), we account for the $d\theta/ds$ term you mentioned, although it is hidden in θ^* . The melting point is a function of salinity and pressure, but since the ice thickness is constant there is no change in pressure along the channel ($dP_i/ds = 0$) and the only change in the melting point is related to the change in the salt concentration ($d\theta/ds$). It is written slightly differently than in Röhlisberger 1972 but it is essentially the same. In our equation the derivative is with respect to time and multiplied by S (m^3) instead in his model where there is a derivative with respect to space multiplied by Q (m^3/s), but it is referring to the same quantity of water per time.

- How Eq15, also featuring the melt rate, then ties in with this is also not clear to me.

In Section 4.3, we assume that the brine temperature remains equal to the melting point (instead of the ice temperature). Therefore, there is some energy needed to raise the temperature of the brine, so Eqn 15 replaces Eqn 7 for this section.

The lake has only ever a simple geometry, thus I would recommend to just keep it simple and shorten that part. A simple "refer to Kingslake 2015 for a treatment of more complicated lake hypsometries."

Okay, thank you. We will remove the dimensionless parameter p to simplify Eqs. 10 11 and 12 and change line 155 to say, "We assume a box-shaped lake which gives the lake hypsometry

(Eq. 10),

where h is the depth of the lake, h_i is the initial lake depth, and V_i is the initial lake volume. For the treatment of more complicated lake geometries, see Kingslake (2013)."

Line by line

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12: nice Introduction

Thank you.

13: (any many others): I prefer incomplete lists of references to be prefaced by a "e.g.". Many instances in the Introduction (l.15, 18, 24, 26, etc.)

Okay, we will add in lines 15, 18, 21, 24, 26.

30: I would prefer to use an SI units throughout instead of "psu". At least give a translation (1psu=1kg/m3).

We agree that remaining consistent would be preferred. However, the units of salinity are not often referred to in salt concentrations of kg/m3 as is required in our model for units to agree. Therefore, to be consistent with other literature we present and discuss the salinity and the melting point and density equations in psu. We will add the conversion to line 147 by saying, "The salt concentration β_{hat} discussed above is in [kg/m³] in order to be compatible with the model. These values for salinity are converted to a standard unit for measuring salinity [psu] before calculating the density and melting point in Eqs. 6 and (line 110) respectively using the conversion $\beta_{\text{hat}} [\text{kg/m}^3] = \beta * 1000 / \rho_b$ [psu]. " We will add an equation number to the equation in line 110 to reference. We will also add the definition of the psu (practical salinity units) to line 104.

70: Cite the original too, Röthlisberger (1972). I would write something like: "We construct a lake-drainage model in which the water flows through a subglacial channel (Röthlisberger, 1972; Nye, 1976). We follow the implementation and notation of Fowler (1999) and Kingslake (2015). We assume..." I also like the term "R-channel" and would use that, but no strong feelings here.

Thank you. We will replace the first two sentences with "We construct a lake-drainage model in which the water flows from a subglacial lake through an R-channel (Röthlisberger, 1972; Nye, 1976). We follow the implementation and notation of Fowler (1999) and Kingslake (2015). In our model, we assume a subglacial conduit on an inclined bed slope beneath ice of constant thickness (Fig. 1)." where Fig. 1 is the schematic you recommended.

74: I disagree with Fowler (1999) here and thus with this manuscript: the given definition is the "negative basic hydraulic potential". I think it would be good to state the "negative" somewhere. Similarly Eq2 gives the negative potential gradient.

Okay, we change both of the definitions to 'negative'.

95: the "(instead of Darcy-Weisbach)" I find confusing as D-W is also for freshwater. Just delete?

Okay, we have deleted "(instead of Darcy-Weisbach)" from this line.

97: I think $0.6m^{-1/3}s$ should be 0.06 (according to the code). Also cite where these values are from (maybe Clarke 2003?) as Fowler 1999 does not provide those values.

Yes, you are correct. Thanks for catching this. It should be $ni = 0.06$ and this value is from Clarke (2003). We will change this and add the citation.

Eq6 and following: I don't think those formulas should be stated with so many digits. At most two significant digits are needed. Also state that for the ρ_b and $\Delta\theta_P$ the 3rd and 2nd order terms are not important. Also, Eq6 is written with β as an argument, the others are without argument: should be consistent.

Agreed. We will reduce the equations to include only two significant digits.

Beta can be up to 200 psu so the second and third order terms in Eq. 6 and the equation in line 110 are not completely negligible.

We will be consistent by removing the argument (beta) from Eq. 6.

We have placed the equations in lines 110, 112, and 114 with one equation to describe the melting point where the pressure dependence is linear. Please see response to later comment for the full change.

Eq6: is this correct? I evaluate $\rho_b(0)=0.1$ but fresh water has a density of 1000kg/m³.

Thank you for catching this. The current equation is presented in g/cm³ even though we state that it is in kg/m³. We will convert this to kg/m³ by multiplying each term by 1000 (as was already done in the code).

Eq6 / l109: I don't understand how the melting point depression can have a constant term. That would mean that it is always depressed! Similarly for the change in melting-point due to pressure. Also, I guess these formula depend on temperature being given in deg-Celsius, that should be stated.

Both of the melting point depression equations are a result of the FREZCHEM model which "...provides access to virtual laboratories where freezing experiments can be conducted for saline water on Earth..." (Wolfenbarger et al., 2022). See Wolfenbarger et al., 2022 referenced in the manuscript for more details. These equations were derived

from fitting a line (with a constant term in the fitting equation) through virtual experimental data with ice at a very large range of pressures, which is not relevant here. Therefore, we will use a new simplified equation that is a function of both salinity and pressure that has been derived to best fit ice pressures where ice thickness is less than 500m and that does not allow for a constant term. We have rerun all simulations and confirmed that changing these equations does not impact our results.

We will replace lines 107-114 with “Using the same FREZCHEM model, we calculate the melting point of ice due to salinity and adjust for the ice-overburden pressure (Chang et al., 2022). The melting point in [°C] of ice in contact with saline fluid at pressure P_i in [Pa] is

$$\hat{\theta} = -5.81 \times 10^{-7} \beta^3 + 1.24 \times 10^{-6} \beta^2 - 6.05 \times 10^{-2} \beta - 7.45 \times 10^{-8} P_i.$$

Your comment has pointed out to us that we have made a mistake in the notation. We have denoted P_i in both bars (in line 113) and in pascal (in line 77). To address this, we will change the melting point equation so that P_i is in Pa to be consistent.

123: cite Röthlisberger here too, drop Kingslake.

Okay.

132: why suddenly salinity in kg m^{-3} instead of "psu"?

The empirical equations from FREZCHEM are given in psu (which is approximately equal to g/kg and ppt) which are standard ways to present salinities. However, in order for the units of salt concentration to match our system of equations we must convert this to kg m^{-3} . The distinction between the salt concentrations with different units are beta in psu and beta_hat in kg m^{-3} . Although we acknowledge that this is slightly confusing, we argue that presenting the salinities and related empirical equations in familiar units is important. As mentioned earlier, in order to help address this confusion we have added, “The salt concentration beta_hat discussed above is in $[\text{kg m}^{-3}]$ in order to be compatible with the model. These values for salinity are converted to a standard unit for measuring salinity [psu] before calculating the density and melting point in Eqs. 6 and (line 110) respectively using the conversion $\text{beta_hat} [\text{kg m}^{-3}] = \text{beta} * 1000 / \rho_b$ [psu]. ” to line 150.

139-143: this can be abbreviated to just state Eq 8. The reader does not need to see a derivation of a conservation equation.

Okay, we replace lines 139-143 with, “Assuming there is no brine added along the channel and there is no accretion on the channel walls, the salt concentration equation is...”

115: doesn't line 72 state that these temperatures are always equal?

No, the ice and brine temperatures are only equal to the melting point at time zero. While the ice and brine temperatures remain constant, the melting point does not. The melting point changes in response to the change in the salinity according to the equation shown in line 110. This seems to be unclear in the manuscript and we see that this has led to confusion about the treatment of the melting point and energy equation. We hope that the changes made to line 115 mentioned earlier makes this more clear.

167: I wouldn't call this a "no flux" boundary condition as there is water flux through the boundary. Just call it a Neumann BC. Also, what does it mean? What is meant with "We do not require the channel to exit at a terminus or end subaerially"? Where does it end? Or at least where does the simulation end and how do we know that there $dN/ds=0$?

We will change the language from "no flux" to "Neumann". What we mean by this statement is that the channel may continue on outside of the model domain and the end of the modeled channel is not necessarily at the glacier terminus. There are no other boundary conditions imposed that suggest that the channel does or doesn't end here. We allow for the possibility that we are only modeling a portion of the lake/channel system and do not enforce a water pressure at the end of the channel but allow it to evolve according to the Neumann boundary condition. However, this is not the reason we chose this BC so we will omit this statement and make the following changes at line 165.

"We impose a Neumann boundary condition at the end of the channel where

(Eq. 13).

We choose this boundary condition (opposed to $N = 0$) in order to solve the system numerically in a more efficient way (see Section 5.4 and Appendix A). Neumann boundary conditions on effective pressure at the end of the channel have been used to solve similar systems of equations without an influence on the qualitative results (Kingslake, 2015; Evatt et al., 2006)."

174: reference the suggested Fig. 1 (the sketch) here.

We will add a reference to Fig. 1 in line 71.

Fig.1: could the temperature as a function of distance also be plotted? Maybe even the different components, i.e. press-melt-point term and salinity-melt-point term.

We have added the change in the melting point from the beginning of the simulation to the end along the channel to Figure 1d. The depression of the melting point from pressure is constant (since the ice thickness is constant) so the change in the melting point is only due to changes in salinity. Please see the updated version of Figure 1 below.

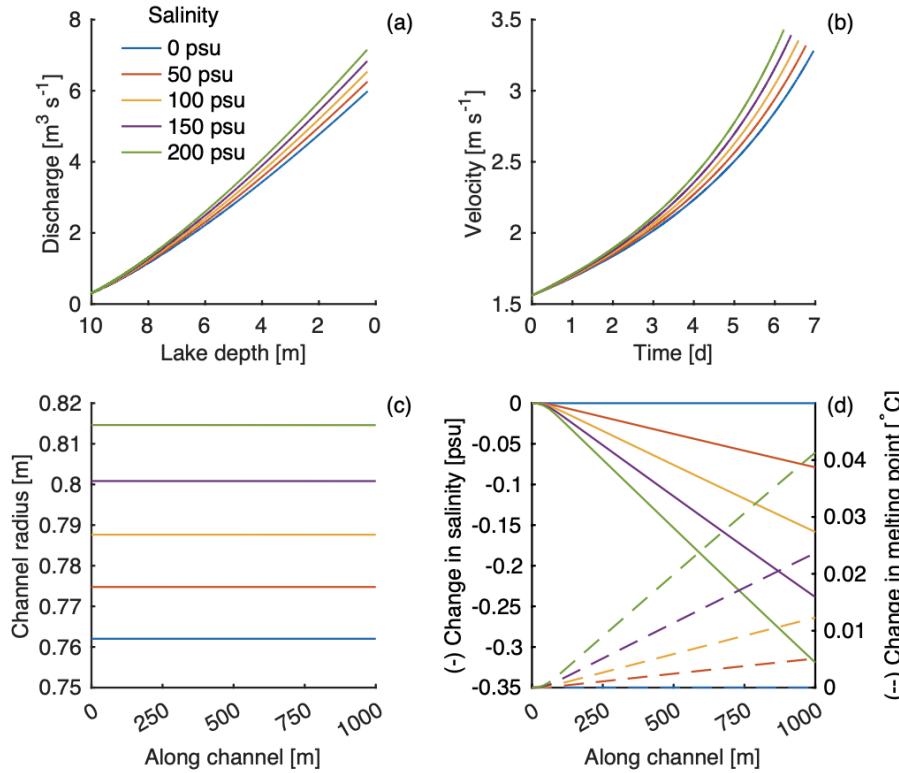


Figure 1: The impact of brine on discharge, velocity, channel radius, salt concentration, and effective pressure for various initial salt concentrations, shown by the colors in (a). (a) Discharge at the lake outlet as the lake drains. (b) Velocity at the lake outlet over time in days. (c) Channel radius along the length of the channel at the time the lake has emptied. (d) The solid lines are associated with the left axis which is the difference between the final salt concentration after the lake has emptied and the initial salt concentrations shown in the legend of (a) along the channel. The right axis (dashed lines) refers to the difference between the melting points at the end of the simulations along the channel and the initial melting point, where the change in the melting point is only due to the change in the salinity (shown in the left axis).

230: I think this is vice versa.

This sentence is stated in a confusing way. Yes, there is more energy available for melting when there are higher salt concentrations which is why we see higher discharge rates. However, energy is needed to raise the temperature of the ice surrounding the channel to the new melting point as the melting point increases with the dilution of the brine which is the point we were trying to make here. We will change this sentence to say, "The presence of salt in the system tends to increase the amount of energy needed to melt the channel walls because the ice temperature must increase to the evolving melting point before melting. As the salinity along the channel changes, the melting point changes and subsequently the energy needed to melt the channel walls, although this change is minimal as seen in Fig. 1d."

242: what are "hyperconcentrated sediment concentrations"? What is this "threshold"?

This phrase is redundant and the sentence is perhaps not worded well. We follow the language used in Maizels (1997) which distinguishes between high sediment concentrations and hyperconcentrated flow. We will reword this to say, "can contain suspended sediment concentrations SSC of up to 70.7 g L⁻¹ (Beecroft, 1983; Old et al., 2005) and in some extreme cases over 400 g L⁻¹ (Maizels, 1997)."

253-259: I recommend to delete these lines as they don't add much

Okay.

288: I concur with Reviewer 1: these N values do not make sense to me. Is this caused by the Neumann BC on N? Note that a, in my opinion, more natural and standard BC $N=p_i$ (or $p_w=0$) leads to dN/ds not equal zero. See for instance in Röthlisberger (1972): Fig.5d shows a setup similar to here and shows $p_w \rightarrow 0$ at the outlet.

We agree that a standard boundary condition to impose at the end of the channels is $p_w = 0$. Another standard boundary condition to impose is $N=0$ at the terminus which is approximately what we observe in our model, despite the Neumann Boundary condition. The values are very slightly negative due to the dynamics discussed in Section 4.3, but are essentially zero. We will add at line 288, "The effective pressure at the end of the channel for all simulations is extremely close to zero ($-200 < N \leq 0$ Pa) and therefore we claim that the sign of the effective pressure is negligible and does not qualitatively affect our results."

A related note: This model does not handle open channel flow and therefore we do expect higher water pressures all the way to the end of the channel.

312: Excellent that the code is provided, thanks! Ideally the authors should also provide a third script that, when run, will reproduce and write to disc all figures used in the publication.

We only provided the main model here for simplicity. However, we agree all files would be useful. We will add all versions of model code (Section 4.2, 4.3, etc.), the script that runs all the simulations at once, and the scripts that create the figures.