I was happy to be provided with the opportunity to review a manuscript that leverages measurements of ice conductivity and radio-wave englacial reflections to determine a specific index of refraction for glacial ice at the Radio Neutrino Observatory Greenland, Summit Station, on the Greenland Ice Sheet. I have limited expertise to understand the specifics of the radar hardware used, and I hope that a more qualified reviewer is able to confirm that the instrument setup was appropriate for the experiment. My suggestions to address comments below are my own personal opinion, and the authors are free to take alternative approaches so long as it is justified. Please consider my comments and suggestions below, and I apologise if I may have missed or misconstrued elements of your manuscript.

TJ Young (University of St Andrews)

Thank you for your review. Your comments were very helpful, and we followed your advice on most things. Point by point answers are below.

31 July 2023

General comments
Overall—a succinct and clearly-written manuscript and I have several comments, of which all are relatively minor.

(a) Further clarity on radio wave properties
Please provide the signal frequencies/wavelengths generated. Even within the radio frequency band, most of the equations that the manuscript results employ rely on assumptions, which may or may not be negligible depending on the nature of the radio wave properties used by the instrument. Frequency affects the permittivity variations, which in turn relates to the index of refraction. Hence, for example, your assumption of negligible polarisation dependence on your calculations (L175) can only be justified if the frequencies used are the same or within a threshold to those used in Aguilar et al. (2022c). Separately, though not affecting the actual calculations, frequency will also affect the strength of observed conductivity-induced englacial reflections (Fujita & Mae 1994).

The lower end of the frequency band is set by the 145MHz highpass filter mentioned in the description of the measurement setup. We added the information that the upper end of the band is at about 500MHz, set by the response of the horn antenna. This is the same band as Aguilar et al. (2022c), so assuming a negligible polarization dependence is justified.

(b) Explicit statement needed that this method assumes additional invariance in several parameters Given that the study calculates a bulk index of refraction, there is then an implicit assumption of a constant permittivity in and density of ice, following Looyenga (1965). At this point, the framework needs to assume that reflections detected by the radio wave is from abrupt contrasts in conductivity and not from permittivity (Fujita & Mae 1994). Note also that englacial reflections can also be caused by changes in density and crystal orientation fabric. Additionally, the speed of the radar wave when travelling through ice also is dependent on
density, temperature, and crystal orientation fabric (the last of which is already mentioned in (a)). Density is addressed for the most part (I think) by absorbing it into the $\Delta T$ free parameter (via the offset in time by firm properties). Within the depth ranges that you are working at, the effects of temperature should be negligible. However, if you extend your analyses to shallower or deeper sections of the ice column, it should be taken into consideration, or at least shown to (still) be negligible.

I would then suggest to make explicit that the method used in this manuscript assumes invariance in these factors (permittivity, temperature) for both radio-wave transmission and reflection. A good citation for this would be Fujita et al. (2000).

We added a brief paragraph at the end of the discussion about uncertainties discussing this. Temperature profiling of the GISP2 borehole shows a constant temperature within 1°C for the upper 2km, so the effect of temperature variations can be safely assumed to be negligible for the range of depths used in this measurement.

As the vast majority of neutrino interactions detected by RNO-G are expected to occur in the upper ~1.5km of the ice sheet, this assumption also holds for the detection volume of RNO-G.

(c) Consideration of the “echo-free zone”
The manuscript presents confident claims that the method can be applied “to greater depths relatively easily”. There is however a common occurrence of an ‘echo-free zone’ (e.g. Drews et al. 2009) in which, for reasons still largely unknown, radars are unable to consistently receive coherent englacial reflections. I would suggest to take this caveat into consideration.

We show in Fig. 5 that the correlation between radar echo strength and changes in ice conductivity holds to at least 1700m, which would allow us to at least double the depth range used for the measurement. That is what we meant by saying that this measurement can easily be extended.

(d) Suggestions to consider employing ice-penetrating data to strengthen the argument
The suggestion that the differences between reflectors at the two different radio-echo sounding sites, as well as their comparison to the GISP2 borehole, can perhaps be verified through visual interpretation of ice-penetrating radar profiles done around the site, depending on exactly where and which directions the measurements were taken relative to the surrounding landmarks. See if the radargrams provided in Jacobel & Hodge (1995) may help towards this suggestion. There may be other radargrams that exist at resolutions that may be too coarse for beneficial use (e.g. the CReSIS datasets).

Thanks you for this suggestion. Jacobel & Hodge explicitly mention that (except for the deepest 300-400m), the internal reflective layers are continuous between GISP2 and GRIP. We now mention this in the paper when we justify using the GRIP conductivity data for GISP2.

(e) Formatting
I have not commented on issues with formatting, but the authors should give the manuscript a thorough proofreading to correct typos and formatting issues, especially with citations (\citep v.
Specific comments (by line L)

10 “Cosmic rays have been of interest to physicists for over a hundred years” ß why? The journal has a wide readership so providing more basis will strengthen your motivation for research.

16-17 Spell out Eev (exa-electronvolts?) It would also perhaps help those not familiar with neutrino physics if you can also mention that the energy produced by cosmic neutrinos can be in this range. Does the detection of high-energy particles then scale proportionally with size, such that the detectors have to be proportionally large?

This is a good point, we expanded the introduction a bit to give some more background on UHE cosmic rays and neutrinos.

70 How “near” was the first measurements to the GISP2 borehole?

This is described a little bit later, where the setup is described as having the antennas 102 meters apart with the GISP2 hole in the middle. We rephrased this a bit to make it clear that this means they are 51m to each side of the hole.

86-92 To just check, were the same antennas used in these two additional measurements and were they also positioned 106m apart? Was the antenna plane aligned in the same direction as the first measurement?

These were the same antennas. They were closer together than the 102m at the GISP2 hole, but for a depth of 550m the change in propagation length due to horizontal separation between the antennas is small (about 2m). Please note that these measurements were not used for the index of refraction measurement, but just to demonstrate that the correlation between radio echos and conductivity holds to larger depths. The antenna alignment was the same as in the first measurement.

86 Which direction were the measurements (here I am assuming the 2nd and 3rd) taken from the GISP2 hole?

It was to the north of the GISP2 hole. We added that information to the paper.

100 Note also, though, that there is considerable variation between conductivity measurements between adjacent ice cores (Wolff et al. 2005, Gautier et al. 2016).

This was our motivation for not using the additional measurements to determine the index of refraction, as we could not guarantee that the reflective layers were still at the same depths further from the borehole.

105 Suggest taking this first sentence out, this is your opinion.

Done
Eq. 4 From this equation it is now evident that \( z_0 \) is the vertical distance that also takes into account planar distance, and \( z \) is the vertical distance without this deviation. However Eq. 3 implies the opposite (that \( z_0 \) is instead the vertical distance with no horizontal deviation).

The notation is indeed a bit inconsistent here. We fixed it

124 Quantify how “negligible” is this effect.

The difference in signal travel time when including raytracing is less than 1ns. We added this information.

125 A statement that firn variations around the GISP2 site is negligible would support your case here, if there is a study that exists.

157 Same argument as my comment for L125: this statement is true only if there is no areal variation in firn density if you want to include measurements taken from different locations.

The time offsets between the different measurements are corrected for (described around line 100). So if a change in the firn properties between the different sites caused a time delay, this would be corrected here as well. Therefore we do not need to assume a uniform firn.

Fig. 3 I’m not quite sure where the values for the bottom plot are coming from. Are these max correlation values for a given index of refraction regardless of which time offset they represent, or for the specific time offset that gives the maximum correlation value of 1.778?

Yes, the time offset is left to vary for each value of \( n \). We added this information to the figure description.

Figs. 4, 5 This is my personal opinion, but I can get all the information that I need from Figure 5, which renders Figure 4 unnecessary.

We think that Figure 4 makes it easier to see details and is less cluttered, so we would prefer to keep it.

165 Please give the citation that provides the 0.5m uncertainty measurement.

The 0.5m uncertainty was given in one of the publications we already cited for the depth correction between GRIP and GISP2. We repeated the citation here again.

170 I am not certain that simply adding the uncertainty measurements on \( \Delta z \) and \( \Delta t \) is the correct way to produce a corresponding uncertainty for \( n \) especially given their placement in Eq. 1.
For products and quotients, the errors can be propagated by adding the squares of the relative errors of the individual components. In our case this is:

\[(\sigma_{z/z})^2 = (\sigma_{t/t})^2 + (\sigma_{z/z})^2\]

So our calculation is correct.

174 Give units for 1.6 ± 3.3

Done

176 As far as I understand, there are three measurements, one taken “near the GISP2 borehole” (L70-71) and two taken “550m away from the GISP2 borehole” (L86) – which of these are the “two measurements” that you are referring to?

This is referring to the two measurements of n, one using only the data taken at GISP2, and the other one also including the data from near the Bally building.

183 “… and radio reflections should hold…” ß given that you have not shown this evidence yet

Done